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Existence of Global Generalized Solutions for a Generalized Zakharov Equation

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This paper considers the existence of the generalized solution to the initial vale problem for a generalized Zakharov equation by a priori integral estimates and Galerkin method, one has the existence of the global generalized solution to the problem

1. Introduction

The Zakharov system is one of the basic plasma models, introduced by V. Zakharov in 1972 to describe the propagation of Langmuir waves in an ionized plasma (Zakharov (1972)).
This system attracted many scientists' wide interest and attention. Merle (1998) consider the blow-up solution of the Zakharov equations. An analytic technique, namely the homotopy analysis method is applied to obtain approximations to the analytic solution of the generalized Zakharov equation (Abbasbandy et al (2009)). Khan et al (2011) obtained new soliton solutions of the generalized Zakharov equations by the well-known He's variational approach. Ginibre et al (1997) studied the local Cauchy problem in time for the Zakharov system. Merle (1996) first established a viriel identity for such equations and then proved a blow-up result for solutions with a negative energy. You (2015) considered the existence of the generalized solution to the initial vale problem for quantum Zakharov equation in dimension three. By considering the modified Adomian decomposition method, exact and numerical solutions are calculated for the generalized Zakharov equation which is an imaginary equation, with initial condition (Wang et al (2007)). An exact 1-soliton solution is obtained by the ansatz method (Suarez etal (2011)). Masselin (2001) consider a blow-up solution of the Zakharov system in dimension 3. Guo et al (2013) considered the global dynamics below the ground state energy for the Zakharov system in the 3D radial case and obtained dichotomy between the scattering and the growup. Guo et al (2010) dealt with the existence and uniqueness of smooth solution for a generalized Zakharov equation and established local in time existence and uniqueness in the case of dimension two and three. You (2009) studied a generalized Zakharov equation and obtained the existence and uniqueness of the global solutions to initial value problem.
In this paper, we are interested in studying the following generalized Zakharov system in dimension two.

$$i\varepsilon_t + \Delta\varepsilon - n\varepsilon + \alpha|\varepsilon|^p\varepsilon = 0, \tag{1}$$

$$n_{tt} - \Delta n = \Delta|\varepsilon|^2, \tag{2}$$

with initial data

$$\varepsilon(x,0) = \varepsilon_0(x),\ n(x,0) = n_0(x),\ n_t(x,0) = n_1(x). \tag{3}$$

where $\varepsilon = (\varepsilon_1,\cdots,\varepsilon_N)$ is complex valued unknown function, n is real valued unknown function, $x = (x_1, x_2) \in \mathbb{R}^2$, α and $p > 0$ are real constants.

Now we state the main results of the paper.

Theorem 1. Suppose that $\varepsilon_0 \in H^1$, $n_0 \in L^2$, $n_1 \in H^{-1}$ and $0 < p \le 2$ with $\|\varepsilon_0(x)\|_{L^2}$ small. Then there exists a global generalized solution of the initial problem (1)-(3).

$$\varepsilon_m(x,t) \in L^\infty(\mathbb{R}^+; H^1) \cap W^{1,\infty}(\mathbb{R}^+; H^{-1}), \qquad n(x,t) \in L^\infty(\mathbb{R}^+; L^2) \cap W^{1,\infty}(\mathbb{R}^+; H^{-1}),$$

$$n_t(x,t) \in L^\infty(\mathbb{R}^+; H^{-1}) \cap W^{1,\infty}(\mathbb{R}^+; H^{-2}).$$

To study generalized solution of the generalized Zakharov system, we transform it into the following form

$$i\varepsilon_t + \Delta\varepsilon - n\varepsilon + \alpha |\varepsilon|^p \varepsilon = 0, \tag{4}$$

$$\varphi_t - (n + |\varepsilon|^2) = 0, \tag{5}$$

$$n_t - \Delta\varphi = 0, \tag{6}$$

with initial data

$$\varepsilon(x,0) = \varepsilon_0(x), \; n(x,0) = n_0(x), \; \varphi(x,0) = \varphi_0(x). \tag{7}$$

where φ_0 satisfying $\nabla^2\varphi_0 = n_1$.

For the sake of convenience of the following contexts, we use C to represent various constants that can depend on initial data.

The paper is organized as follows. In section 2, we establish a priori estimations. In section 3, we state the existence of global generalized solution.

2. A priori estimations

In this section, we will derive a priori estimations for the solution of the system (4)-(7).

Lemma 1. Suppose that $\varepsilon_0(x) \in L^2(\mathbb{R}^2)$. Then for the solution of problem (4)-(7) we have

$$\| \varepsilon(x,t)\|^2_{L^2(\mathbb{R}^2)} = \| \varepsilon_0\|^2_{L^2(\mathbb{R}^2)}.$$

Proof. Multiplying Eq(4) by $\bar{\varepsilon}$, and integrating the imaginary part of the result, we get the lemma.

Lemma 2. Suppose that $\varepsilon_0 \in H^1, \; n_0 \in L^2, \; \varphi_0 \in H^1$. Then for the solution of problem (4)-(7) we have

$$\mathrm{H}(t) = \mathrm{H}(0).$$

where

$$\mathrm{H}(t) = \| \nabla\varepsilon\|^2_{L^2} + \int_{\mathbb{R}^2} n|\varepsilon|^2 \, dx + \frac{1}{2}\| \nabla\varphi\|^2_{L^2} + \frac{1}{2}\| n\|^2_{L^2} - \frac{2\alpha}{p+2}\| \varepsilon\|^{p+2}_{L^{p+2}}.$$

Proof. Taking the inner product of Eq(4) and ε_t, it follows that

$$\left(i\varepsilon_t + \Delta\varepsilon - n\varepsilon + \alpha|\varepsilon|^p \varepsilon, \; \varepsilon_t\right) = 0 \tag{8}$$

Since

$$\mathrm{Re}(i\varepsilon_t, \varepsilon_t) = 0, \quad \mathrm{Re}(\Delta\varepsilon, \varepsilon_t) = -\frac{1}{2}\frac{d}{dt}\| \nabla\varepsilon\|^2,$$

$$\begin{aligned}\mathrm{Re}(-n\varepsilon, \varepsilon_t) &= -\frac{1}{2}\int_{\mathbb{R}^2} n|\varepsilon|^2_t \, dx \\ &= -\frac{1}{2}\frac{d}{dt}\int_{\mathbb{R}^2} n|\varepsilon|^2 \, dx + \frac{1}{2}\int_{\mathbb{R}^2} n_t |\varepsilon|^2 \, dxn \\ &= -\frac{1}{2}\frac{d}{dt}\int_{\mathbb{R}^2} n|\varepsilon|^2 \, dx + \frac{1}{2}\int_{\mathbb{R}^2} \Delta\varphi(\varphi_t - n) dx \\ &= -\frac{1}{2}\frac{d}{dt}\int_{\mathbb{R}^2} n|\varepsilon|^2 \, dx - \frac{1}{4}\frac{d}{dt}\| \nabla\varphi\|^2_{L^2} - \frac{1}{4}\frac{d}{dt}\| n\|^2_{L^2},\end{aligned}$$

$$\mathrm{Re}\left(\alpha|\varepsilon|^p \varepsilon, \varepsilon_t\right) = \frac{\alpha}{p+2}\frac{d}{dt}\int_{\mathbb{R}^2} |\varepsilon|^{p+2} \, dx = \frac{\alpha}{p+2}\frac{d}{dt}\| \varepsilon\|^{p+2}_{L^{p+2}},$$

thus from Eq(8) we get

$$\frac{d}{dt}\left[\| \nabla\varepsilon\|^2_{L^2} + \int_{\mathbb{R}^2} n|\varepsilon|^2 \, dx + \frac{1}{2}\| \nabla\varphi\|^2_{L^2} + \frac{1}{2}\| n\|^2_{L^2} - \frac{2\alpha}{p+2}\| \varepsilon\|^{p+2}_{L^{p+2}}\right] = 0.$$

Letting

$$\mathrm{H}(t) = \| \nabla\varepsilon\|^2_{L^2} + \int_{\mathbb{R}^2} n|\varepsilon|^2 \, dx + \frac{1}{2}\| \nabla\varphi\|^2_{L^2} + \frac{1}{2}\| n\|^2_{L^2} - \frac{2\alpha}{p+2}\| \varepsilon\|^{p+2}_{L^{p+2}}.$$

It follows that

$$\mathrm{H}(t) = \mathrm{H}(0).$$

Lemma 3 (Gagliardo-Nirenberg inequality (Friedman (1969))). Assume that $u \in L^q(\mathbb{R}^n)$, $D^m u \in L^r(\mathbb{R}^n)$, $1 \le q, r \le \infty, 0 \le j \le m$, we have the estimations

$$\| D^j u\|_{L^p(\mathbb{R}^n)} \le C \| D^m u\|^{\alpha}_{L^r(\mathbb{R}^n)} \| u\|^{1-\alpha}_{L^q(\mathbb{R}^n)},$$

where C is a positive constant, $0 \le \frac{j}{m} \le \alpha \le 1$,

$$\frac{1}{p} = \frac{j}{n} + \alpha\left(\frac{1}{r} - \frac{m}{n}\right) + (1-\alpha)\frac{1}{q}.$$

Lemma 4. Suppose that $\varepsilon_0 \in H^1,\ n_0 \in L^2,\ \varphi_0 \in H^1$ and $0 < p \le 2$ with $\| \varepsilon_0(x)\|_{L^2}$ small. Then we have $\| \nabla\varepsilon\|^2_{L^2} + \| \nabla\varphi\|^2_{L^2} + \| n\|^2_{L^2} \le C.$

Proof. By Hölder inequality, Young inequality and Gagliardo-Nirenberg inequality, there holds

$$\left|\int_{\mathbb{R}^2} n\,|\varepsilon|^2\, dx\right| \le \| n\|_{L^2} \| \varepsilon\|^2_{L^4} \le \frac{1}{4}\| n\|^2_{L^2} + \| \varepsilon\|^4_{L^4} \le \frac{1}{4}\| n\|^2_{L^2} + C\| \nabla\varepsilon\|^2_{L^2} \| \varepsilon\|^2_{L^2}. \tag{9}$$

Using Gagliardo-Nirenberg inequality, we write

$$\frac{2|\alpha|}{p+2}\| \varepsilon\|^{p+2}_{L^{p+2}} \le C\| \nabla\varepsilon\|^p_{L^2} \| \varepsilon\|^2_{L^2}. \tag{10}$$

Note that from lemma 2 and Eq(9), (10), one has

$$\| \nabla\varepsilon\|^2_{L^2} + \frac{1}{2}\| \nabla\varphi\|^2_{L^2} + \frac{1}{4}\| n\|^2_{L^2} \le \left|\mathrm{H}(0)\right| + C\| \nabla\varepsilon\|^2_{L^2} \| \varepsilon\|^2_{L^2} + C\| \nabla\varepsilon\|^p_{L^2} \| \varepsilon\|^2_{L^2}.$$

Note that $0 < p \le 2$ and $\| \varepsilon_0\|_{L^2}$ small, we thus get the lemma.

Lemma 5. Suppose that $\varepsilon_0 \in H^1,\ n_0 \in L^2,\ \varphi_0 \in H^1$ and $0 < p \le 2$ with $\| \varepsilon_0(x)\|_{L^2}$ small. Then we have $\|\varepsilon_t\|_{H^{-1}} + \|\varphi_t\|_{L^2} + \|n_t\|_{H^{-1}} \le C.$

Proof. Taking the inner product of Eq(4) and V, Eq(5)-(6) and v, it follows that

$$\left(i\varepsilon_t + \Delta\varepsilon - n\varepsilon + \alpha\,|\varepsilon|^p\, \varepsilon, V\right) = 0, \tag{11}$$

$$\left(\varphi_t - (n + |\varepsilon|^2), v\right) = 0, \tag{12}$$

$$\left(n_t - \Delta\varphi, v\right) = 0. \tag{13}$$

where $\forall\ v, v_i \in H_0^1\ (i = 1,\cdots,N)$, $V = (v_1,\cdots,v_N)$.

By Hölder inequality, it follows from Eq(11) that

$$\begin{aligned}\left|(\varepsilon_t, V)\right| &\le \left|(\Delta\varepsilon, V)\right| + \left|(n\varepsilon, V)\right| + \left|(\alpha\,|\varepsilon|^p\, \varepsilon, V)\right| \\ &= \left|(\nabla\varepsilon, \nabla V)\right| + \left|(n\varepsilon, V)\right| + |\alpha|\left|(|\varepsilon|^p\, \varepsilon, V)\right| \\ &\le \|\nabla\varepsilon\|_{L^2} \|\nabla V\|_{L^2} + \|n\|_{L^2} \|\varepsilon\|_{L^4} \|V\|_{L^4} + |\alpha| \|\varepsilon\|^{p+1}_{L^{2(p+1)}} \|V\|_{L^2}.\end{aligned} \tag{14}$$

By Gagliardo-Nirenberg inequality, we know that

$$\|\varepsilon\|_{L^4} \le C\|\nabla\varepsilon\|^{\frac{1}{2}}_{L^2} \|\varepsilon\|^{\frac{1}{2}}_{L^2}, \quad \|\varepsilon\|^{p+1}_{L^{2(p+1)}} \le C\| \nabla\varepsilon\|^p_{L^2} \| \varepsilon\|_{L^2},$$

$$\|V\|_{L^4} \le C\|\nabla V\|^{\frac{1}{2}}_{L^2} \|V\|^{\frac{1}{2}}_{L^2} \le C\left(\|\nabla V\|_{L^2} + \|V\|_{L^2}\right).$$

Hence from Eq(14) we get

$$\left|(\varepsilon_t, V)\right| \le C\|V\|_{H_0^1}. \tag{15}$$

Using Hölder inequality, from Eq(12), there is

$$\left|(\varphi_t, v)\right| \le \left|(n, v)\right| + \left|\left(|\varepsilon|^2, v\right)\right| \le \|n\|_{L^2} \|v\|_{L^2} + \|\varepsilon\|^2_{L^4} \|v\|_{L^2} \le C\|v\|_{L^2}. \tag{16}$$

From Eq(13) and Hölder inequality, we have

$$\left\|(n_t, v)\right\| = \left\|(\Delta\varphi, v)\right\| = \left\|(\nabla\varphi, \nabla v)\right\| \le \|\nabla\varphi\|_{L^2} \|\nabla v\|_{L^2} \le C\|v\|_{H_0^1}. \tag{17}$$

Hence from Eq(15)-(17), one obtain

$$\|\varepsilon_t\|_{H^{-1}} + \|\varphi_t\|_{L^2} + \|n_t\|_{H^{-1}} \le C.$$

3. The existence of global generalized solution

In this section, we formulate the proof of Theorem 1. First we give the definition of generalized solution for problem (4)-(7).Firstly, we give two lemmas (Lions (1969)).

Lemma 6. Let B_0, B, B_1 be three reflexive Banach spaces and assume that the embedding $B_0 \to B$ is compact. Let

$$W = \left\{ V \in L^{p_0}((0,T);B_0), \frac{\partial V}{\partial t} \in L^{p_1}((0,T);B_1) \right\}, \quad T<\infty, 1<p_0, p_1<\infty.$$

W is a Banach space with norm

$$\|V\|_W = \|V\|_{L^{p_0}((0,T);B_0)} + \|V_t\|_{L^{p_1}((0,T);B_1)}.$$

Then the embedding $W \to L^{p_0}((0,T);B)$ is compact.

Lemma 7. Let Ω be an open set of $\mathbb{R}^n$ and let $g, g_\varepsilon \in L^p(\mathbb{R}^n)$, $1<p<\infty$, such that $g_\varepsilon \to g$ a.e. in Ω and $\|g_\varepsilon\|_{L^p(\Omega)} \le C$. Then $g_\varepsilon \to g$ weakly in $L^p(\Omega)$.

Now, one can estimate the following theorem.

Theorem 2. Suppose that $\varepsilon_0 \in H^1$, $n_0 \in L^2$, $\varphi_0 \in H^1$ and $0<p\le 2$ with $\|\varepsilon_0(x)\|_{L^2}$ small. Then there exists global generalized solution of the initial value problem (4)-(7).

$$\varepsilon_m(x,t) \in L^\infty(\mathbb{R}^+;H^1) \cap W^{1,\infty}(\mathbb{R}^+;H^{-1}), \qquad n(x,t) \in L^\infty(\mathbb{R}^+;L^2) \cap W^{1,\infty}(\mathbb{R}^+;H^{-1}),$$

$$\varphi(x,t) \in L^\infty(\mathbb{R}^+;H^1) \cap W^{1,\infty}(\mathbb{R}^+;L^2).$$

Proof. By using Galerkin method, choose the basic periodic functions $\{\omega_j(x)\}$ as follows:

$$-\Delta\omega_j(x) = \lambda_j\omega_j(x), \ \omega_j(x) \in H_0^1(\Omega), \ j=1,2,\cdots,l.$$

The approximate solution of problem (4)-(7) can be written as

$$\varepsilon^l(x,t) = \sum_{j=1}^{l} \alpha_j^l(t)\omega_j(x), \quad \varphi^l(x,t) = \sum_{j=1}^{l} \beta_j^l(t)\omega_j(x), \quad n^l(x,t) = \sum_{j=1}^{l} \gamma_j^l(t)\omega_j(x),$$

where

$$\varepsilon^l = \left(\varepsilon_1^l, \cdots, \varepsilon_N^l\right), \quad \alpha_j^l(t) = (\alpha_{j1}^l(t), \cdots, \alpha_{jN}^l(t)).$$

and Ω is a 2-dimensional cube with $2D$ in each direction, that is, $\bar{\Omega} = \{x=(x_1,x_2) \mid |x_i| \le 2D, i=1,2\}$.

According to Galerkin's method, these undetermined coefficients $\alpha_j^l(t)$, $\beta_j^l(t)$ and $\gamma_j^l(t)$ need to satisfy the following initial value problem of the system of ordinary differential equations.

$$\left(i\varepsilon_{mt}^l, \omega_\kappa\right) - \left(\nabla\varepsilon_m^l, \nabla\omega_\kappa\right) - \left(n^l\varepsilon_m^l, \omega_\kappa\right) + \alpha\left(|\varepsilon^l|^p \ \varepsilon_m^l, \omega_\kappa\right) = 0, \ m=1,\cdots,N, \tag{18}$$

$$\left(\varphi_t^l, \omega_\kappa\right) - \left(n^l, \omega_\kappa\right) - \left(\left|\varepsilon^l\right|^2, \omega_\kappa\right) = 0, \tag{19}$$

$$\left(n_t^l, \omega_\kappa\right) + \left(\nabla\varphi^l, \nabla\omega_\kappa\right) = 0, \ \kappa = 1,2,\cdots,l. \tag{20}$$

with initial data

$$\varepsilon^l|_{t=0} = \varepsilon_0^l(x), \quad n^l|_{t=0} = n_0^l(x), \quad \varphi^l|_{t=0} = \varphi_0^l(x), \tag{21}$$

Suppose

$$\varepsilon_0^l(x) \xrightarrow{H^1} \varepsilon_0(x), \quad n_0^l(x) \xrightarrow{L^2} n_0(x), \quad \varphi_0^l(x) \xrightarrow{H^1} \varphi_0(x), \quad l \to \infty.$$

Similarly to the proof of lemma 1-5, for the solution $\varepsilon^l(x,t)$, $n^l(x,t)$, $\varphi^l(x,t)$ of problem (18)-(21), we can establish the following estimations

$$\|\varepsilon^l\|_{H^1} + \|\varphi^l\|_{H^1} + \|n^l\|_{L^2} \le C, \tag{22}$$

$$\|\varepsilon_t^l\|_{H^{-1}} + \|\varphi_t^l\|_{L^2} + \|n_t^l\|_{H^{-1}} \le C. \tag{23}$$

where the constant C is independent of l and D. By compact argument, some subsequence of $\left(\varepsilon^l, n^l, \varphi^l\right)$, also labeled by l, has a weak limit $(\varepsilon, n, \varphi)$. More precisely

$$\varepsilon^l \to \varepsilon \quad \text{in} \quad L^\infty(\mathbb{R}^+; H^1) \quad \text{weakly star}, \tag{24}$$

$$n^l \to n \quad \text{in} \quad L^\infty(\mathbb{R}^+; L^2) \quad \text{weakly star}, \tag{25}$$

$$\varphi^l \to \varphi \quad \text{in} \quad L^\infty(\mathbb{R}^+; H^1) \quad \text{weakly star}.$$

Eq(23) imply that

$$\varepsilon_t^l \to \varepsilon_t \quad \text{in} \quad L^\infty(\mathbb{R}^+, H^{-1}) \quad \text{weakly star}, \tag{26}$$

$$n_t^l \to n_t \quad \text{in} \quad L^\infty(\mathbb{R}^+, H^{-1}) \quad \text{weakly star},$$

$$\varphi_t^l \to \varphi_t \quad \text{in} \quad L^\infty(\mathbb{R}^+, L^2) \quad \text{weakly star}.$$

Moreover, let us note that the following maps are continuous.

$$H^1(\mathbb{R}^2) \to L^4(\mathbb{R}^2), \quad u \mapsto u,$$

$$H^1(\mathbb{R}^2) \times H^1(\mathbb{R}^2) \to L^2(\mathbb{R}^2), \quad (u,v) \mapsto uv.$$

It then follows from Eq(24) and (25) that

$$\left|\varepsilon^l\right|^2 \to w \quad \text{in} \quad L^\infty(\mathbb{R}^+, L^2) \quad \text{weakly star}, \tag{27}$$

$$n^l \varepsilon^l \to z \quad \text{in} \quad L^\infty(\mathbb{R}^+, L^2) \quad \text{weakly star}. \tag{28}$$

First, we prove $w = |\varepsilon|^2$. Let Ω be any bounded subdomain of $\mathbb{R}^2$. We notice that the embedding $H^1(\Omega) \to L^4(\Omega)$ is compact and for any Banach space X, the embedding $L^\infty(\mathbb{R}^+, X) \to L^2(0,T;X)$ is continuous. Hence, according to Eq(24),(26) and lemma 6, applied to $B_0 = H^1(\Omega),\ B = L^4(\Omega),\ B_1 = H^{-1}(\Omega)$, and says that some subsequence of $\varepsilon^l|_\Omega$ (also labeled by l) converges strongly to $\varepsilon|_\Omega$ in $L^2(0,T;L^4(\Omega))$. So we can assume that

$$\varepsilon^l \to \varepsilon \text{ strongly in } L^2(0,T;L^4_{loc}(\Omega)), \tag{29}$$

and thus $\varepsilon^l \to \varepsilon$ a.e. in $[0,T]\times\Omega$. Then, using lemma 7 and Eq(27) imply that $w = |\varepsilon|^2$.

Second, we prove $z = n\varepsilon$. Let ψ be some test function in $L^2(0,T;H^1)$, $\operatorname{supp}\psi \subset \Omega \subset \mathbb{R}^2$

$$\int_0^T \int_{\mathbb{R}^2} (n^l\varepsilon^l - n\varepsilon)\psi\, dxdt = \int_0^T \int_\Omega n^l\left(\varepsilon^l - \varepsilon\right)\psi\, dxdt + \int_0^T \int_\Omega \left(n^l - n\right)\varepsilon\psi\, dxdt.$$

On one hand

$$\left|\int_0^T \int_\Omega n^l\left(\varepsilon^l - \varepsilon\right)\psi\, dxdt\right| \le \left\|n^l\right\|_{L^\infty(0,T;L^2(\Omega))} \left\|\varepsilon^l - \varepsilon\right\|_{L^2(0,T;L^4(\Omega))} \|\psi\|_{L^2(0,T;L^4(\Omega))}.$$

Since Ω is bounded, we deduce from Eq(25) and (29) that

$$\int_0^T \int_\Omega n^l\left(\varepsilon^l - \varepsilon\right)\psi\, dxdt \to 0 \quad (l \to +\infty).$$

On the other hand, let us note that $\varepsilon\psi \in L^1(0,T;L^2)$. In fact

$$\|\varepsilon\psi\|_{L^1(0,T;L^2)} \le \|\varepsilon\|_{L^2(0,T;L^4)} \|\psi\|_{L^2(0,T;L^4)} < \infty.$$

Therefore we deduce from Eq(25) that

$$\int_0^T \int_\Omega (n^l - n)\varepsilon\psi\, dxdt \to 0 \quad (l \to +\infty).$$

Thus $n^l\varepsilon^l \to n\varepsilon$ in $L^2(0,T;H^{-1})$. So $z = n\varepsilon$.

Hence taking $l \to \infty$ from Eq(18)-(21), by using the density of ω_j in $H_0^1(\Omega)$ we get the existence of local generalized solution for the periodic initial value problem (4)-(7). letting $D \to \infty$, the existence of local solution for the initial value problem (4)-(7) can be obtain. By the continuation extension principle and a prior estimates, we can get the existence of global generalized solution for problem (4)-(7).

We thus complete the proof of theorem 2. Hence one can get the theorem 1.

4. Conclusions

This paper considers the existence of the generalized solution to the initial vale problem for a generalized Zakharov equation by a priori integral estimates and Galerkin method, one has the existence of the global generalized solution to the problem

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