

Research on β Spline Curve Application of the Running Trajectory Simulation in Driveway Transform Process

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Changing lanes will produce traffic conflict in the running process, is extremely easy to cause traffic accidents. According to the characteristics of the driveway transform expected trajectory, the driveway transform process is divided into four stages, at the same time β spline curve is introduced, on the basis of the given boundary conditions, determine β spline curve reverse algorithm. Under the condition of the vehicle corner, the corner change rate and the driving behaviour of the vehicle, the relationship between the speed of lane changing and the length of the lane is established, and then trajectory parameters are calculated. The trajectory curve is calculated by Mat lab simulation and compared with the measured data; the validity of the model is verified by the results of the analysis.

1. Introduction

With the development of traffic micro simulation, the micro model of lane changing has become a hot research topic at home and abroad. The desired trajectory of lane changing is an important part of the micro model; it determines whether the vehicle is safe, smooth and fast in driveway transform process. At the same time, it has important significance to improve the road traffic capacity and reduce the vehicle delay and congestion. Domestic and foreign scholars have carried out different levels of research on the path of lane changing, focusing on data acquisition and curve fitting two aspects (Ma and Chang, 2006; Gao and Yang, 2002; Liu and Xu, 2007; Iakovos and Masayosi, 2003; Ammoun and Nashashibi, 2007). In the aspect of data acquisition and application short distance photogrammetry principle, by a linear transformation to obtain the ground coordinates. The method has the advantages of simple operation, access to data with high precision; in curve fitting, the curvature change of the vehicle running is neglected by the simplified model, which leads to the variation of the curvature of the trajectory. Therefore, this paper introduces β spline curve to establish an expected operating trajectory model which is curvature continuous.

2. Analysis of driveway transform process

According to the change of steering wheel angle, (Van 1999) has been divided the driveway transform into 3 stages: Rotates from the pilot the steering wheel starts to arrive at rotation most wide angle is the first stage, from rotation most wide angle to the corner is zero is the second stage, and angle of zero to negative maximum Angle is the third stage. (Yang, 2005) and others revised tradition 2 and 3 types lane-changing model, the process is divided into the Torsion Angle, move closer, close Angle and adjustment 4 stages. In the process of modelling, the steering angle of front wheel, the duration of the twist angle and the heading angle are introduced.

According to the characteristics of the desired trajectory of lane changing, the trajectory curve is continuous and there is no inflection point. This article assumes that the curvature change rule of vehicle running is linear. Under the initial condition, the vehicle is traveling at a speed of V , the acceleration is zero, and the vehicle is in the end of the line, so the curvature is zero.

The vehicle at the end of the terminal has the same driving characteristics at the same initial position, but a change has occurred in the plane position. Therefore, the whole lane changing of the desired trajectory can be

considered as the connection of the 4 segments β spline curve, as shown in figure 1. First stage is from the initial lane changing point to a maximum curvature point; The second stage from curvature maximum value to the traffic lane boundary, here regards as the vehicles has the front corner particle; The third stage is the vehicles from the traffic lane boundary (curvature is zero) to the curvature maximum value spot; The fourth stage is from curvature maximum value to changing the magical skill to complete the end point.

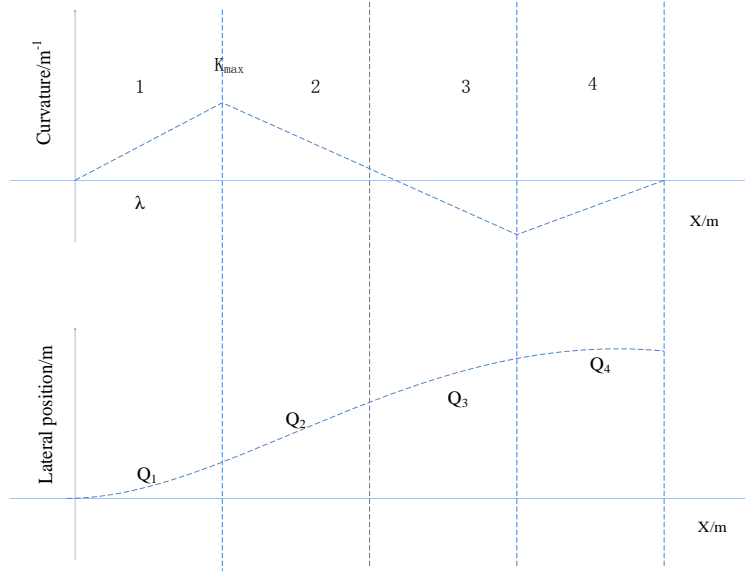


Figure 1: Driveways transform process

3. Establishment of the changing-lane desired trajectory model

3.1 β Spline curve equation

β spline curve is governed by a set of points [], these points are referred to as the control points and control points of control polygon. The β spline curve is simulated in the form of curves. The controlled polygon can produce $m-2$ segment curve, and the section curve has 4 control points and each point on the curve is the weighted value of the control points. As shown in Figure 2, the control polygon that is composed of 7 control points can produce 4 segment β spline curve. The expression of the β spline curve is (Cong, 2007):

$$q_i(u) = \sum_{r=-1}^2 b_r(\beta_1, \beta_2, u)u_{i+r} \quad 0 \leq u \leq 1 \quad i = 1, 2, 3, \dots, m - 2 \quad (1)$$

And take $\beta_1 > 0, \beta_2 \geq 0$.

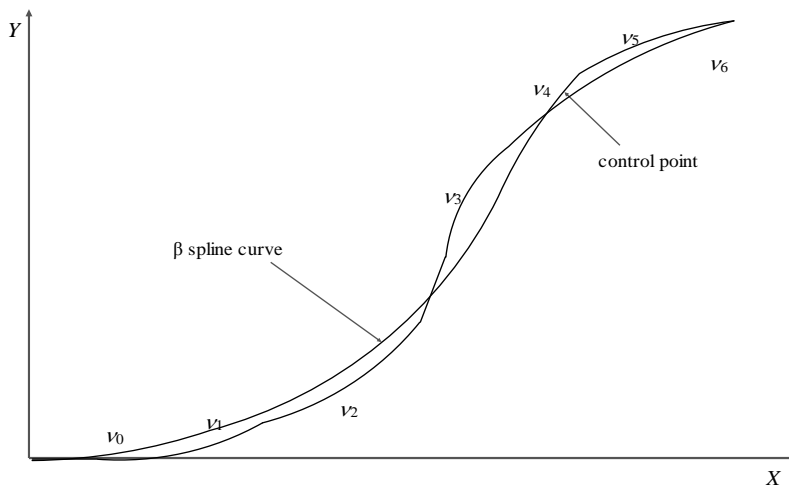


Figure 2: β spline curve and control points

3.2 Constraint condition

Due to the β spline curve with the advantages of sectional treatment, the segment o only be determined by the adjacent four control points, therefore, the change of characteristic polygon a vertex, will only influence and the vertex of the adjacent 4 segment curve and other parts of the curve does not change. At the same time, the shape of the curve is not moved to control the vertex and only change the value of the shape parameter β_1 and β_2 , can achieve the purpose of modifying the curve. This property shows that the β spline curve e is more flexible and more widely used. Applying the β spline curve to produce the desired trajectory has the following constraints.

3.2.1 Vehicle corner constraint

As shown in Figure 3, the turning radius of the vehicle is known as the turning radius of the vehicle, which is referred to as the distance from the center to the outer turning wheel. The radius of turn is smaller, the location that then motor turning needs is smaller, its mobility is better. The current steering angle is the maximum φ_{max} , minimum turning radius is R . The relationship between the minimum turning radius R_{min} and φ_{max} , in the ideal case:

$$R_{min} = \frac{l}{\tan\varphi_{max}} \quad (2)$$

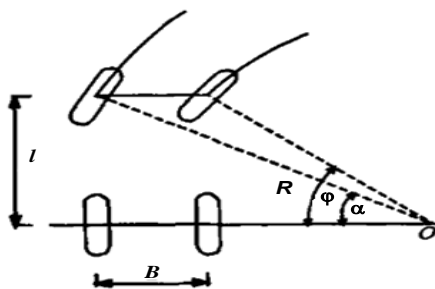


Figure 3: Relationship between deflection angles of steering wheel

In order to study the convenience, ignores regarding α angle and β angle's difference, the unification regards as is an angle. Literature (Yang, 2005) proposed in the vehicles model the biggest front wheel deflection angle is 25.9° , and calculates the smallest radius of turn of vehicles according to the different vehicle type's spread of axis. The small vehicle minimum turning radius through the computation is 5~10 m, namely the maximum curvature value is $(0.1\sim 0.2) \text{ m}^{-1}$. In the actual operation of the vehicle, the change of the curvature radius is continuous. Therefore, the curvature of the trajectory curve should be continuous. Therefore, it is demanded that the β spline curve function has two order continuous and differentiable properties. So there is $q_{i+1}''(0) = q_i''(1)$, $q_{i+1}'(0) = q_i'(1)$, $\rightarrow \beta_1 = 1, \beta_2 = 0$.

3.2.2 Vehicle driving behavior restriction

By vehicle operating characteristics, the curvature of the starting point of the trajectory is zero. The calculation formula of curvature is as follows:

$$\kappa = \frac{|q''|}{(1+q'^2)^{3/2}} \quad (3)$$

Therefore, the second derivative is zero, which is be derived from $\kappa=0$, namely $q_1''(0) = q_n'(1) = 0$. So the boundary conditions are (Hu,2015):

$$\begin{cases} 2\beta_1^3 v_0 - (2\beta_1^3 + 2\beta_1^2 + \beta_2)v_1 + (2\beta_1^2 + \beta_2)v_2 = 0 \\ (2\beta_1 + \beta_2)v_n - (2\beta_1 + \beta_2 + 2)v_{n+1} + 2v_{n+2} = 0 \end{cases} \quad (4)$$

3.2.3 Vehicle angular change rate constraint

According to figure 1, assuming that the curvature change is linear, there is:

$$\frac{dk}{ds} = \frac{\kappa_{max}}{\lambda} \quad (5)$$

When curvature is maximum, λ is longitudinal displacement of the vehicle. Because of $v = \frac{dk}{dt}$, curvature change rule along the spline curve is:

$$\dot{\kappa} = \frac{d\kappa}{dt} = \frac{d\kappa}{ds} \cdot \frac{ds}{dt} = \frac{\kappa_{max}}{\lambda} \cdot v \tag{6}$$

By vehicle turning characteristics $\kappa = \frac{1}{R} = \frac{\tan\varphi}{l}$, Type: φ is for front wheel angle.

$$\dot{\kappa} = \frac{1+\tan^2\varphi}{l} \dot{\varphi} \tag{7}$$

Type: $\dot{\kappa}$ is the maximum angular acceleration of the wheel. Each point of view φ corresponds to a maximum curvature change rate $\dot{\kappa}_{max}$. Therefore, there is the following relationship:

$$\dot{\kappa}_{max} = \frac{1+\tan^2\varphi}{l} \dot{\varphi}_{max} \geq \frac{\dot{\varphi}_{max}}{l} \tag{8}$$

So, along the β spline curve, the vehicle can satisfy the change rate of curvature $|\dot{\kappa}| \leq |\dot{\varphi}|/l$, and then there is $v \leq \lambda \cdot \frac{\dot{\varphi}_{max}}{1-\kappa_{max}}$. Type: λ is the distance in the X direction which vehicle run along the curve, when the curvature changes from the zero to the maximum. And because under the limit state, when κ reach maximum, namely immediately decreases, and the approximate $\lambda_{max} = L/k$, k value obtained from experimental observation, scope for (3.5, 3.5). In conclusion, the actual values of $\lambda : v \cdot l \cdot \kappa_{max} / \dot{\varphi}_{max} \leq \lambda \leq L/k$, So the relationship between the speed of the vehicle and the distance of the lane is deduced:

$$L \geq \frac{k \cdot v \cdot l \cdot \kappa_{max}}{\dot{\varphi}_{max}} \tag{9}$$

3.3 Trajectory solution

In this paper, the curvature change has been assumed to be linear, after the curvature reaches the maximum value, reduces immediately, and neglected the curvature is zero stage, only as spurt value consideration. The lane changing process is divided into 4 stages, each stage corresponding to a section of the spline curve, so the lane changing of the desired trajectory is composed of 4 segments. From the basic characteristics of spline curves, the 4 section curves will produce 5 nodes and 7 control points. The solving process is divided into 2 steps: ①combined with boundary condition and node equation established equations solving the coordinates of control points; ②on the basis of the characteristics of the driveway transform selection reasonable β_1 and β_2 values, to calculate trajectory curve generation into the formula. Given the 4 section curves, 5 nodes are $P_0, P_1, P_2, P_3,$ and P_4 . And: $P_0(0,0), P_4(L,N)$ is the beginning and end point; $P_2(L/2,N/2)$ is vehicles and lane boundary intersection, $P_1(\lambda, \gamma), P_3(L-\lambda, N-\gamma)$ is the curvature maximum value spot, by the massive observation data knew that the y value range is $(N/8, N/4)$; L is the traffic lane transformation length; N is traffic lane width.

Suitable trajectory parameter L, N, λ and γ are selected, and the Mat-lab program is written to get the control polygon, as shown in Figure 4, and the lane changing is expected to run the trajectory curve, as shown in Figure 5, now take $\beta_1=1, \beta_2=0$. Figure 4 is the control polygons which are composed of 7 control points. Figure 5 is expectation movement path spline curve based on the traffic lane transformation, and can be seen from the graph that the trajectory curve is smooth, continuous and no inflection point.

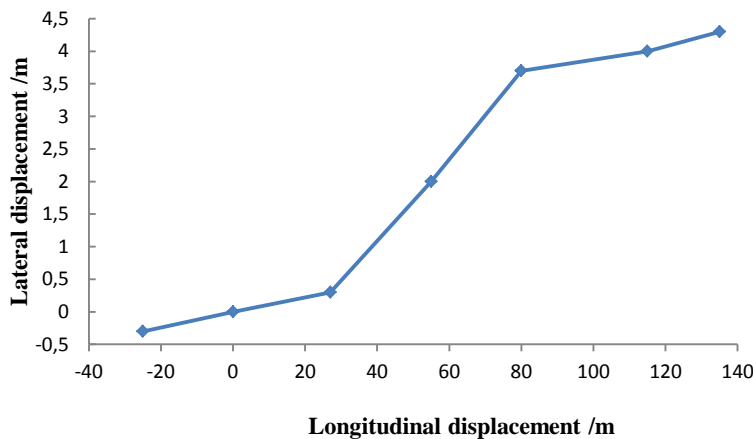


Figure 4: Trajectory curve control polygon

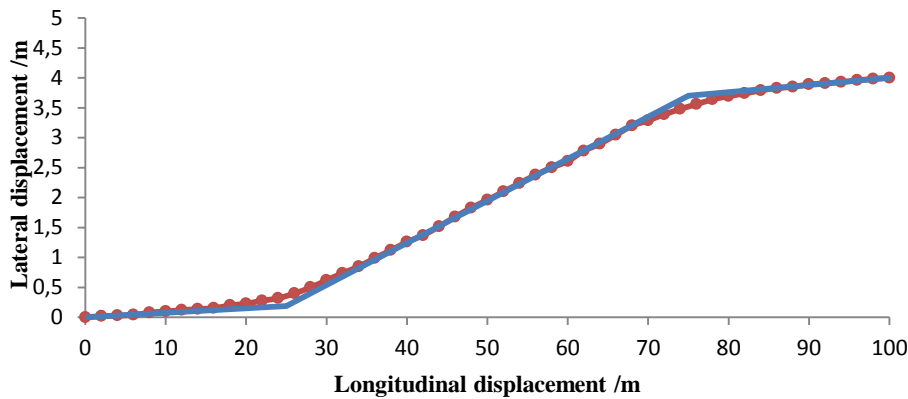


Figure 5: Expected trajectory curve

4. Example

A video file with typical lane changing behavior is selected as the raw data to test the curve. Through VTC system which is self-dependant developed by research group, can be extracted the coordinates of the ground trace when the vehicle is transformed from the video file.

In the actual observation, the lane change is divided into left and right side. In the program, $N > 0$ is left to the left, $N < 0$ to the right side of the road. According to the selected typical lane change form, here is $N = -3.25m$. Finally, the fitting effect is shown in Figure 6, the visible β spline curve can be well approximated by the actual lane changing operation.

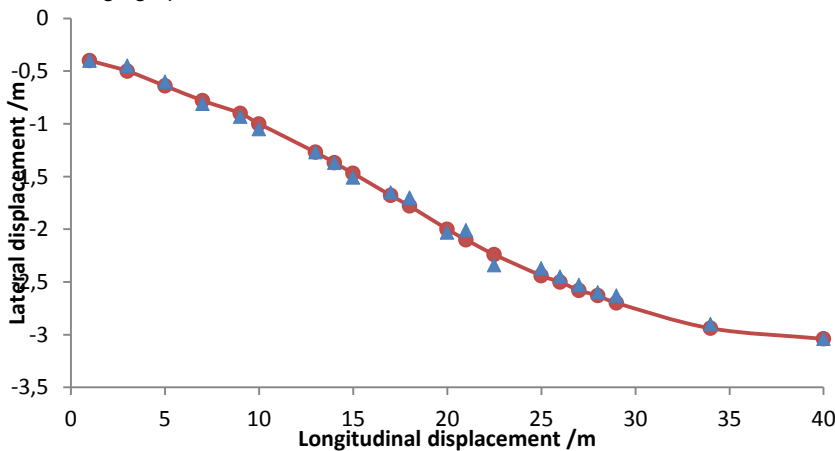


Figure 6: Spline curve fitting effect

Through the SPSS statistical analysis software, the error is analysed by using the single sample t test. The goal is the use the sampled data from overall, infers this overall the average value whether has the remarkable difference with the established examination values, namely confirmation error $\bar{e} = 0$ average value. First proposed that zero supposed H_0 is: The error mean and examination value do not have the remarkable difference, the indication is $u = u_0 = 0$, Choice significance fiduciary level $\alpha = 0.05$, software statistical analysis like Table 1 listed.

Table 1: Erroneous sample t-test result

	t-value	Freedom	Double tail probability p	Mean difference	95% confidence intervals for mean and test values	
					Minimum value	Maximum value
e	-0.037	24	0.971	-0.001	-0.032	0.031

The problem is based on double tail test, so the comparison between $\alpha/2$ and $p/2$, which is the comparison of α and P . Observation table 1, due to the $P = 0.971$, $\alpha = 0.05$, and the average error is only 0.1%, so it should not be rejected zero hypothesis, that the error means and 0 have no significant difference. 95% confidence intervals explained: Possible error's average value has 95% probabilities to fall in $-0.032 \sim 0.031$. The 0 value is included in the confidence interval, which also confirms the above inference.

5. Conclusion

In this paper, the β spline curve is introduced into the desired trajectory of lane changing. The feasibility and rationality of the model are verified by theoretical and error analysis. The curve engagement is smooth, and does not have the inflection point, the path curve that this method extracts better tallies with the actual situation, and spline curve equation may change into the matrix pattern, can be easily solved.

This article considered that the pilot is not interfered by other vehicles and road environment conditions, the cars have freedom to choose driveway transform expected trajectory. The control polygon is obtained by the node inverse control point, and the desired trajectory curve is obtained. In the actual operation of the vehicle, according to the different situations, such as the restriction of the collision point and the obstacle point, the control polygon is first determined, and then extracts conforms to the actual traffic circulation condition path curve. This curve is flexible, may be suitable for the different traffic condition.

However, this paper set up the driveway transform expected trajectory there exist the following problems:

- (1) The desired trajectory curve of this article is applied to the behavior of the vehicle at high speed.
- (2) Cars running on curve in the speed should be a change, but this article is a constant. Model in the process of building the speed change on the influence on the result, this aspect needs to be further discussed.

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