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Research On Nonlinear System Tuning Based On Virtual Reference Feedback

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To avoid the complex modeling process of the system object and design the controllers for nonlinear systems fast and effectively, the design of the feedback controller of the nonlinear system draws lessons from the thinking of the virtual reference feedback tuning method. On this basis, in order to improve the system stability and eliminate the uncertain disturbance, adding a linear controller in the feedback loop can improve the tracking performance of the system, and the design is converted into the parameter identification of the nonlinear function under the expansion equation of a basis function. The simulation has verified the effectiveness of the proposed algorithm.

1. Introduction

Nonlinearity is the most common phenomenon in nature and engineering technical fields, and the research of nonlinear systems has also achieved rapid development and breakthrough. Thereinto the exact linearization method developed by means of differential geometry has received the extensive attention. The development of modern computer technology has enhanced the further understanding of nonlinear systems.

2. Design of the two degree of freedom controller for nonlinear systems

Drawing on the thought of the virtual reference feedback tuning method, the nonlinear controller is designed, which is converted into the identification problem of the nonlinear function under the expansion equation of a basis function. In order to further enhance the stability of the system, a linear controller is added to the system, which makes the system have a good tracking performance. For the parameterized linear controller, the recursive least square method is used to identify the parameters of the nonlinear controller(Song et al., 2010).
Consider that under the condition of the closed-loop, a nonlinear controller k_{lin} is added to the closed-loop system S, based on the nonlinear controller k_{nl}. The closed-loop system structure of the two degree of freedom controller is designed, as shown in Figure 1.

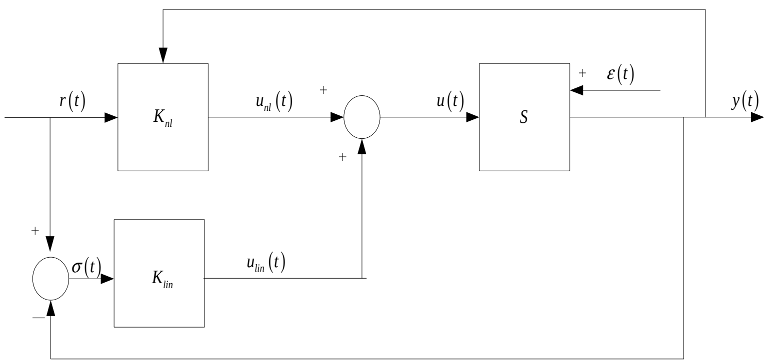

Figure 1: Structure of the closed-loop feedback control system

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S is a nonlinear system; k_{nl} is a nonlinear controller; k_{lin} is a linear controller; *u(t)* denotes the input variable; *y(t)* denotes the output variable; $\varepsilon(t)$ denotes an external disturbance variable applying to the system object. *r(t)* is the input reference signal of the closed-loop system; $u_{nl}(t)$ is the output variable of the nonlinear controller k_{nl}; $u_{lin}(t)$ is the output variable of the linear controller k_{lin}; is the tracking error(Chaturvedi et al., 2015).
The nonlinear system object model *S* in Figure 1 is described as:

$$y(t+1) = g\left(y(t), u(t), \varepsilon(t)\right) \tag{1}$$

From the Figure 1, *u(t)* is obtained by adding the output of the nonlinear controller k_{nl} and the output of the linear controller k_{lin} (Xu. et al., 2013).

$$u(t) = u_{nl}(t) + u_{lin}(t)$$

Two degree of freedom controllers are described as

$$\begin{cases} u_{nl}(t) = K_{nl}\left(r(t)\right) \\ u_{lin}(t) = K_{lin}\sigma(t) = K_{lin}\left(r(t) - y(t)\right) \end{cases} \tag{2}$$

The nonlinear controller k_{nl} is located in the feedback control, which aims at stabilizing the closed-loop system, while the linear controller k_{lin} in the feedback control loop can reduce the closed-loop error through the feedback error, so that the closed-loop system owns the good real-time tracking performance(Dai. et al., 2015).

3. The design of nonlinear controller

3.1 Shortcomings of the model inverse method

The nonlinear controller can be obtained directly by using the model inverse control method. It assumes that the identification model $\hat{y}(t+1)$ of the nonlinear function *g* is

$$\hat{y}(t+1) = f\left(y(t), u(t)\right) \tag{3}$$

If the identification model $\hat{y}(t+1)$ is obtained, the output variable $u_{nl}(t)$ of the nonlinear controller k_{nl} can be designed through the online inverse relationship, that is, the output variable $u_{nl}(t)$ is designed by solving the following inverse relationship (H. Kim. et al., 2015).

$$\begin{cases} u_{nl}(t) = \arg\min\limits_{u(t)} J\left(u(t)\right) \\ J\left(u(t)\right) = \dfrac{1}{\rho_y}\left[r(t+1) - f\left(y(t), u(t)\right)\right]^2 + \dfrac{\mu}{\rho_u} u^2(t) \end{cases} \tag{4}$$

Where standard constants ρ_y and ρ_u are respectively defined as

$$\begin{aligned} \rho_y &= \left\| \left(y(1) \quad y(2) \cdots y(N)\right) \right\|_2^2 \\ \rho_u &= \left\| \left(u(1) \quad u(2) \cdots u(N)\right) \right\|_2^2 \end{aligned} \tag{5}$$

$u \geq 0$ is the design parameter.
Take the inverse of the above formula, and define a virtual input reference variable as(Zhang et al., 2014)

$$\bar{r}(t) = M^{-1} y(t) \tag{6}$$

3.2 Parameter identification under the basis function

For the nonlinear controller k_{nl} *(y(t))*, its parametric basis function expansion form is

$$\hat{K}_{nl}\left(y(t)\right) = \sum_{i=1}^{M} \theta_i \psi_i\left(y(t)\right) \tag{7}$$

Where ψ_i is the Lipschitz continuous function; the coefficient $\theta_i \in R$ is the unknown parameter to be identified, and M is the total number of basis functions.
The parametric form of formula (7) is rewritten as the linear regression form:

$$u_{nl}(t) = \hat{K}_{nl}(y(t)) = \varphi^T(t)\theta + e(t) \tag{8}$$

Where $e(t)$ is the forecast error. From the formula (8), we can see that the design of the controller can be transformed into the identification problem of the unknown parameter vector θ under the condition of basis functions (I. Raptis. et al., 2011).
Define the objective optimization criteria function as

$$J_1(\theta) = \frac{1}{N}\sum_{t=1}^{N}\left(u(t) - \sum_{i=1}^{M}\theta_i\psi_i(y(t))\right)^2 = \frac{1}{N}\sum_{t=1}^{N}\left(u(t) - \varphi^T(t)\theta\right)^2 \tag{9}$$

The unknown parameter is obtained as follows by solving the optimal solution of the above formula.

$$\hat{\theta} = \min_{\theta} J_1(\theta) \tag{10}$$

By processing the above formula, the estimated value of the unknown parameter vector, namely $\hat{\theta}$, is shown as follows.

$$\hat{\theta} = \left[\sum_{t=1}^{N}\varphi(t)\varphi^T(t)\right]^{-1}\left[\sum_{t=1}^{N}\varphi(t)u_{nl}(t)\right] = \left(\frac{1}{N}\Phi^T\Phi\right)^{-1}\frac{1}{N}\Phi^T U_{nl} \tag{11}$$

3.3 The design of linear controller

For the design of the linear controller k_{lin}, the method of virtual reference feedback tuning in the data-driven is adopted, i.e., use the collection and observation sequence at both ends of the controller k_{lin} to tune the linear controller. The relevant part is extracted from the closed-loop feedback system shown in Figure 1, and the local part only contains the closed-loop structure of the linear controller k_{lin} and the nonlinear system object module S, as shown in Figure 2.

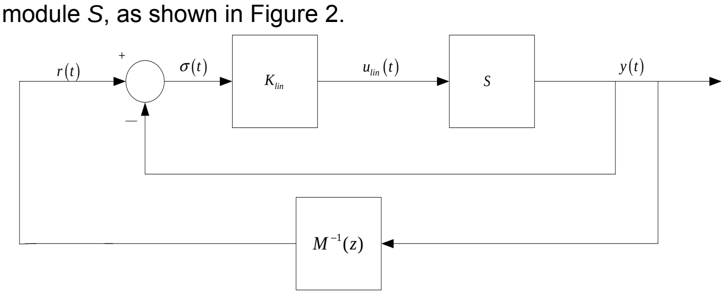

Figure 2: Design of linear controller in feedback loop

The closed-loop transfer function M which is given an expectation is included in Figure 2. When collecting the output data $y(t)$, define a virtual reference signal $\overline{r}(t)$ which satisfies the equation(Su.et al., 2013):

$$y(t) = M\overline{r}(t) \tag{12}$$

If the output obtained by applying the virtual signal $\overline{r}(t)$ to the closed-loop system is consistent with the collected output data $y(t)$, the design goal of the controller can be achieved. Define the reference tracking error:

$$\sigma(t) = \overline{r}(t) - y(t) = \left(M^{-1} - 1\right)y(t) \tag{13}$$

The reference tracking error is applied to the output signal of the controller k_{lin}:

$$u_{lin}(t) = K_{lin}\sigma(t) \tag{14}$$

Then with reference to Figure 2, $u_{lin}(t)$ can be expressed as

$$u_{lin}(t) = u(t) - u_{nl}(t) = \delta u(t) \tag{15}$$

The input data $\sigma(t)$ and the output data $\delta u(t)$ of the controller k_{lin} are obtained, respectively. The parameterized form of the linear controller k_{lin} is given in advance.

$$K_{lin}(\eta) = \beta^T \eta$$
$$\beta = [\beta_1 \quad \beta_2 \quad \cdots \quad \beta_n]^T, \eta = [\eta_1 \quad \eta_2 \quad \cdots \quad \eta_n]^T \tag{16}$$

Where β is the linear discrete transfer function which is known in the controller, and η is the parameter vector to be solved. By using the input and output observation data $\{\sigma(t), \delta u(t)\}_{t=1}^{N}$ and the parameterized linear controller, the unknown parameter vector η can be solved through the following optimization problem (Zhao et al., 2009).

$$\min_{\eta} J_{VR}^{N}(\eta) = \frac{1}{N}\sum_{t=1}^{N}\left[\delta u(t) - K_{lin}(\eta)\sigma(t)\right]^2 = \frac{1}{N}\sum_{t=1}^{N}\left[\delta u(t) - K_{lin}(\eta)\left[M^{-1}-1\right]y(t)\right]^2 \tag{17}$$

3.4 Analysis of the closed-loop stability

The nonlinear closed-loop feedback system shown in Figure 2 is described as

$$\begin{cases} y(t+1) = g\left(y(t), u(t), \varepsilon(t)\right) \\ u(t) = u_{lin}(t) + u_{nl}(t) \\ u_{nl}(t) = K_{nl}\left(r(t), y(t), u_{nl}(t-1)\right) \\ u_{lin}(t) = K_{lin}\left(r(t) - y(t), u_{lin}(t-1)\right) \end{cases}$$

(18)

Where the nonlinear controller k_{nl} and the linear controller k_{lin} all satisfy Lipschitz continuity conditions, and the nonlinear function g also satisfies Lipschitz continuity. Under the assumption of Lipschitz continuity, the nonlinear function g can be described as (Che.Z. et al., 2011).

$$g\left(y(t), u(t), \varepsilon(t)\right) = g_0\left(y(t), u(t)\right) + g_\varepsilon \varepsilon(t)$$

Where

$$g_0\left(y(t), u(t)\right) = g\left(y(t), u(t), 0\right)$$
$$\left\| g_\varepsilon \varepsilon(t) \right\|_\infty \le \gamma_\varepsilon$$

Define a residual function as

$$\Delta\left(y(t), u(t)\right) = g_0\left(y(t), u(t)\right) - f\left(y(t), u(t)\right) \tag{19}$$

And the residual function all satisfies Lipschitz continuity, i.e., there is a nonnegative constant γ_y that for different y and y', there is

$$\left\| \Delta(y, u) - \Delta(y', u) \right\| \le \gamma_y \left\| y - y' \right\|_\infty \tag{20}$$

4 Simulations

The nonlinear system object is described as

$$y(t) = -0.8u(t-1) + f(u(t)) + \varepsilon(t)$$

The nonlinear function f is

$$f^0(x) = 2x^2$$

The nonlinear controller k_{nl} is

$$u_{nl}(t) = K_{nl}(r(t)) = 2(r(t))^2$$

The linear controller k_{lin} is

$$u_{lin}(t) = K_{lin}\sigma(t) = K_{lin}(r(t) - y(t))$$

$$= \begin{bmatrix} \sigma(t) & \sigma(t-1) & \sigma(t-2) & \sigma(t-3) & \sigma(t-4) \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \end{bmatrix}$$

In the simulation process, since the basis function φ_1 is explicitly fixed, the parameter η_2 can be directly identified, while the parameter η_1 cannot be estimated directly, and only function $K_{nl} = \eta_1 \varphi_1^T$ can be calculated. Use the radial basis function and the obtained dual vector to express the nonlinear function k_{nl} as the weighted sum form of the kernel function, whose comparison with its actual situation is shown in Figure 3. The continuous curve in the figure is the real and practical curve corresponding to nonlinear functions. The asterisk curve is the nonlinear function curve of identification estimation. From this figure, we can see that the identification curve and the actual curve are basically overlapped, indicating that the kernel function of identification estimate can be infinitely close to the original nonlinear function.

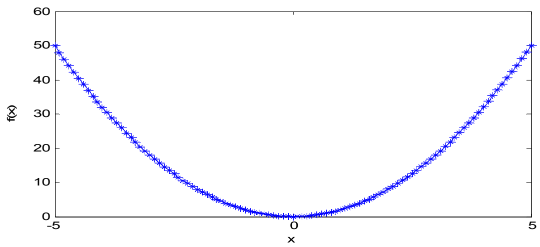

Figure 3: Nonlinear function estimate

The parameterized linear controller k_{lin} has 5 parameters needed to be further identified, and they are solved by the recursive estimation algorithm. The changing process of the values of the 5 unknown parameters with the change of iterations is shown in Figure 4. The controller parameter values gradually tend to be steady and converge to the true values with the increase of the number of iterations.

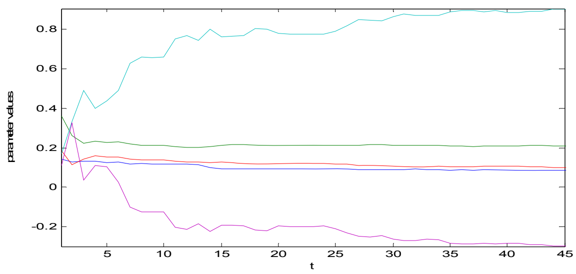

Figure 4: The convergence curves of the estimated values of the 5 unknown parameters

After designing the controller of the closed-loop system, in order to further verify the control effect of the controller, the command signal *r(t)=0.5sin(2πt/200)+0.2sin(2πt/50)* is applied to the closed-loop system. As shown in Figure 5, the simulation shows that there is a certain deviation between the system output produced by the inverse control of the model and the command. This is because that the nature of the model inverse control is an open-loop control whose anti-disturbance ability is not strong, at the same time, that the process of building the system inverse model is more complex and easy to produce deviation is also a reason. In this paper, we add a linear controller based on the nonlinear control feedback controller, but it is not a simple addition of linear controllers. In order to enhance the stability of the system, making up the deviation which arises in the design of the nonlinear controller can further restrain the system disturbance.

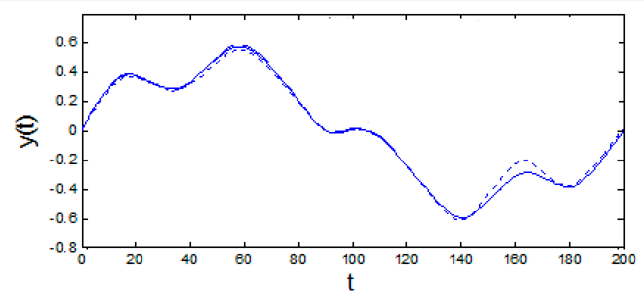

Figure 5: The comparison of the system output curves (----the two degree of freedom controller design method;—the model inverse control)

5 Conclusions

In this paper, the design of nonlinear system controller is studied. The design of nonlinear controller can be transformed into the identification problem of the nonlinear function under the expansions of a certain kind of basis functions, and the SVM kernel function is used to replace the product operation of the linear regression matrix. On this basis, in order to improve the tracking performance of the closed-loop system, a linear controller is added, and the recursive least squares method is adopted to identify the parameters of linear controller. The design of the input signals of the closed-loop system draws lessons from the virtual reference feedback tuning method. Finally, the stability of the closed-loop system is analyzed, and the simulation verifies the validity of the algorithm.

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