CHEMICAL ENGINEERING TRANSACTIONS
VOL. 51, 2016

A publication of
AIDIC
The Italian Association
of Chemical Engineering
Online at www.aidic.it/cet

Guest Editors: Tichun Wang, Hongyang Zhang, Lei Tian

ISBN 978-88-95608-43-3; **ISSN** 2283-9216

DOI: 10.3303/CET1651157

Linear Bayesian Inference Theory Applied in Complex Analysis of Economic Forecasting and Management

Chunyang Bai

Henan Institute of Science and Technology, Henan, Xinxiang 453003, China
happylover618@126.com

This paper systematically studied the Bayesian theory and methods of modern economic management in the multivariate models, including three linear models: single equation model, multi-equation model systems and Bayesian vector VAR (p) model. Prediction tectonic theory and its application were more general Bayesian classification methods. Risk decision solutions random array parameters, Bayesian study single equation model estimation theory, including the design matrix singular linear model coefficients assumptions Bayesian test method. Random error sequence Bayesian diagnostic unit root test autocorrelation. At the same time, the Bayesian single equation model is applied to quality control, constructing Bayesian variance with mean and variance control chart the mean standard deviation unknown a Bayesian control charts. VAR model and Bayesian VAR model effects were compared, the results showed that the effect of prediction Bayesian VAR model is better than other types of predictive models.

1. Introduction

With the application of statistical theory and methods, Bayesian theory was welcomed and has been developing rapidly, especially in decision-making problems. It occupy an increasingly important position in statistical applications, and decision-making issues, first using the experience of knowledge is essential. Compared with purely theoretical problems, such problems are often subjective probabilities formulation more natural, while reflecting the degree of decision-makers access to information, and therefore easy to accept the Bayesian point of view (Miller et al., 2013). Bayesian that: prior distribution reflects the understanding of the overall parameters of the test before the distribution, after obtaining sample information, people of this understanding has changed (Buteler et al., 2009). The result is reflected in the posterior distribution is a combination of the posterior distribution prior distribution parameters and sample information. It can be seen that the frequency of school statistical inference is "from scratch" process before a trial on the case of unknown parameters is ignorant, and after the test is somewhat understood, but there is no common understanding of how much presentation methods in practice depends on the statistics used by the targeted (Kelly et al., 2013). Bayesian inference is not, it is a "from there to the" process, and the results clearly natural, in line with people's habits of mind obtained according to an information revise its previous opinion, not necessarily from scratch (Gruber et al., 2016).
From highly theoretical point of view, we must pay attention to such a basic point: do not have the statistical inference is inference under incomplete information (Weaver et al., 2009). The information available is not enough to determine the unique solution of the problem, which provides the establishment of more possible kinds of theoretical system and methods (Krolzig et al., 2013). In fact, in addition to the frequency and Bayesian school, there are other methods based on theory and ideology of these have failed to develop into a school with two rival school, mainly due to apply too narrow, or fail to develop a set of common, there is inherent harmony of theory and square feet like (Wikle et al., 2015). However, these theories, in some cases also have successfully applied the two schools have their successes and deficiencies, there are broad prospects for development, it is practically complementary.
This article will explore the structure of the model parameter prior distribution, including diffusion and conjugate prior distribution structure prior distribution. Risk decision solutions random array parameters, Bayesian study single equation model estimation theory, including the design matrix singular linear model

Please cite this article as: Bai C.Y., 2016, Linear bayesian inference theory applied in complex analysis of economic forecasting and management, Chemical Engineering Transactions, 51, 937-942 DOI:10.3303/CET1651157

coefficients assumptions Bayesian test method, and random error sequence Bayesian diagnostic unit root test autocorrelation. At the same time, the Bayesian single equation model is applied to quality control, constructing Bayesian variance with mean and variance control chart the mean standard deviation unknown a Bayesian control charts. Bayesian theory constructed more general classification methods. Will be based on sufficient statistics to study various parameters generally follow a normal distribution, the prior distribution parameters were taken to diffuse prior distribution and normality when conjugate prior, how to use the forecast to be sentenced density function of the sample, after construction posterior probability ratio and the corresponding classification rule, treat accordingly sentenced to classify the sample.

2. Bayesian inference theory

2.1 Prior distribution

The conjugate distribution Bayesian analysis commonly used in another type of prior distribution, its ideological foundation is the law of a priori and a posteriori laws consistent, concrete is the prior distribution and posterior distribution of this requirement to belong to the same distribution family (Lenk et al., 2013). For each specific distribution, it has its conjugate distribution, use the following formula like factoring and sufficient statistics and other natural function analysis method to construct the conjugate prior distribution required herein (D'Agostino et al., 2016).

$$\prod_{i=1}^{n} f\left(\theta | Y_i\right) = g_n\left(t_n, \theta\right) h\left(Y_1, Y_2, \cdots, Y_n\right) \; DD = (V-D)/V_\sigma \tag{1}$$

If the definition of risk function

$$R(\theta,\delta) \hat{=} E_\theta\left[L\left(\theta,\delta(Y)\right) dP_\theta(Y)\right] = \int_Y L\left(\theta,\delta(Y)\right) dP_\theta(Y) \tag{2}$$

Then called $R(\delta) \hat{=} \int_\Theta R(\theta,\delta)\pi(\theta)d\theta$ is Bayes risk. Bayesian decision function and Bayesian posterior decision function is equivalent to, even if the minimum posterior risk decision function also makes Bayes minimum risk (D'Agostino and Graefe, 2013). On the contrary, the minimum risk Bayes decision function also makes posteriori minimum risk, therefore, according to the posterior risk criteria to seek solutions to Bayesian decision (Capinera et al., 2015).

2.2 The Bayesian estimation theory

Assuming random variables x1, x2, ...,, xm follows a linear relationship exists between y and independent variables:

$$\begin{aligned} & y_1 = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_m x_m + \varepsilon_1, \\ & \varepsilon_1 \sim i.i.d\ N\left(0,\sigma^2\right), i = 1,2,\cdots,n \end{aligned} \tag{3}$$

The aforementioned single equation model using the following matrix form:

$$Y = X\beta + \varepsilon, rank(X) \sim N\left(0,\sigma^2 I_n\right), \sigma^2 > 0 \tag{4}$$

In it: $$X = \begin{pmatrix} 1 & x_{11} & x_{21} & \cdots & x_{m1} \\ 1 & x_{12} & x_{22} & \cdots & x_{11} \\ & \cdots & \cdots & \cdots & \\ 1 & x_{1n} & x_{2n} & \cdots & x_{mn} \end{pmatrix} Y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}, \beta = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_m \end{pmatrix}, \varepsilon = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{pmatrix} \tag{5}$$

Where increased matrix X column full rank condition; before making the following analysis, this paper first presents the definition required for the introduction of the use of multivariate t distribution (Knodel et al., 2009).

2.3 The correlation coefficient of the posterior distribution

Before the autocorrelation coefficient as Φ Bayesian analysis should be subject to a given prior distribution of unknown parameters in the model. Obviously, for a given β and σ, in the absence of a priori information, the autocorrelation coefficients can be considered closed interval must obey uniform distribution [-1,1] on. Joint prior distribution density function is:

$$\pi(\phi,\beta,\sigma)=\pi(\phi|\beta,\sigma)\pi(\beta,\sigma)\propto 1/\sigma \tag{6}$$

According to Bayes' theorem, parameter was Φ, β and σ. The joint posterior density function

$$\begin{aligned}&\pi(\beta,\sigma|Y,X)\propto L(\phi,\beta,\sigma)\cdot\pi(\phi,\beta,\sigma)\\&\propto\frac{1}{\sigma^n}\exp\left\{-\frac{1}{2\sigma^2}\left\|Y_2-\phi Y_1-X_2\beta+\phi X_1\beta\right\|^2\right\}\end{aligned} \tag{7}$$

Their specific expression is as follows:

$$\begin{cases}c_1(\beta)=(Y_1-X_1\beta)^T(Y_1-X_1\beta)\\c_2(\beta)=(Y_2-X_2\beta)^T(Y_2-X_2\beta)\\c_3(\beta)=(Y_1-X_1\beta)^T(Y_2-X_2\beta)\\\mu_\phi(\beta)=c_3(\beta)/c_1(\beta)\end{cases} \tag{8}$$

3. Experiments and results

3.1 Bayesian model simulation analysis

In the prior distribution parameter settings, the principle is that when parameters are determined at a certain value (such as zero value), the model parameters close to this value to determine the orientation and not locked, as long as the necessary data to support, then this method can get a more accurate estimate.
After the model parameters in order to obtain the Bayesian estimation, we can calculate the formula to get the exact model parameters conditional posterior distribution. In practice, we can use simulation methods to obtain approximate posterior distribution of their condition, which can avoid cumbersome and complex Bayesian calculations more quickly and easily obtain the parameters' Bayesian estimation. Here we use MCMC methods and WinBuGS packages using WinBUGS packages AR (2) model of Doodie Bayesian analysis (Figure 1) as follows:

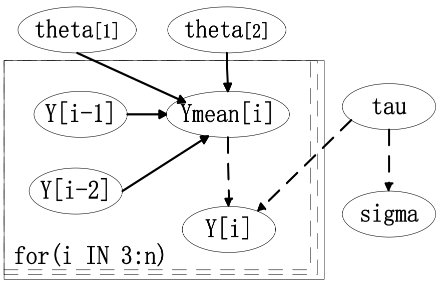

Figure 1: Model Doodle

During the analysis of the model, MCMC convergence diagnosis is very important, must not simply through a lot of iterations of iterations as the pre-simulation. Analyzing convergence in terms of, WinBuGs was through the chain of multi-parameter iterative analysis. Determine that the multi-input set of initial values to form a multilayer iterative chain, when the parameters of the model convergence, tend to coincide with the iteration graphical results.
We use a time-series simulation MA to carry out our Bayesian analysis using SAS software to generate 500 data from a model simulation of MA (2) model:

$$y_t=u_t-0.5u_{t-1}-0.3u_{t-2} \tag{9}$$

Wherein ut-N (0,1) of data listed in Table 1.

Table 1: MA(2) model simulation

obs	y				
1	1.4837	1.3622	0.1142	-0.4396	0.4735
6	-0.3714	-0.4030	-0.3972	-0.25687	0.3621
11	0.1684	-0.3824	-0.1127	0.2107	1.1184
16	1.1388	-0.4293	2.3779	1.2268	0.4808
21	-0.8252	-1.7040	-0.7217	-0.6414	0.6162
…	…	…	…	…	…
476	-1.5981	1.7895	-0.6281	0.6379	0.8349
481	0.8521	-0.7006	-0.5161	-0.7638	0.6658
486	0.3104	-0.5994	-1.7979	-2.6976	-0.6203
491	-1.4563	0.4837	1.3188	1.1554	-0.1597
496	1.2030	-0.4362	0.2017	0.42571	-0.19740

3.2 Numerical examples

Sims used the likelihood determine the optimal lag order model number ratio test statistic, model VAR (p1) model VAR (P2) of the likelihood ratio test statistic is:

$$LR = \left(N - \tilde{N} \right)\left(\ln\left|\hat{\Omega}_{P_1}\right| - \ln\left|\hat{\Omega}_{P_2}\right| \right),\ p_1 < p_2 \qquad (10)$$

Here are the Ώ model VAR (P1) and VAR (P2) residual covariance matrix, N is the sample size, N is the correction factor term, which is equal to VAR (p2) factor regression model number. Table 2 lists the VAR model 3rd order lag lag fifth-order hypothesis test results, test results in favor of shorter lag order; and the third-order lag lag order 4 hypothesis testing, test results in favor of 3 lags number, so after determining the VAR model of order 3.

Table 2: The LR test result of optimal delay degree

	H0:p=3 $\longleftrightarrow$ H1:=5		H0:p=3 $\longleftrightarrow$ H1:=4	
	LR	Probability	LR	Probability
GDP growth model	10.89	0.281	29.57	0.002
Inflation Model	9.21	0.423	34.92	0.000
Import & Export Type model	13.56	0.346	23.47	0.019

Bayesian vector autoregression VAR (p) prediction model and its application in economic forecasting, including Tony Bayes restriction VAR (p) model predictive inference and non-limiting VAR (p) model prediction Bayesian inference, the structure Mirmesota total reel prior distribution and the prior distribution analysis Bayesian inference under VAR (p) forecasting model, combined with empirical analysis and forecasting of AR model, VAR model and Bayesian VAR model effects were compared, the results showed that: the effect of prediction Bayesian VAR model is better than other types of predictive models.

3.3 Evaluation of the results and the accuracy of the prediction model

After obtaining the estimated model parameters, you can use Bayesian VAR (p) model yt prediction analysis step of predicting a specific prediction step AR model and VAR model is basically the same, may be a step ahead forecast, prediction or two steps ahead step ahead forecast. Model prediction accuracy can be evaluated from many aspects, such as the absolute mean square error, mean absolute percentage error, U Theil statistic this paper to analyze the predictive accuracy of Bayesian VAR (p) model, U statistic is based on the root mean square error of prediction model research-based ratio of root mean square error of prediction random walk model, namely:

$$\hat{U} = \frac{\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(F_{M,i} - A_i\right)^2}}{\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(F_{RW,t} - A_t\right)^2}} \tag{11}$$

Where At represents the time series at the actual value t, where,, represents the predicted value based on the prediction method M, and where the foot is based on random walk model predictions. If the value of U is equal to 1.0, then the accuracy of the prediction method of prediction accuracy M random walk model is basically the same; if the value of U is less than 1.0, then the prediction accuracy of the prediction accuracy of the method M is more than the random walk model high: If the value of U is greater than 1.0, then the accuracy of the prediction method of material low prediction accuracy than the random walk, then the method cannot improve the prediction accuracy of time series, which is also not suitable for prediction purposes. Simulation of ɵ1and ϕ1 posterior distribution is shown in Figure 2.

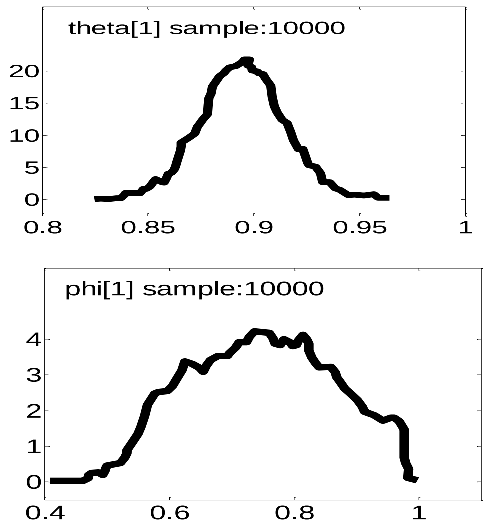

Figure 2: Simulation of ɵ1and ϕ1 posterior distribution

4. Conclusions

Risk decision solutions random array parameters, Bayesian study single equation model estimation theory, including the design matrix singular linear model coefficients assumptions Bayesian test method, and random error sequence Bayesian diagnostic unit root test autocorrelation. At the same time, the Bayesian single equation model is applied to quality control, constructing Bayesian variance with mean and variance control chart the mean standard deviation unknown a Bayesian control charts. Bayesian theory constructed more general classification methods. Will be based on sufficient statistics to study various parameters generally follow a normal distribution, the prior distribution parameters were taken to diffuse prior distribution and normality when conjugate prior, how to use the forecast to be sentenced density function of the sample, after construction posterior probability ratio and the corresponding classification rule, treat accordingly sentenced to classify the sample.

Reference

Buteler M., Weaver D.K., Peterson R.K.D., 2009, Oviposition behavior of the wheat stem sawfly when encountering plants infested with cryptic conspecifics, Environmental Entomology, 38, 1707–1715, pmid:20021767, DOI: 10.1603/022.038.0624.

Capinera J.L., 2005, Relationships between insect pests and weeds: an evolutionary perspective, Weed Science, 53, 892–901, DOI: 10.1614/ws-04-049r.1.

D’Agostino A., Giannone D., Lenza M., Modugno, M., 2016, Nowcasting Business Cycles: A Bayesian Approach to Dynamic Heterogeneous Factor Models, Dynamic Factor Models (Advances in Econometrics, Volume 35) Emerald Group Publishing Limited, 35, 569-594.

D'Agostino A., Giannone D., Lenza M., Modugno, M., 2015, Nowcasting Business Cycles: a Bayesian Approach to Dynamic Heterogeneous Factor Models, Available at SSRN.

Graefe A., Küchenhoff H., Stierle V., Riedl B., 2015, Limitations of Ensemble Bayesian Model Averaging for forecasting social science problems, International Journal of Forecasting, 31(3), 943-951.

Gruber L., West M., 2016, GPU-accelerated Bayesian learning and forecasting in simultaneous graphical dynamic linear models, Bayesian Analysis, 11(1), 125-149.

Kelly R. A., Jakeman A. J., Barreteau O., Borsuk M. E., ElSawah S., Hamilton S. H., ... Van Delden H., 2013, Selecting among five common modelling approaches for integrated environmental assessment and management, Environmental modelling & software, 47, 159-181.

Knodel J.J., Beauzay P.B., Eriksmoen E.D., Pederson J.D. 2009, Pest management of wheat stem maggot (Diptera: Chloropidae) and wheat stem sawfly (Hymenoptera: Cephidae) using insecticides in spring wheat. Journal of Agricultural and Urban Entomology 26, 183–197, DOI: 10.3954/1523-5475-26.4.183.

Krolzig H. M., 2013, Markov-switching vector autoregressions: Modelling, statistical inference, and application to business cycle analysis, Springer Science & Business Media, 454

Ledenyov D. O., Ledenyov V. O., 2015, On the tracking and replication of hedge fund optimal investment portfolio strategies in global capital markets in presence of nonlinearities, applying Bayesian filters: 1. Stratanovich–Kalman–Bucy filters for Gaussian linear investment returns distribution and 2. Particle filters for non-Gaussian non-linear investment returns distribution. Stratanovich–Kalman–Bucy Filters for Gaussian Linear Investment Returns Distribution and, 2.

Lenk P. J., Allenby G. M., Rossi P. E., 2013, When BDT in Marketing Meant Bayesian Decision Theory: The Influence of Paul Green’s Research. Marketing Research and Modeling: Progress and Prospects: A Tribute to Paul E. Green, 14, 16.

Miller Z.J., Menalled F.D., Burrows M., 2013, Winter annual grassy weeds increase over-winter mortality in autumn-sown wheat, Weed Research, 53, 102–109, DOI: 10.1111/wre.12007.

Weaver D.K., Buteler M., Hofland M.L., Runyon J.B., Nansen C., Talbert L.E., Lamb P., Carlson G.R., 2009, Cultivar preferences of ovipositing wheat stem sawflies as influenced by the amount of volatile attractant, Journal of Economic Entomology, 102, 1009–1017, pmid:19610414, DOI: 10.1603/029.102.0320

Wikle C. K., Stroud J. R., Katzfuss M., 2015, Approximate Inference for High-Dimensional Spatio-Temporal Systems Using the Ensemble Kalman Filter.