

# Sequential Initialisation Technique for Synthesising Multi-Period Multiple Utilities Heat Exchanger Networks

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This paper presents a new method for synthesizing heat exchanger networks involving multi-period operations and multiple utilities. Since process parameters, such as streams' flowrate and supply/target temperatures, in multi-period operations usually fluctuate around some nominal conditions, the synthesis models developed for their optimisation need to ensure that the heat load requirement of every stream in every period of operation is satisfied. Solving problems of this nature is not a trivial task due to the presence of multiple periods of operations. The complexity is further compounded in situations where multiple options of utilities are involved in the optimisation task. Hence in this paper, a new synthesis method which involves firstly, solving single period models for each period of operation in the problem, after which the selected matches from each of the single period solution are systematically used to initialise the multi-period superstructure. Contrary to existing methods where utilities are placed or appended only in the first and last stage of the multi-period superstructure model, the newly presented method positions utilities in all stages of the superstructure where process streams of the opposite kind exist. The new approach is applied to two examples and the solutions obtained demonstrate the benefits of the method.

## 1. Introduction

Energy using processes such as those found in chemical and petroleum industries are under pressure to reduce the amount of energy consumed in their operations. This is due to the highly uncertain costs and availability of fossil based fuels. One other key reason for which energy usage should be reduced, especially those that are fossil based, is the associated environmental impact which is gradually having its toll on the world's environment. A large number of studies have been carried out on ways of achieving the aforementioned reductions in energy usage and one of them is through designing efficient networks of heat exchangers. However, most papers presented in this area have been based on the assumption that process operating parameters are fixed, as done in the paper presented by Short et al. (2015). However, in reality, process stream parameters do vary within certain ranges due to issues such as changes in environmental conditions, plant start-ups/shut downs, changes in feed/product quality, etc. Hence, this implies that heat exchanger network synthesis (HENS) design methods need to be capable of handling the aforementioned changes in process operating conditions. Networks, that fulfil these criteria, are called multi-period networks. The work of Verheyen and Zhang (2006) addressed the synthesis of multi-period heat exchanger networks using a modified version of the stage-wise superstructure (SWS) of Yee and Grossmann (1990). Isafiade and Fraser (2010) also presented an approach for solving multi-period problems using their interval based superstructure approach. Other studies which addressed the multi-period design problem are that of Kang et al. (2015), where a 2-step approach was used, and that of Jiang and Chang (2015), where the time sharing scheme was used. Although these methods addressed the multi-period HENS problem, however, they are limited in that they considered only one option of utility, and this utility was placed or appended to the first and last intervals (for hot and cold utilities respectively) of the stage-wise superstructure. The work of Isafiade et al. (2015), which also used the multi-period SWS model, however considered multiple utilities in solving HEN multi-period problems. This approach was limited due to the fact that it placed all hot utilities only in the first interval of the multi-period superstructure and all cold utilities only in the last interval. In as much as this

superstructure definition approach has the benefit of simplifying the solution generation process by virtue of the fact that the number of potential discrete variables are reduced, its main drawback is that for cases where the multiple utilities have significantly different temperature ranges, they will not have the opportunity to exchange heat with streams of the opposite kind through an optimal use of driving forces. Hence in this paper, a new approach which positions every utility in all intervals of the multi-period superstructure where process streams of the opposite kind exist, is presented. The authors of this paper are aware that this superstructure definition approach has the tendency to increase the computational intensity involved in solving the problem, as it will result in an increase in the degree of non-linearity of the model, as well as increase in the number of discrete variables. However, in order to circumvent this potential problem, a systematic initialisation procedure, which is newly generated in this paper is adopted.

## 2. Problem statement

The problem solved in this paper can be stated as follows: Given a set HP of hot process streams, and a set CP of cold process streams, with their supply temperatures, target temperatures and flowrates at specified P periods of operations. The hot and cold streams have to be cooled and heated respectively. Given also are a set of hot (HU) and cold (CU) utilities which can be used in any of the specified periods of operations. The task is to synthesize a heat exchanger network which would satisfy the heat demand of all process streams in all periods of operations while using utilities and heat exchangers in a cost effective manner.

## 3. Model development

Figure 1 shows the modified multi-period stage-wise superstructure used in this paper. This representative superstructure involves 2 hot process streams, 2 cold process streams, 2 hot utilities and 2 cold utilities. The process streams are represented as thick lines while the utility streams are represented as dashed lines in the figure. What is unique to this superstructure is that hot utilities 1 and 2 are both made to participate in stages 2, 3 and 4, where cold process streams 1 and 2 exist. Also, cold utilities 1 and 2 are made to participate in stages 1, 2 and 3 where hot process streams 1 and 2 are present. However, the hot and cold process streams run from their starting intervals, as shown in the figure, to the end of the superstructure, while the utilities may or may not cross intervals. The utilities will not cross intervals in cases where they do not have a significant temperature change, e.g. steam, while they will cross intervals in cases where they have significant temperature change, e.g. hot oil. This new representation of multi-period stage-wise superstructure implies that for the specific case of Figure 1, process heat exchangers can only exist in stages 2 and 3 (represented as two black circles connect with a line) while utility exchangers (represented as two white circles connect with a line) can exchange heat in all stages of the superstructure (stages 1, 2 and 3 for cold utilities and stages 2,3 and 4 for hot utilities). The same assumption of equal temperature mixing for split streams, as presented by Yee and Grossmann (1990) for the single period scenario, is also adopted in this paper for both single and multi-period scenarios. It should be known that Figure 1 is illustrated for a single period case, however in order to convert it to a multi-period model, the index  $p$ , which represents each period of operation, is included in the single period model equations.

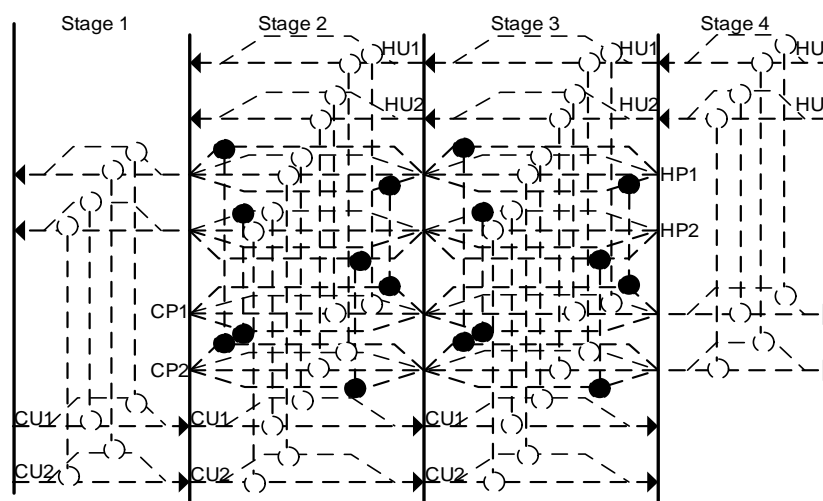


Figure 1: Modified multi-period stage-wise superstructure used in this work

Due to space limitation, the detailed set of model equations will not be shown here, however, the reader can get the details from the work of Verheyen and Zhang (2006) and later Isafiade et al. (2015). Only the objective function used is shown in this paper which is illustrated in Eqs(1) and (2).

$$A_{i,j,k} \geq \frac{q_{i,j,k,p}}{(LMTD_{i,j,k,p}) (U_{i,j})} \quad (1)$$

$$\min \left\{ \sum_{p \in P} \left( \frac{DOP_p}{\sum_{p=1}^{NOP} DOP_p} \sum_{i \in HP} \sum_{j \in CU} \sum_{k \in K} CUC_j \cdot q_{i,j,k,p} + \frac{DOP_p}{\sum_{p=1}^{NOP} DOP_p} \sum_{i \in HU} \sum_{j \in CP} \sum_{k \in K} HUC_i \cdot q_{i,j,k,p} \right) \right. \\ \left. + AF \left( \sum_{i \in H} \sum_{j \in C} \sum_{k \in K} CF_{ij} \cdot z_{i,j,k} + \sum_{i \in H} \sum_{j \in C} \sum_{k \in K} AC_{ij} \cdot A_{i,j,k}^{ACI} \right) \quad \forall i \in HP, j \in CP, k \in K, p \in P \quad (2)$$

In Equation 1,  $A_{i,j,k}$  represents the area ( $m^2$ ) of the heat exchanger that connects hot stream  $i$  and cold stream  $j$  in interval  $k$  of the superstructure,  $LMTD_{i,j,k,p}$  represents the logarithmic mean temperature difference (K) between hot stream  $i$  and cold stream  $j$  in stage  $k$  and period  $p$  of the superstructure,  $q_{i,j,k,p}$  represents the amount of heat (kW) exchanged between hot stream  $i$  and cold stream  $j$ , in period  $p$ ,  $U_{i,j}$  represents overall heat transfer coefficient between hot stream  $i$  and cold stream  $j$  ( $0.1 \text{ W/m}^2\text{°C}$ ). In Eq(2), which is the objective function,  $DOP_p$  represents duration of period  $p$  while  $NOP$  represents the number of periods,  $CUC_j$  represents cost of cold utility ( $\$/(\text{kW}\cdot\text{y})$ ),  $HUC$  represents cost of hot utility ( $\$/(\text{kW}\cdot\text{y})$ ),  $AF$  represents the annualisation factor (0.2),  $CF_{ij}$  represents the cost of installing each unit of heat exchanger (8,333.3 \$),  $z_{i,j,k}$  represents the binary variable whose purpose is to show the existence, or otherwise, of a match between any hot and any cold streams in the superstructure,  $AC_{ij}$  represents area cost of heat exchangers ( $641.7 \text{ \$/m}^2$ ) while  $ACI$  represents heat exchanger area cost index. Equations 1 and 2 show that the maximum area approach as used by Verheyen and Zhang (2006) and later by Isafiade et al. (2015) is also used in this paper. The maximum area ensures that the area used for the calculation of heat exchanger sizes in the objective function is the area of the maximum of heat exchanger areas exchanging heat between the same pair of streams existing in different periods of the superstructure for interval  $k$ . Apart from the modified multi-period stage-wise superstructure used in this paper, another unique feature of the new method is the solution approach adopted. The solution technique entails basically using a systematic approach to initialise the multi-period superstructure at various times, so as to obtain a good solution in reasonable time. This is necessary because apart from the nonlinear expressions in the model, the presence of multiple periods of operations also complicates the solution generation process. A further complexity inherent in the model is the fact that the number of discrete variables are significantly increased since the utilities are made to participate in every interval where process streams exist. However these difficulties are overcome in this paper using the solution procedure outlined next.

Generate a single period superstructure model for each period of operation using the modified single period SWS superstructure shown in Figure 1.

- i. Solve each of the single period models generated in the first step (using the same number of superstructure stages for each period) as a mixed integer non-linear program (MINLP) and identify the selected matches. Matches which are selected in more than one period should be represented only once in the list of identified matches.
- ii. Use the set of identified matches in the second step to initialise the multi-period model, which is a multi-period superstructure version of Figure 1. It should be known that despite the fact that the multi-period model at this stage is initialised with a set of matches, the model is still solved as an MINLP because the initialising matches represent the binary variables. The multi-period model at this stage should have the same number of stages as used in the second step for the individual single period models.
- iii. Systematically add or remove matches in the reduced multi-period superstructure of the third step while solving until an optimal solution is obtained.

It is worth stating that Isafiade et al. (2015) used a somewhat similar solution generation approach to that outlined above. However, their method restricted utilities to only the first and last stages of the superstructure. Also, Isafiade et al. (2015) did not solve individual single period models so as to generate the set of initialising matches, instead they solved a full multi-period model a number of times, and then identified the selected matches in about 2 or 3 of the best solutions depending on the size of the problem. The identified matches were then used to initialise the multi-period model. A key shortcoming of this approach is that since the utilities are restricted to only one interval of the superstructure, there may not be an optimal use of driving forces,

especially for intermediate utilities. Furthermore, chances of getting good solutions at the stage of solving the full multi-period model may be reduced due to the highly non-linear nature of MINLP multi-period multiple utility models.

#### 4. Examples

The newly developed method is applied to 2 examples both having equal period durations. The first example is taken from Isafiade et al. (2015) while the second example is generated in this paper so as to fully demonstrate the benefits of the new approach.

##### 4.1 Example 1

The process stream data for this example are shown in Table 1, while the cost data for its utilities are: HU1 = 70 €/((kW·y)), HU2 = 50 €/((kW·y)), HU3 = 40 €/((kW·y)), CU1 = 1.3 €/((kW·y)). Other capital cost data are presented with Equation 1. Applying the 1<sup>st</sup> and 2<sup>nd</sup> steps of the solution procedure gives the set of matches (and total annual costs, TAC) for each of periods 1, 2 and 3 shown in Table 2. Based on these set of selected matches, a new set of matches which are common to periods 1 to 3, or present in at least one of the periods, were chosen for use in the 3<sup>rd</sup> step to initialize the multi-period model. These new set of matches, known as initializing matches, are shown in the last column of Table 2. Using these set of initializing matches in the multi-period model greatly reduces the number of discrete variables, hence simplifies the solution generation process. Solving the model at this stage gave a TAC of € 4,416,756 with 13 units. This solution was obtained in 2.12 s of CPU time. The last step was then carried out by removing matches (HP1,CP1,3) and (HP1,CP2,3) and the model was solved again. At this last step, a solution with a TAC of € 4,413,854, having 9 units, was obtained in 1.67 s of CPU time. It is worth stating that exactly the same TAC and number of units was obtained by Isafiade et al. (2015), however the structures of the solution networks are different. Further excluding matches (HP2,CP2,2) and (HP3,CP1,3) does not have any effect on the solution obtained. However, if any of the other matches were excluded, the TAC goes up. This shows the importance of having the right set of matches, alongside other initializations for key variables, at the initialization stage, in solving multi-period problems in a mathematical programming environment. Systematically initializing the model at the third step, which is the step at which the multi-period model is solved, helps position the solver within the right region of finding good solutions in reasonable time. The structure obtained using the method of this paper is shown in Figure 2.

Table 1: Process stream data for Example 1

Streams	Periods								
	Period 1			Period 2			Period 3		
	T <sup>s</sup> (°C)	T <sup>t</sup> (°C)	F (kW/°C)	T <sup>s</sup> (°C)	T <sup>t</sup> (°C)	F (kW/°C)	T <sup>s</sup> (°C)	T <sup>t</sup> (°C)	F (kW/°C)
HP1	393	60	201.6	406	60	205	420	60	208.5
HP2	160	40	185.1	160	40	198.8	160	40	175.2
HP3	354	60	137.4	362	60	136.4	360	60	134.1
CP1	72	356	209.4	72	365	210.3	72	373	211.1
CP2	62	210	141.6	62	210	141.0	62	210	140.5
CP3	220	370	176.4	220	370	175.4	220	370	174.5
CP4	253	284	294.4	250	290	318.7	249	286	271.2
HU1	490	490	-	490	490	-	490	490	-
HU2	438	438	-	438	438	-	438	438	-
HU3	375	375	-	375	375	-	375	375	-
CU1	0	10							

In this figure, the representative areas (i.e. the maximum areas) for each exchanger is shown above the exchanger. It can be seen in the figure that HU3 and CU1 are both used in intermediate stages, which is stage 2. This is unlike the solution of Isafiade et al. (2015) where utilities are only used in the first and last intervals, which are the only available utility stages. In the solution of Isafiade et al. (2015), only HU2 and HU3 were used as hot utilities, while CU1 was used as cold utilities.

##### 4.2 Example 2

The process stream data for this example are shown in Table 4 while the cost data for its utilities are HU1 = 70 €/((kW·y)), HU2 = 30 €/((kW·y)), HU3 = 20 €/((kW·y)), CU1 = 1.3 €/((kW·y)), CU2 = 1.0 €/((kW·y)). Other capital cost data are presented with Equation 1. Note that this example has 2 cold utilities, unlike Example 1 which has 1 cold utility. Applying the 1<sup>st</sup> and 2<sup>nd</sup> steps of the solution procedure gives the set of matches, for each of the periods, shown in Table 3. It can be seen that the set of matches obtained for each of the periods in step 2

contains a large number of selected matches that are unique to each of the periods. Therefore, when the initializing matches were identified from the pool of matches obtained in step 2, a total of 23 initializing matches was obtained. Since this is a large number for a moderately sized problem, a systematic approach was used in reducing these set of initializing matches. The approach used entails solving a multi-period version of the problem using the modified multi-period SWS model but with all multi-period binary variables present in the model. A solution having the following matches (HU1,CP1,1), (HU1,CP3,2), (HU2,CP1,2), (HU2,CP4,2), (HU3,CP2,1), (HP1,CP2,2), (HP1,CU2,4), (HP2,CP2,3), (HP2,CU1,4), (HP3,CP1,3), (HP3,CU2,4) with a TAC of € 6,331,908 was obtained. After a series of repeated addition/elimination of matches, using the resulting selected matches from the conventional solution approach as a guide, the set of initializing matches were ultimately reduced to those shown in the last column of Table 3. Solving the multi-period model with these final set of initializing matches gave a solution having 11 units with a TAC of € 6,240,266. This solution was obtained in 0.97 s of CPU time. Solving this problem using the conventional method gives a solution having a TAC of € 7,507,781 with 10 units. The solution was obtained in 7.49 s of CPU time. Figure 3 shows the final solution network obtained for Example 2. This solution has 2 process heat exchangers, 7 heaters, and 3 coolers. It can be seen that 4 of the heaters are used in the intermediate stages. This implies that process streams such as CP1, CP3 and CP4, all have the opportunity of having their intermediate temperatures determined as a result of heat exchange with the intermediate placement of hot utilities. This would not have been possible if the utilities had been restricted to only the first and last stages of the superstructure.

Table 2: Selected matches for individual periods for Example 1

Period 1	Period 2	Period 3	Selected matches for step 3
HU2,CP3,1	HU2,CP1,1	HU1,CP1,1	HU2,CP1,1
HU3,CP4,3	HU3,CP4,3	HU1,CP3,1	HU2,CP3,1
HP1,CP1,2	HP1,CP1,2	HU2,CP1,2	HU3,CP4,3
HP1,CP1,3	HP1,CP2,3	HU2,CP3,2	HP1,CP1,2
HP1,CU1,4	HP1,CP3,2	HU2,CP4,1	HP1,CP1,3
HP2,CU1,3	HP1,CU1,4	HP1,CP1,3	HP1,CP2,3
HP3,CP1,3	HP2,CU1,2	HP1,CU1,4	HP1,CP3,2
HP3,CP2,3	HP2,CU1,3	HP2,CP2,3	HP1,CU1,4
HP3,CU1,4	HP3,CP1,2	HP2,CU2,4	HP2,CU1,2
	HP3,CP1,3	HP3,CP2,2	HP2,CU1,3
	HP3,CU1,4	HP3,CU2,3	HP3,CP1,2
	HP3,CP3,2		HP,CP1,3
			HP3,CP2,3
TAC: € 4,801,926	TAC: € 4,877,509	TAC: € 6,377,848	HP3,CP3,2
			HP3,CU1,4

Table 3: Selected matches for individual periods for Example 2

Period 1	Period 2	Period 3	Selected matches for step 3
HU1,CP1,1	HU1,CP1,1	HU2,CP3,1	HU1,CP1,1
HU1,CP3,2	HU1,CP3,1	HU3,CP4,3	HU1,CP3,1
HU2,CP1,2	HU2,CP1,2	HP1,CP1,3	HU2,CP1,2
HU2,CP3,3	HU2,CP3,2	HP1,CU1,4	HU2,CP3,3
HU2,CP4,2	HU2,CP4,3	HP2,CU1,2	HU2,CP4,2
HP1,CP2,3	HU3,CP2,1	HP2,CU1,3	HU3,CP2,1
HP1,CU1,4	HP1,CP2,2	HP3,CP2,3	HP1,CP2,2
HP2,CU2,2	HP1,CU2,3	HP3,CP3,2	HP1,CU1,4
HP3,CP2,2	HP2,CU2,2	HP3,CU1,4	HP2,CU1,4
HP3,CU1,4	HP3,CP1,3		HP2,CU2,2
	HP3,CU2,4		HP3,CP1,3
			HP3,CU1,4
TAC: € 6,390,959	TAC: € 6,461,437	TAC: € 4,636,293	

## 5. Conclusions

Presented in this paper is a new synthesis method for multi-period heat exchange network problems involving multiple utilities. The method positions every utility in the problem in every stage of the superstructure where process streams of the opposite kind exist, so that there can be an optimal use of available driving force. Using this superstructure generation approach increases the complexity of solution generation, therefore the new method uses a solution approach where the multi-period problem is initialized using selected matches from each period's optimal solution network. This new method overcomes some of the drawbacks associated with current multi-period synthesis techniques that are based on the SWS model, in that good solutions can be obtained in reasonable times. One of the main drawbacks associated with this new method is that there are no specific rules to follow concerning addition/removal of matches in the last step of the new method, hence this step may become tedious to handle for large problems.

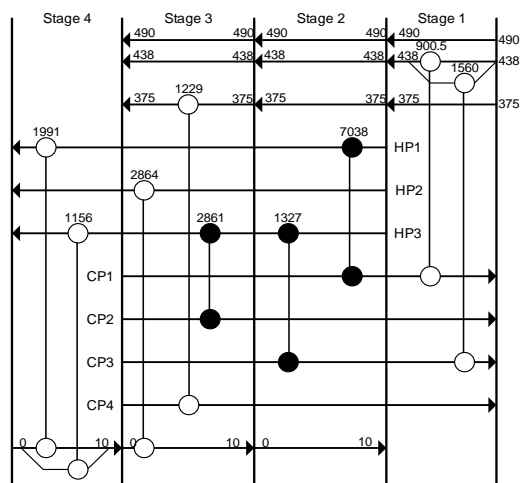


Figure 2: Final multi-period network obtained for Example 1

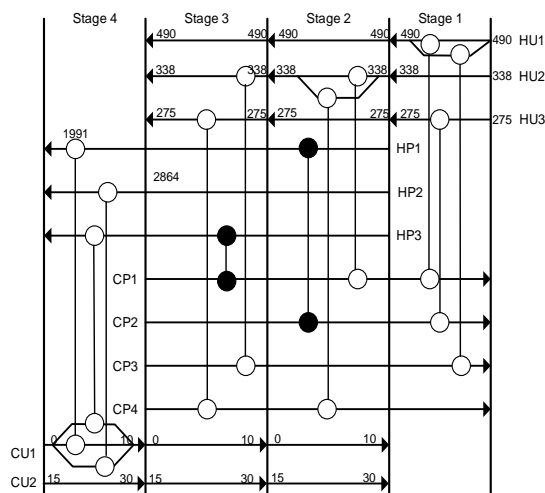


Figure 3: Final multi-period network obtained for Example 2

Table 4: Process stream data for Example 2

Streams	Periods								
	Period 1			Period 2			Period 3		
	T <sup>s</sup> (°C)	T <sup>t</sup> (°C)	F (kW/°C)	T <sup>s</sup> (°C)	T <sup>t</sup> (°C)	F (kW/°C)	T <sup>s</sup> (°C)	T <sup>t</sup> (°C)	F (kW/°C)
HP1	213	60	201.6	206	60	205	220	60	208.5
HP2	160	40	185.1	160	40	198.8	160	40	175.2
HP3	254	60	137.4	262	60	136.4	260	60	134.1
CP1	72	356	209.4	72	365	210.3	72	373	211.1
CP2	62	210	141.6	62	210	141.0	62	210	140.5
CP3	220	370	176.4	220	370	175.4	220	370	174.5
CP4	253	284	294.4	250	290	318.7	249	286	271.2
HU1	490	490	-	490	490	-	490	490	-
HU2	338	338	-	338	338	-	338	338	-
HU3	275	275	-	275	275	-	275	275	-
CU1	0	10							
CU2	15	30							

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