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DOA Estimation of Coherent Sources under Small Number of Snapshots

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A novel method is presented to estimate the direction of arrival (DOA) for coherent signals in the environment of small number snapshots. According to the statistical characteristics of the signal source, the Bootstrap algorithm reconstructs the new Eigen-space, and expands element numbers of covariance matrix. The amount of information to be extended after the reconstruction, and to maximize the use of the noise subspace and signal subspace information. At the same time, Block sampling for coherent signal has good filtering to coherence function, which solves the problem of coherent signal smoothing. The simulation results show that the method in a small number of snapshots of direction of arrival estimation has good estimation performance.

1. Introduction

DOA estimation algorithm is widely used in sonar, radar, electronic warfare and other aspects in order to locate the signal source location information. After several decades of development, DOA estimation has a large number of high resolution algorithms, such as MUSIC proposed by Schmidt (1986) and Ralph, ESPRIT by Roy (1989) et al. With different application environment, more and more algorithms emerge in endlessly, and the DOA estimation of coherent sources is an important research subject. In order to solve the problem of rank deficiency in coherent signal source, many algorithms like the classical dimension reduction algorithm which was proposed by Shan (1985) et al. Mao (2013) extended it to 2D-DOA. The matrix reconstruction algorithm was employed by Chen (2010) et al, Chen and Zhang (2013) proposed the Virtual array transformation. Chen (2014) employed the Toeplitz method.
The dimension reduction algorithm is simple and has good applicability, but it can cause the loss of the array aperture. Although the method of dimension reduction has not that shortcomings, but it often needs to construct a special matrix for a specific environment which reported by Wang (2004) et al. All the above algorithm are often requires a large number of snapshots support, but there are a large number of passive and active jamming signal in today's complex electromagnetic environment especially under electronic warfare environment, the possibility of radar capture a large number of snapshots smoothly is very low (Errati et al. (2013) reported). Therefore, the study of coherent signal estimation problem in the small number of snapshot background has important significance.
In recent years, the compressed sensing algorithm (Wang et al. (2009) reported), the overlap matrix algorithm (Blunt et al. (2011) reported), the particle swarm optimization (Errati et al. (2013) reported), the reconstruction Method by Toeplitz (Chen et al. (2014) reported) were proposed to optimized algorithm. Despite of their great interests, these methods suffer from serious drawbacks such as weak applicability that it is necessary to construct special arrays or for specific applications. For example, although the greed algorithm (Huang (2013) reported) can detected the direction of coherent signal in a small number of snapshots, but at least more than 25 array elements are needed to acquire good performance. Xie (2010) et al. use single snapshot data to construct the Toeplitz matrix and finding the direction of coherent signal successfully, but the signal must satisfy the requirements of zero initial phase, However it is difficult to achieve in practical application.

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2. Mathematical model of signal

2.1 mathematical model of uniform linear array (ULA)

Assuming that a far field narrow-band signal is incident to a spatially uniform array antenna array. The array is composed of an array element, the array element spacing $d = \lambda / 2$, and the of a narrow band signal are respectively. So the array received signal is $\mathbf{X}(t) = [\mathbf{x}_1(t), \mathbf{x}_2(t), ..., \mathbf{x}_M(t)]$ at some time. So the l th array received data at the snapshot *t* is given by:

$$x_l(t) = \sum_{l=1}^{N} s_i(t - \tau_{li}) + n_l(t) \tag{1}$$

Where, $s_i(t)$ is incidence signal, τ_{li} is time delay between i th sources in the arrays, $n_i(t)$ is the noise and Interference at t time. Formula (1) can be written as vector matrix:

$$\mathbf{X}(t) = \mathbf{A}\mathbf{S}(t) + \mathbf{N}(t) \tag{2}$$

Where *X(t)* is the *M*×1 received matrix; *S(t)* is the *N*×1 signal matrix; A is the *M*×*N* steering matrix, and:

$$\begin{cases} \mathbf{X}(t)=[x_1(t),\cdots,x_M(t)]^{\mathrm{T}} \\ \mathbf{A}=[\mathbf{a}(\theta_1),\cdots,\mathbf{a}(\theta_N)] \\ \mathbf{S}(t)=[s_1(t),\cdots,s_K(t)]^{\mathrm{T}} \\ \mathbf{N}(t)=[n_1(t),\cdots,n_M(t)]^{\mathrm{T}} \end{cases} \tag{3}$$

2.2 MUSIC algorithm

MUSIC algorithm is a classical high resolution DOA direction finding method, the main principle is that the signal subspace and noise subspace is not related to each other.
Define the covariance matrix of *X(t)* is R_x:

$$R_x = E[X(t)X^{\mathrm{H}}(t)] = AR_S A^{\mathrm{H}} + \sigma^2 I \tag{4}$$

So the characteristic value decomposition of R_x is *R*:

$$R = U_S \Sigma_S U_S^H + U_N \Sigma_N U_N^H \tag{5}$$

Where the U_s is the feature vector signal subspace corresponding to the large eigen-values; the U_n is the noise subspace corresponding to the small eigen-values. Ideally, the noise subspace and the signal subspace are mutually orthogonal, that is, the steering vector $a(\omega)$ is orthogonal to U_n:

$$a^{\mathrm{H}}(\omega)U_N = 0 \tag{6}$$

However, the two are not completely orthogonal in the actual situation, so the problem of DOA is transformed into a minimum optimal search problem:

$$P_{MUSIC} = 1/(a^{\mathrm{H}}(\omega)U_N U_N^{\mathrm{H}} a(\omega)) \tag{7}$$

3. The proposed algorithm

The Bootstrap algorithm is a new statistical method that was proposed by professor Efron in 1977 on the basis of summarizing others algorithm. The main idea of the method is to use the analogy; it repeated sampling from the small sample, using the sampling data to estimate the parameters to the overall parameters. Repeating the same process more than 200 times can be obtained data by analogy thought we want.
But the bootstrap that was proposed by Efron is mainly aimed at is independent identically distributed data, so it is invalid when data is dependent or related. Kunsch (1989) and Hans proposed the Block bootstrap method for the independent data based on the Efron's algorithm, the main idea is put the associated data in a Block as samples.

According to characteristics of coherent signal we proposed a data processing method based on block sampling in small sample environment. The small sample data of coherent are re-sampled to re-built covariance matrix and the coherent solution be performed at the same time, then the Music algorithm is used to estimate DOA. The specific method is as follows:
As shown in figure 1, set up a set of coherent signal incident to the space array that the element number is 8, the snapshot number is 5, then get a 8×5 data matrix. In order to avoid destroying the correlation between the data, we use a column of data as a unit to re-sample. Each block size $L = 3$. It is divided into five block ($M = 5$) from the first column, so we can getting the matrix which size is 3×5. Using the moving block bootstrap method can sample sliding from 1 to 5 block data. The sampling number more than more than 200 times.

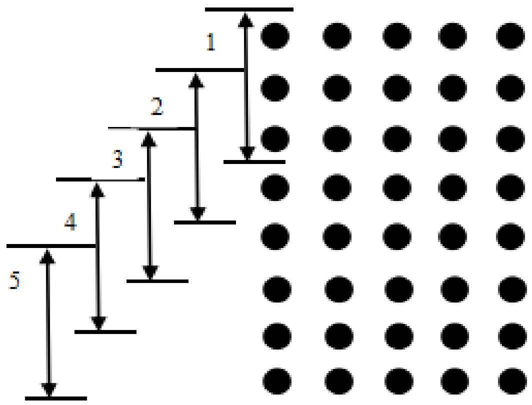

Figure 1: The schematic of block sampling for ULA

The sampling processes are similar to the spatial filtering. Superposition the matrix that is blocked and calculated covariance matrix, then can get a full rank matrix. (Mao (2013) et al. reported). By the nature of the full rank matrix: $rank(\mathrm{k}A) = rank(A)$ (k is constant), after the re-sampling that the data can be put back again, the linear superposition of matrix is the same as the original matrix rank. So the data matrix is augmented after re-sampling and the sampling matrix parameter approximate to real data according to the large data of the bootstrap sampling theorem.

4. The algorithm simulation

Simulation 1:
Suppose there have two coherent signal, the incident direction angles are -20°and 40°, signal-to-noise ratio is 10dB, the number of array elements is 8, array element spacing $d = \lambda / 2$, Change the number of snapshot from 5 to 100. If the difference of absolute value between the incidence angle and estimate value less than 1°,then we can define to estimation is success. Using spatial smoothing algorithm for 200 times Monte - Carlo experiments compared with the proposed algorithm.

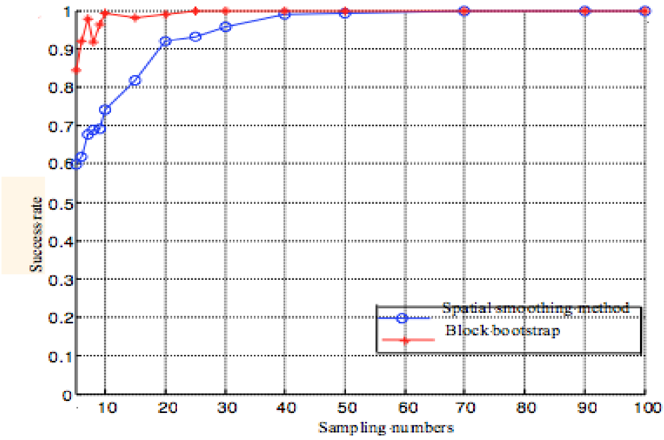

Figure 2: The contrast figure of the two method

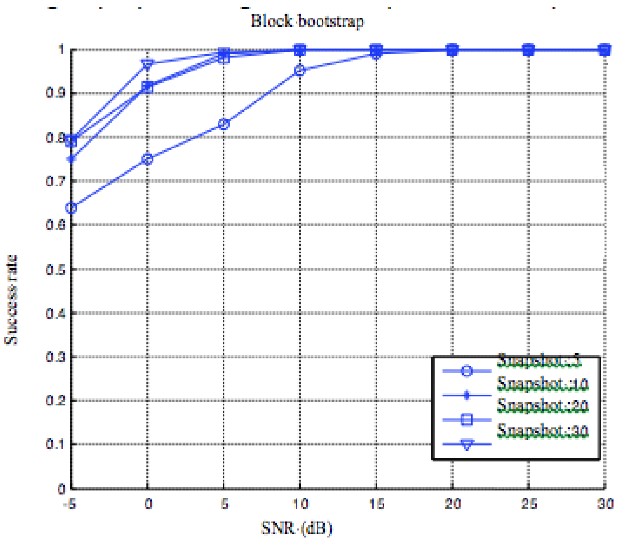

Figure 3: The Probability of success in different SNR for block method

The proposed method is superior to spatial filtering method in the condition of snapshot number less than 30, the advantage is more obviously especially under the number of 10. Proposed method can reach 90% accuracy basically after 6 number of snapshot, while the spatial filtering method can reach more than 90% until the snapshot number more than 20. After 40 snapshots, both they can accurate direction basically.

Simulation 2:

Other conditions remain unchanged, the snapshot numbers are respectively: 5, 10, 20, 30. The performance testing of proposed algorithm compared to the spatial filtering method under the condition of different SNR.

It can be seen from the figure 3, the success rate of the block sampling method is improved with the increase of SNR. The success rate close to 1 when the L=10 and the SNR more than 5 db basically. When the L= 5 and the SNR≧10 dB, the success rate increase to more than 95%.

In figure 4 spatial filtering method of direction finding success rate with the increase of the signal-to-noise ratio is monotone increasing, when the snapshot number is greater than 10 above, also more than 10 db SNR over time, the success rate is close to 1. When the L=5, the success rate tend to 99% only the SNR is greater than 15 db.

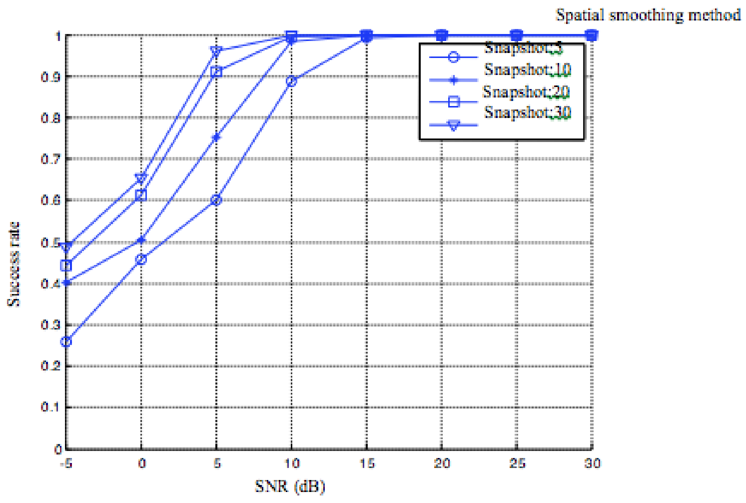

Figure 4: The Probability of success in different SNR for spatial filtering method

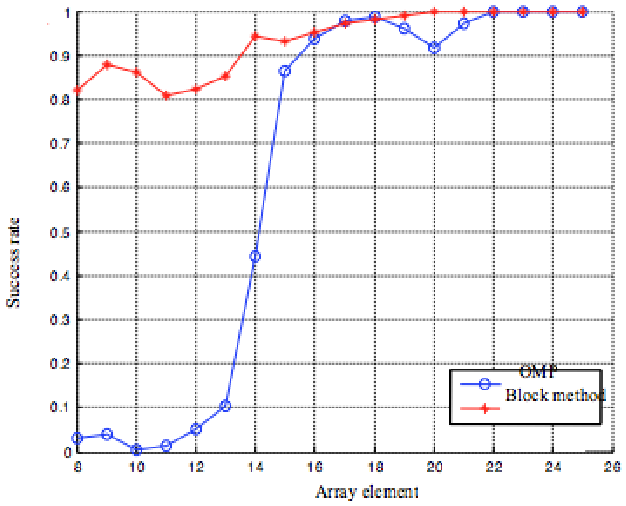

Figure 5: The Probability of success in different array for OMP and Bootstrap method

Simulation 3:
Suppose there are two coherent signals, the direction of the incident angle are 20 and 40 degrees, the signal-to-noise ratio is 10 dB, uniform linear array, the array element spacing $d = \lambda / 2$, the number of snapshots is 5. The orthogonal matching pursuit (OMP) algorithm based on compressed sensing (Wang (2013) reported) is used to compare the proposed method.
The block size is 4 in figure 5, the number of the array elements changed from 8 to 25, after 200 Monte-Carlo experiments to obtain the success probability of the two methods. The OMP algorithm even directed the arrival (DOA) estimation of coherent source in a small number of snapshots, but against the array elements higher requirements. Obviously you can see the proposed method in array utilization rate is better than that of the OMP algorithm.

5. Conclusions

This paper mainly studies the small snapshot number of coherent signal direction finding problems, through the use of sampling technology, for the reconstruction of the coherent signal sampling and coherent. The experimental results show that in the ($\mathrm{SNR} < 10$) under the condition of low SNR, success rate of the proposed method is superior to the spatial filtering method. At a high signal-to-noise ratio ($\mathrm{SNR} \geq 10$), the

success rate of this method is quicker than spatial filtering method converges to 1. In the sample number of fast, less than 10, $\mathrm{SNR} \geq 10$ cases, this method basically can reach more than 90% success rate.
But it can be seen from the figure 2 and figure 5, the success rate curve is not monotone increasing because of the selection of bootstrap's confidence interval. So we will solve the size of block and the partition problem. The sampling data model fitting and adaptive classification will be employed in order to further improve the performance of direction finding in the next step.

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