CHEMICAL ENGINEERING TRANSACTIONS
VOL. 46, 2015
A publication of AIDIC
The Italian Association of Chemical Engineering
Online at www.aidic.it/cet
Guest Editors: Peiyu Ren, Yancang Li, Huiping Song

ISBN 978-88-95608-37-2; **ISSN** 2283-9216
DOI: 10.3303/CET1546215

The Ebola Outbreak Control Model Based on the Differential Equation

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Recently, it has been a hot topic in the field of medical and disease how to slow down or control the speed of the spread of Ebola. To solve this problem, we divided the whole stage of Ebola spread into three stages: Free-Spread stage, Isolation-Control stage and Prevent-Treatment stage. Based on these stages, we get the corresponding SEID ordinary differential equations method, SEIQD method and SEIQRD method respectively. And the feasible projects can be worked out at different stages to control Ebola. In further discussion, we study the problem of drug delivery, and delivery method was got in order to make the best use of medicine which includes three steps: 1) finding the best location of transfer station; 2) carrying out the clustering analysis of the affected regions; 3) finding the preliminary delivery systems of any one of affected regions. From the above steps, possible feasible delivery systems can be made from the above steps. Finally, the simulation analyses of the series of methods were made. The numerical results indicate that our method can be used to simulate the Ebola spread.

1. Introduction

Mathematical model of infectious diseases has been studied again while the out broke of the Ebola virus among the word. In the early 1927, Kermark and Mckendrick established the famous SIR warehouse room model, and studied the propagation law and popular trend of the virus then proposed the Threshold Theory [1] .They laid a solid foundation for other people's further research. In the subsequent period of time, more scholars paid attention to the study of this model. Jianquan Li and Zhen Ma thought that the most important factors were vaccination, temporary immunity and population scale change, so they set up SIS epidemic model[2]; but in the opinion of Mei Song and Guirong Jiang, after considered the influence of birth rate, so they build their own epidemic model [3~4]; Naoki Yoshida and Tadayuki Hara hold the view that the model should consider the transmission density and man's birth rate and death rate, so they build their model according to the elements[5]; But Franceschetti, Pugliese,Jianquan Li, Lei Shi still took their attitude that the best precept is the different influence of einwanderung [6~8]. Later, epidemic model with vertical transmission and SIRS model with quarantine were built based on SIR model [9~12]; But Ladi Wang thought the epidemic model can be established on the influence of some factories, such as: birth, death, einwanderung, and even the disease [13]. Some effective strategies for disease control based on the stability of its model were showed by the above documents. Recently, Guihua LI and Zhen JIN had a more realistic model based on the isolated populations, latent populations and even the contact rate are considered [14]; The global stability of SEIQR model has been researched by Wenxiong Xu et al [15], and Xinli Hu studied the global stability of SEIQR model with the effect of Isolated and latent populations [16].
On the basis of reading a large number of references, understand the specific significance of various factors in different epidemic model, and then connecting with the reality, the series method were figured out that can eradicate Ebola.

2. Methods Introduction

In order to control the spread of the disease much better, the proliferation of Ebola virus were divided into three stages: Free-Spread stage, Isolation-Control stage and Prevent-Treatment stage.

Please cite this article as: Li Y.L., Wang J., Huang B., Liu W.X., 2015, The ebola outbreak control model based on the differential equation, Chemical Engineering Transactions, 46, 1285-1290 DOI:10.3303/CET1546215

Free-Spread stage. In the early spread of Ebola virus without any protection and treatment, considering the relative contact rate and mortality rate due to illness, we divide the population in the affected areas into several classes and build differential equations by analyzing the relationship between the classes. After solving the differential equations and fitting the function of the number of infected people changes over time, we get the spread law of the Ebola virus in the case of natural conditions.

Isolation-Control stage. After discovered the spreading of the Ebola disease, the government will quarantine the infected persons. At this stage, we formulate a differential model by considering the isolation rate which based on the capabilities of local government. After solving the differential equation, we gain the case of Ebola virus to be controlled and give its curve measures.

Prevent-Treatment stage. After the drug development and delivery, differential equations were settled by considering the rate of cured persons compared with susceptible persons and the rate of susceptible persons obtained the immunity through the vaccination. At this stage, in the case of the drug is sufficient in order to control the epidemic, we give the minimum quantity of the drug to make the cure rate is greater than the infection rate, then consider about the difficulty of production and the cost of production. In the case of drugs is lacked, we put drugs in batches then calculate how long it will take and put the total amount of drugs to control the epidemic.

We get the corresponding SEID ordinary differential equations method, SEIQD method and SEIQRD method respectively based on three stages: Free-Spread stage, Isolation-Control stage and Prevent-Treatment stage. The feasible projects can be worked out at different stages to control Ebola by analyzing the different situations. Finally, the simulation and error analysis of the series of models are made. The numerical results indicate that our model can be used to simulate the Ebola spread.

We assume that natural birth rate and mortality rate is equal and the population mobility can be neglected in the affected region during the stage of Ebola spreading. We believe that ease the disaster of an region needs a relatively long time when compare to treat the affected person, so we can assume that natural birth rate and mortality rate is equal during the stage of Ebola spreading.

2.1 SEID method of Free-Spread stage

Considering the incubation period of the Ebola, during this stage, the population was divided into four parts: Suspected (S), Exposed (E), Infectious (I) and Dead (D). Because they can change as the time goes, then analyses the relation between them, we get SEID ordinary differential equations method (SEID method) as follows:

$$\begin{cases} \dfrac{dS(\mathrm{t})}{dt} = -\beta S(\mathrm{t})(1-\gamma)\mathrm{I}(\mathrm{t}) \\ \dfrac{dE(\mathrm{t})}{dt} = \beta S(\mathrm{t})(1-\gamma)\mathrm{I}(\mathrm{t}) - \theta \mathrm{E}(\mathrm{t}) \\ \dfrac{\mathrm{d}I(\mathrm{t})}{dt} = \theta E(\mathrm{t}) - \gamma I(\mathrm{t}) \\ \dfrac{dD(\mathrm{t})}{dt} = \gamma \mathrm{I}(\mathrm{t}) \end{cases} \tag{1}$$

In the type, t is the day that Ebola has spread; is the effective contact rate; 1/ is the proportion of people who is from Infectious to Exposed; is the day of incubation period; is lethality rate.

We assume that the effective contact rate is affected by temperature and population density slightly, so it is considered as a constant parameter during process of solving the model.

2.2 SEIQD method of Isolation-Control stage

During this stage, the government will take isolation measures to slow the spread of the disease, so the population is divided into five parts: Suspected(S), Exposed (E) [4], Infectious (I), Quarantine (Q) and Dead (D). Because they can change as the time goes, we analyses the relation between them, then get SEIQD ordinary differential equations method (SEIQD method) based on the isolated rate as follows:

$$\begin{cases} \dfrac{dS(\mathrm{t})}{dt} = -\beta S(\mathrm{t})(1-\delta-\gamma)\,\mathrm{I}(\mathrm{t}) \\ \dfrac{dE(\mathrm{t})}{dt} = \beta S(\mathrm{t})(1-\delta-\gamma)\,\mathrm{I}(\mathrm{t}) - \theta E(\mathrm{t}) \\ \dfrac{dI(\mathrm{t})}{dt} = \theta E(\mathrm{t}) - (\delta+\gamma)\,\mathrm{I}(\mathrm{t}) \\ \dfrac{dQ(\mathrm{t})}{dt} = \delta\,\mathrm{I}(\mathrm{t}) - \varepsilon Q(\mathrm{t}) \\ \dfrac{dD(\mathrm{t})}{dt} = \gamma\,\mathrm{I}(\mathrm{t}) + \varepsilon Q(\mathrm{t}) \end{cases} \tag{2}$$

In the type, t is the day that Ebola has spread; β is the effective contact rate; θ is the proportion of people who is from Infectious to Exposed; δ is the day of incubation period; δ is the lethality rate; γ is the proportion of people who is from Quarantine to Dead.

We assume that affected person in quarantine cannot infect other people, especially the medical staff. We believe that affected persons and quarantine persons are two different groups, the former can infect other people still, but the latter cannot, so that we can better to analyze the influence of the government's control efforts to the model's result.

2.3 SEIQRD method of Prevent-Treatment stage

During this stage, medicines are put into use; the medicines divided are into vaccines and Drugs. There is two points to be considered carefully, one is cure rate; and the other is the rate of vaccines works. So the population is divided into six parts: Suspected (S), Exposed (E), Infectious (I), Quarantine (Q), Recovery (R) and Dead (D). Because they can change as the time goes, we analyses the relation between them, then get SEIQRD ordinary differential equations method (SEIQRD method) based on speed of manufacturing of the vaccines or drugs as follows:

$$\begin{cases} \dfrac{dS(t)}{dt} = -\beta(1-\alpha)\mathrm{S(t)}(1-\delta-\gamma-\rho)\mathrm{I(t)} \\ \dfrac{dE(t)}{dt} = \beta(1-\alpha)\mathrm{S(t)}(1-\delta-\gamma-\rho)\mathrm{I(t)} - \theta\,\mathrm{E(t)} \\ \dfrac{dI(t)}{dt} = \theta\,\mathrm{E(t)} - (\delta+\gamma+\rho)\mathrm{I(t)} \\ \dfrac{dQ(t)}{dt} = \delta\mathrm{I(t)} - (\varepsilon+\rho)\,\mathrm{Q(t)} \\ \dfrac{dR(t)}{dt} = \rho(\mathrm{I(t)} + \mathrm{Q(t)}) \\ \dfrac{dD(t)}{dt} = \gamma\mathrm{I(t)} + \varepsilon\,\mathrm{Q(t)} \end{cases} \tag{3}$$

In the type, t is the day that Ebola has spread; β is the effective contact rate; $1/\theta$ is the proportion of people who is from Infectious to Exposed; θ is the day of incubation period; δ is the isolated rate; γ is the lethality rate; ε is the proportion of people who is from Quarantine to Dead; ρ is the cure rate ; α is the rate of vaccines work.

3 Case studies

3.1 Data sources and study area

Instance analysis has been carried out by the data of Liberia which is published by WHO in 2013.We can analyze the change of infected person, figure out the best value of isolate rate and the most economical solution of drug. Finally, we can work out the concrete proposal which can eradicate Ebola.

3.2 Result and Analyses

For differential equations (1), (2), (3), we work out the ordinary differential equations and draw change curves based on Runge-Kutta method [17~18].

According to the figure 1, without any isolation measures and drugs treatment, a Liberia with4, 294,000 people as the center of Ebola, and about 51 days the disease will infect the whole country.

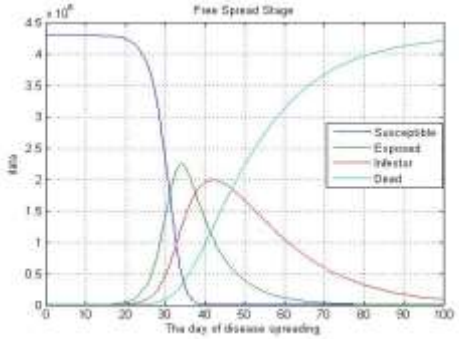

Figure 1: Data changes in Free Spread stage

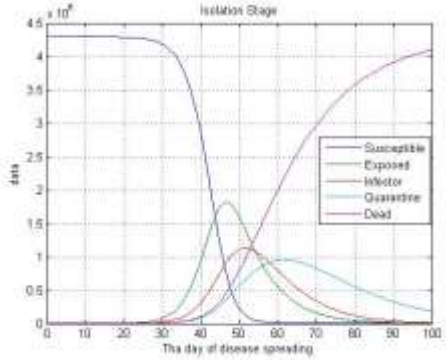

Figure 2: Data changes in Isolation stage

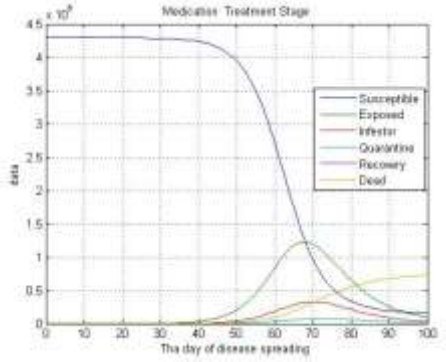

Figure 3: Data changes in Prevent-Treatment stage

According to the figure 2, since the government takes isolation measures, the spread speed has slow down compare with the former speed, and so is the dead rate. And we can safely make the conclusion that the government just delayed the spread of the Ebola, but cannot eradicated Ebola.
According to the figure 3, Ebola can be controlled by some medicines include vaccines and drugs been used, and with more and more medicines be used as the time goes, cure rate grows slowly and infectious rate is slow down, so Ebola will disappear finally. At this stage, there is two situations, one is medicines are enough so that we can figure out the minimum cost and the minimum medicines to control Ebola; the other situation is medicines are not enough under this circumstances, the speed of manufacturing of the vaccines and drugs is needed to consider then we can safely draw the conclusion that how long Ebola will disappear and gross of medicines are need.

3.3 Progressive Relationship between Three Models

In the process of solving the problem, three steps are considered during the first day and disappear day, we get three methods step by step. In the process of work out the optimal solution scheme and drug delivery to restrain Ebola of an area, we need to consider the infection range without governmental social control and medicines use by the infection rate; and then governmental social control will be taken, we can calculate how the infectious rate will changes; finally medicine will be use and then we shall work out how to use the minimum medicines to control or even eradicate Ebola.

4. Simulation results

In order to improve our model's practicality in real situation and solve the eradicate problem of the Ebola better, we carry on the simulation by using Liberia's data.
After calculation three stages respectively, we find that the Infected person is growing rapidly in the Free-Spread stage, and because the total population is fixed, there will be only Dead and Infected person in this country as time goes by. So we can make the conclusion that this country will be destroyed by the Ebola if any measures are not been took.
As for the Isolation-Control stage, the growth speed of infected person will slow down as the government takes quarantine measures. While the Isolation rate and Infection rates are equal, the Ebola's spread will be controlled and the Isolation rate is 0.578 after calculated. But there is a problem that the mortality was still

high. So the isolation will be very effective at delaying the Ebola, but the country will still perished if we do not use drugs.

In the Prevent-Treatment stage, the infected person will be cure by the medicine and we can use vaccines to susceptible people in order to help them develop antibodies. Although the drugs are very shortage, but the number of infected person will decrease day by day and Ebola will be eradicated finally. Point O means the moment when the cure rate and the diffusion rate are equal. During the entire treatment process, when the drugs are enough we deliver the drugs for about 3871 people needed and vaccines for about 4211 people needed so that we can control Ebola by 381 days; and if not, we can deliver the drugs for about 7735 people needed and vaccines for about 8312 people needed so that we can cure Ebola by 947 days.

Finally we draw various infections curves of different stages and it is shown in figure 4:

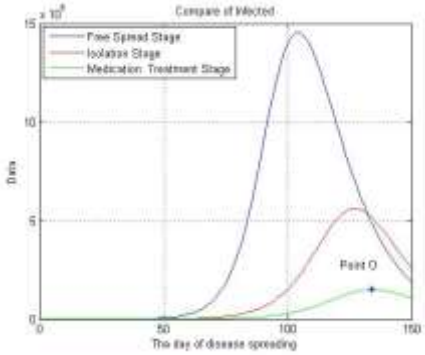

Figure 4: Emulation and Comparison

5. Further discussion of the method

When consider the manufacture's speed of the vaccine or drug, the medicine will be delivered to the best locations in order to make the best use of it. And there are three steps we should do as follows.

Step One: Find the Best Location of Transfer Station. In reality, when consider the drug is delivery through transit station, so we find the best location to build transit station which its distance from the entire affected region is the minimum, so that the economic input is least.

Step Two: Carry Out the Clustering Analysis of Affected Region. In this step, the clustering analysis of diseased region is carried out according to the speed and time of spread in order to divide it into different types. And then choose the best delivery location.

Step Three: Find the Preliminary Delivery Systems of Any One of Affected Region.

Once the medicine arrived in the region, then considered the Ebola spread speed and the area which a rescue station could treat, so we can work out where to build the rescue station and how many medicines are needed can make the Ebola under control as the time goes by..

From what we have discussed above, the conclusion of a delivery systems which is possible and feasible (including when, where and what we should do).

6. Conclusions

Through the example analysis and analogue simulation, it is easy to know that if you want to achieve the goal of eradicating the Ebola, we need to identify the parameter's values of the specific areas (For example, the transmission rates in different regions are different), so that the changes of different groups will beget accurately get in the region and it make the model digital completely. In current situation which government's control is limited and the drug is precious, we can get minimum isolation rate of the epidemic control, minimum usage of drugs and vaccines. Further, the total required time and the total amount of drug will be getting when the epidemic in the region is eradicated.

In the further discussion, we assume that the population in the affected areas is evenly distributed when the delivery system is established; however there is a deviation with actual situation. But if we can add some random process simulation so that simulate the real life human activities, a more accurate result will be get in the process of solving.

The process will be divided into the three stages according to when the epidemic starts and when the epidemic been eradicated, then a series of methods will be worked out to eradicate Ebola based on the implement of quarantine measures and drug. This series method has high feasibility after derivation.

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