

# Robust Model Predictive Control of Heat Exchangers in Series

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This study investigates using of robust model based predictive control (MPC) algorithms for optimal operating of heat exchangers in series from the stability and economic viewpoints. For the advanced controller design, the influence of uncertain parameters was taken into account. In order to design the robust MPC, the optimization problem with constraints was formulated in the form of linear matrix inequalities and then the convex optimization problem was solved using the semidefinite programming. The designed robust MPC strategies were based on the worst-case optimization and on the additional control input saturation. We investigated a case study with two various significant disturbances in the temperature of the input stream in the heat exchangers in series. Results revealed that the robust MPC improved control performance and ensured energy savings during the heat exchanger network operation.

## 1. Introduction

Shell-and-tube heat exchangers (HEs) attract the interest of specialist in chemical engineering and process control. The heat losses can rise up to 50 % and therefore it is necessary to implement advanced control strategies and to optimize the operation of HEs. In (Doodman et al., 2009) a robust stochastic approach for optimization design of air cooled heat exchangers is studied and the results reveal that the harmony search algorithm converges to optimum solution with higher accuracy in comparison with genetic algorithms. The work of Vasičkaninová et al. (2011) shows that using the neural network predictive control (NNPC) structure for control of heat exchangers can lead to energy savings. The robust model predictive control (RMPC) for passive building thermal mass and mechanical thermal energy storage was designed and its features were deeply investigated in Kim (2013). In our previous work Bakošová et al. (2014) we investigated RMPC design of a heat exchanger network (HEN), and we designed an alternative RMPC procedures for HEN in paper Oravec et al. (2015). This paper investigates the use of RMPC algorithms for optimal operating of a HEN from the economic viewpoint. In order to design RMPC, the optimization problem with constraints is formulated in the form of LMIs and then the convex optimization problem is solved using the semi-definite programming (SDP). RMPC strategies are based on the worst-case scenario optimization (Kothare et al., 1996) and the additional control input saturation (ACIS, Cao et Lin, 2005). To demonstrate the effectiveness of RMPC a case study is considered. Robust stability, violation of the constraints, total energy savings, and overall computational complexity are analysed. Simulation results reveal that RMPC ensures the optimal operation of the HEN with uncertainty.

## 2. Robust MPC of heat exchanger network

Three counter-current shell-and-tube heat exchangers in series form the controlled simple heat exchanger network. The investigated HEN is a part of the kerosene hydrotreating technology in a refinery (Figure 1). Hydrogenated kerosene is the component of diesel and is produced in a reactor. The light fractions arise during the hydrogenation and have to be removed from the product.

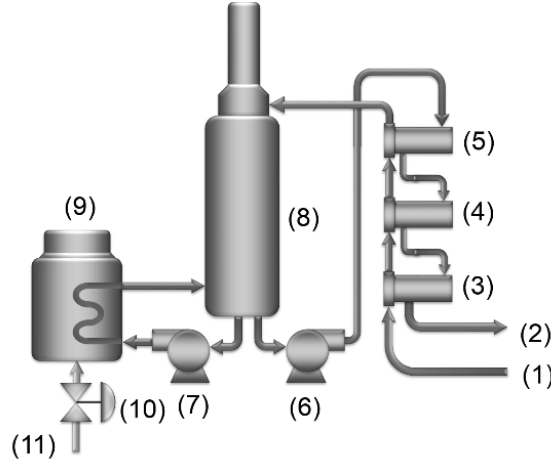


Figure 1: Scheme of the HEN: (1) input stream, (2) product, (3) 3rd HE, (4) 2nd HE, (5) 1st HE, (6) and (7) pumps, (8) stabilizer, (9) furnace, (10) valve, (11) natural gas

The hydrogenated kerosene – product has to be stabilized in a stabilizer. The stream inputs to the stabilizer from the cold separator – storage tank. The temperature of the input stream changes from 14 to 27 °C. The input stream to the stabilizer is preheated in the HEN by the product stream that leaves the bottom of the stabilizer. The other stream leaving the bottom of the stabilizer is heated in the furnace and returned to the bottom of the stabilizer. The temperature of this stream determines the temperature at the bottom of the stabilizer and also the temperature of the product. The light fractions leave the top of the stabilizer. The input stream to the stabilizer flows through the tubes of HEs, and the heating stream flows over the tubes through the shell of HEs. The tubes of the HEs are made from steel. The objective is to pre-heat the input stream to the reference value 182 °C and to minimize the energy consumption measured by the total consumption of natural gas in the furnace. The control input is the temperature at the bottom of the stabilizer, which determines the natural gas consumption in the furnace. The inlet temperature of the heating stream to the HEN is the same. The controlled output is the temperature of the input stream to the stabilizer.

The mathematical model of the HEN was derived using the heat balances under the following simplification (Ingham, 2007): the thermal capacities of the metal walls are neglected; the HEs are well insulated; heat loss to the surroundings and mechanical work effects are negligible; the technological parameters are either constant or vary in some intervals. The heat balances for the HEN lead to the six first-order ordinary differential equations given by

$$V_1 \rho_1 c_{p,1} \frac{dT_1^j(t)}{dt} = \frac{A_h U}{2} \left( (T_2^j(t) - T_1^{j+1}(t)) + (T_2^{j-1}(t) - T_1^j(t)) \right) + q_1 \rho_1 c_{p,1} (T_2^j(t) - T_1^{j+1}(t)), \quad (1)$$

$$V_2 \rho_2 c_{p,2} \frac{dT_2^j(t)}{dt} = \frac{A_h U}{2} \left( (T_2^j(t) - T_1^{j+1}(t)) + (T_2^{j-1}(t) - T_1^j(t)) \right) + q_2 \rho_2 c_{p,2} (T_2^j(t) - T_1^{j+1}(t)), \quad (2)$$

where  $T_1^i(0) = T_{1,0}^i$ ,  $T_2^i(0) = T_{2,0}^i$  are initial conditions and the superscript  $j = 1, 2, 3$ , stands for the 1st, 2nd, and the 3rd heat exchanger, respectively. The subscripts 1 and 2 indicate the heated and the heating stream, respectively. In Eqs(1)–(2),  $V$  is the volume,  $\rho$  is the density,  $c_p$  is the specific heat capacity,  $t$  is the time,  $T(t)$  is the time-varying temperature,  $q$  is the volumetric flow rate,  $A_h$  is the heat transfer area and  $U$  is the overall heat transfer coefficient. The initial conditions  $T_{1,0}^j$  and  $T_{2,0}^j$  in Case I are 169.7 °C, 119.0 °C, 67.1 °C, 180.7 °C, 130.3 °C, 78.6 °C, and in Case II are 173.6 °C, 123.6 °C, 72.4 °C, 181.4 °C, 131.7 °C, 80.7 °C. Case I and Case II represent the situations for two studied scenarios. The Case I represents the situation with a disturbance in the inlet stream. The inlet temperature of the input stream changed from 20 °C to 14 °C. The Case II represents the situation with the other disturbance in the inlet stream. The inlet temperature changed from 20 °C to 27 °C. The values of technological parameters and the steady-state values of the temperatures are summarized in Table 1. Here  $n$  is the number of the HE's tubes,  $l$  is the length of the HE,  $d_{in,1}$  is the inner diameter of the tube,  $d_{out,1}$  is the outer diameter of the tube,  $d_{in,2}$  is the inner diameter of the shell,  $T_{1,in} = T_1^4$  is the temperature of the inlet stream of the heated fluid to the 1st HE and  $T_{2,in} = T_2^0$  is the inlet temperature of the heating stream to the 3rd HE. The superscript S denotes the steady-state value, and the steady-state temperatures  $T_1^{j,S}$ ,  $T_2^{j,S}$ ,  $j=1, 2, 3$ , were computed for the inlet temperature of the heated stream  $T_1^{4,S} = 20$  °C

and of the heating stream  $T_2^{0,S} = 230$  °C from Eqs.(1)–(2) with zero derivatives. These steady state temperatures represent the reference values for control of the HEs.

Table 1: Technological parameters and steady-state values of variables in HEs

Variable	Unit	Value	Variable	Unit	Value
$n$	1	216	$A_h$	m <sup>2</sup>	89.57
$l$	m	6	$T_1^{1,S}$	°C	171.3
$d_{in,1}$	m	$19 \times 10^{-3}$	$T_1^{2,S}$	°C	122.0
$d_{out,1}$	m	$25 \times 10^{-3}$	$T_1^{3,S}$	°C	71.6
$d_{in,2}$	m	$850 \times 10^{-3}$	$T_2^{1,S}$	°C	182.0
$q_1$	m <sup>3</sup> s <sup>-1</sup>	$35.5 \times 10^{-3}$	$T_2^{1,S}$	°C	133.0
$q_2$	m <sup>3</sup> s <sup>-1</sup>	$24.0 \times 10^{-3}$	$T_2^{1,S}$	°C	82.8
$c_{p,1}$	J kg <sup>-1</sup> K <sup>-1</sup>	2570	$T_{1,in}^S = T_1^{4,S}$	°C	20.0
$c_{p,2}$	J kg <sup>-1</sup> K <sup>-1</sup>	2684	$T_{2,in}^S = T_2^{0,S}$	°C	230.0

Further, three uncertain parameters are considered in the controlled system: the overall heat-transfer coefficient and the densities of the heated and the heating stream. The values of these parameters are given in the Table 2, where  $U$  is the overall heat transfer coefficient,  $\rho_1$  is the density of the input stream, and  $\rho_2$  is the density of the product. The well-known approach of parametric uncertainties handling was used to describe the HEN in the form of a polytopic uncertain system, see (Kothare et al., 1996). Therefore, the set of eight vertex systems was generated for all variations of boundary values of three uncertain parameters (Table 2).

Table 2: Uncertain parameters of HEs.

Variable	Unit	Minimal Value	Nominal Value	Maximal Value
$U$	W m <sup>-2</sup> K <sup>-1</sup>	338	375	413
$\rho_1$	kg m <sup>-3</sup>	444	447	654
$\rho_2$	kg m <sup>-3</sup>	633	651	802

Each vertex system was described by 6 ordinary differential equations Eqs.(1)–(2). The control performance of the controlled process was investigated using these eight limit-behaviour models. The nominal system of the HEN was created for the situation with the inlet temperature of the heated stream  $T_1^{4,S} = 20$  °C and the inlet temperature of the heating stream  $T_2^{0,S} = 230$  °C. This model served as the reference system. The non-linear state-space model of the controlled process Eqs(1)–(2) was linearized for the robust controller design using the 1-st order Taylor expansion of nonlinear terms (Mikleš and Fikar, 2007), and the linear state-space model of the HEN was obtained for the nominal system and each vertex system in the form of six ordinary linear differential equations. As RMPC is a discrete-time control strategy, the linear continuous-time models were transformed into the discrete-time domain using the sampling time  $t_s = 1$  s. The value of the sampling time does not directly influence the RMPC design. It has to be chosen so that obtained discrete-time model matches the behaviour of the nonlinear model with sufficient accuracy. Finally, the model of HEN was transformed into the form of a state-space system in the discrete-time domain (Mikleš and Fikar, 2007) described by 8 vertex systems, see (Kothare et al., 1996).

### 3. Results and discussion

The simulation results of robust model predictive control of HEN were obtained using 1.7 GHz and 4 GB RAM. The simulations were done in the MATLAB/Simulink environment; RMPC was managed by our free-available MUP toolbox (Bakošová and Oravec, 2014). The optimization problem of semidefinite programming was formulated by YALMIP toolbox (Löfberg, 2004) and solved by solver MOSEK.

To optimize operation of the HEN with uncertainty we designed RMPC by Kothare et al. (1996), denoted by RMPC<sub>1</sub> (Cao and Lin, (2005) represented RMPC<sub>2</sub>. RMPC was compared with the well-known discrete-time optimal control (LQR) based on the solution of the matrix Riccati equation, see e.g. (Mikleš and Fikar, 2007). LQR was designed with the same conditions as RMPC<sub>1</sub>, RMPC<sub>2</sub> to make the results fully comparable, i.e. the weight matrices of the quadratic quality criterion were  $W_x = \text{diag}([0.1, 0.1, 0.1, 0.1, 0.1, 0.1])$ ,  $W_u = 0.1$ . The input and output constraints were set to keep the control input volumetric flow-rate  $q$  in  $\pm 20$  °C and the system output in  $\pm 10$  °C neighbourhoods of the steady-state values, respectively.

The designed RMPC was studied in the presence of two various disturbances. The first control scenario (Case I) considered that the temperature of the input stream decreased from its steady-state value 20 °C to the temperature 14 °C. The second scenario (Case II) considered increase of the temperature of the input stream from its steady-state value 20 °C to the temperature 27 °C. The initial conditions for the Case I and Case II are in Table 1. We investigated the temperature control of the output stream from HEN in the Case I. The aim of control was to eliminate the influence of the over described disturbance. From the robust control viewpoint, it was not important to assign each control trajectory to the particular vertex system. The main purpose was to point out the range of admissible behaviour of HEN. Results generated by RMPC<sub>1</sub> are shown in Figure 2 a). From the robust control viewpoint, it was not important to assign each control trajectory to the particular vertex system. The main purpose was to point out the range of the admissible behaviour of HEN. RMPC<sub>1</sub> ensured satisfying fast control performance. As a side effect of such behaviour, there was the slight overshoot in some vertex systems. Figure 2 b) shows the results of RMPC<sub>2</sub>. It is obvious, that the control performance is quite similar to RMPC<sub>1</sub>. The mass of the natural gas needed in the furnace for preparing the hot product stream (Figure 1) was also studied. Table 3 summarizes the total consumption of the natural gas during the simulation of control during 1,000 s, where the values of  $m_{LQR}$  were evaluated for LQR,  $m_{RMPC,1}$  for RMPC<sub>1</sub>, and  $m_{RMPC,2}$  for RMPC<sub>2</sub>. All eight vertex systems were considered. The 0-th vertex corresponds to the nominal system. The nominal system can be obtained for the uncertain system with the nominal values of uncertain parameters, see Table 2. We analysed the data in Table 3 also by the relative savings of the natural gas defined as

$$\Delta m_{RMPC,1}^{(v)} = \frac{m_{LQR}^{(v)} - m_{RMPC,1}^{(v)}}{m_{LQR}^{(v)}} \times 100 \%, \quad \Delta m_{RMPC,2}^{(v)} = \frac{m_{LQR}^{(v)} - m_{RMPC,2}^{(v)}}{m_{LQR}^{(v)}} \times 100 \%, \quad (3)$$

where  $v = 0, 1, \dots, 8$ . The Figure 3 shows the relative saving of natural gas in RMPC<sub>1</sub> and RMPC<sub>2</sub> approaches, respectively. As can be seen, except of one vertex, there was ensured the improvement from 0.3 % up to 11.6 % by RMPC<sub>1</sub>, and the improvement from 0.2 % up to 12.1 % by RMPC<sub>2</sub>. On the other hand, the worst values were -0.7 %, -0.5 % using RMPC<sub>1</sub>, RMPC<sub>2</sub>, respectively. We recall that this situation occurred just in one vertex. Compared to LQR-based control, RMPC-based strategies were able to increase savings of natural gas in about 4 % also for the nominal system in the both RMPC strategies. The results of the Case II are further discussed, see Figure 4. We also investigated the control responses of the temperature of the output stream of the HEN. In Case II the temperature also converged to the reference for all vertex systems, cf. Figure 2. RMPC<sub>1</sub> approach also assured the convergence of the output temperature to the required value (Figure 4 a)). The results of RMPC<sub>2</sub> are shown in Figure 4 b) and they are similar to those obtained by RMPC<sub>1</sub>, cf. Figure 2 b).

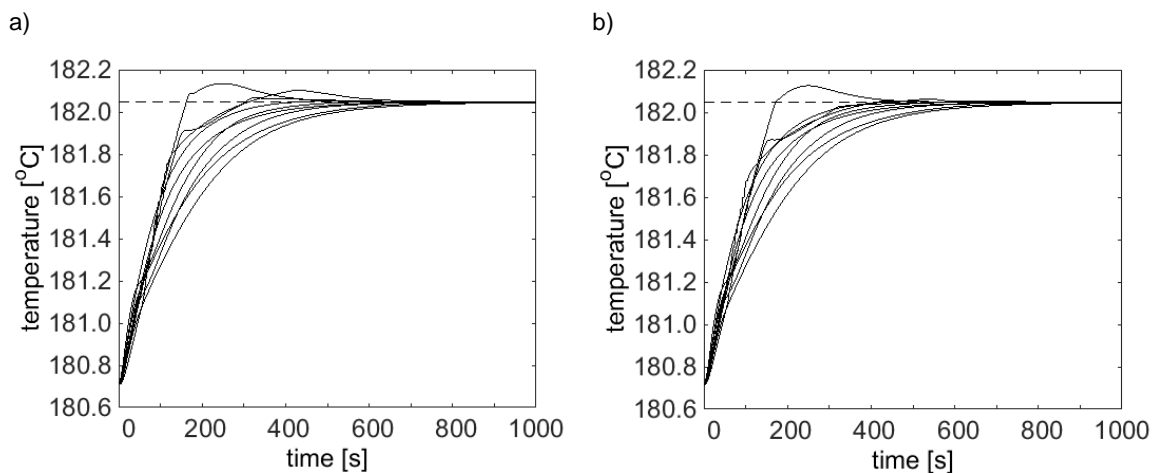


Figure 2: Control responses of the temperature of the outlet stream of the HEN in Case I: a) RMPC<sub>1</sub>, b) RMPC<sub>2</sub>; control trajectories for system vertices (solid) and reference (dashed)

Figure 5 shows the relative saving of natural gas in RMPC<sub>1</sub> and RMPC<sub>2</sub> strategies subject to LQR, respectively. These results are different compared to the Case I, due to the fact that RMPC<sub>1</sub> and RMPC<sub>2</sub> methods ensured the relative improvements for all vertex systems. The relative savings of natural gas in RMPC<sub>1</sub> varied from 0.9 % to 13.2 %. The relative saving for the nominal system was 5.0 % (Figure 5 b)). The relative improvements generated by RMPC<sub>2</sub> varied also from 0.9 % to 13.2 %. The relative saving for the

nominal system was 5.2 % (Figure 5 b)). We analysed the control performance for two admissible disturbances. In both cases, the control responses obtained by  $\text{RMPC}_1$  and  $\text{RMPC}_2$  approaches were quite similar. The highest relative consumption of natural gas  $\Delta m_{\text{RMPC},1}^{(1)} = -0.5\%$  in Case I was less compared to the  $\Delta m_{\text{RMPC},1}^{(1)} = -0.7\%$  in Case II.

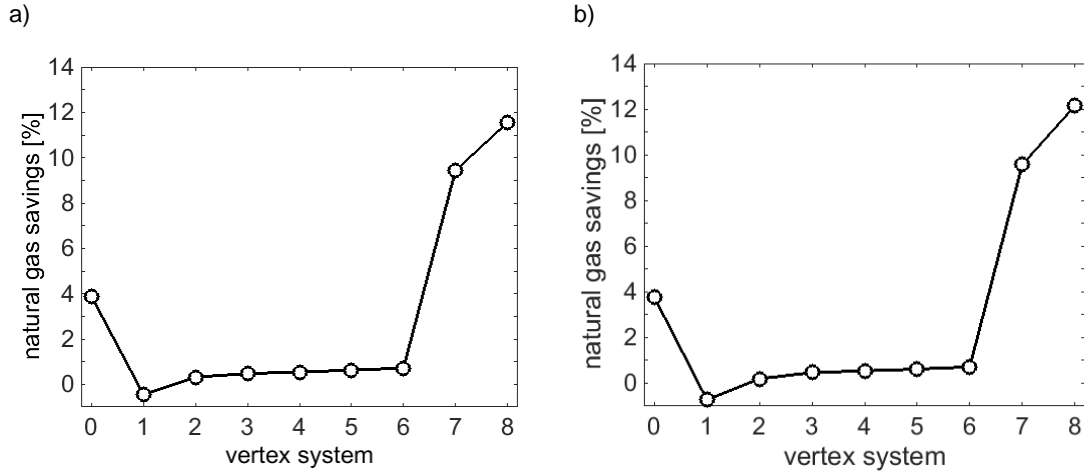


Figure 3: Relative savings of natural gas ensured by RMPC subject to LQR in Case I: a)  $\text{RMPC}_1$ , b)  $\text{RMPC}_2$

Table 3: The total consumption of the natural gas in LQR,  $\text{RMPC}_1$ , and  $\text{RMPC}_2$  approaches in Case I and Case II

Vertex $v$	$m_{\text{LQR}}^{I(v)}$ [kg]	$m_{\text{RMPC},1}^{I(v)}$ [kg]	$m_{\text{RMPC},2}^{I(v)}$ [kg]	$m_{\text{LQR}}^{\text{II}(v)}$ [kg]	$m_{\text{RMPC},1}^{\text{II}(v)}$ [kg]	$m_{\text{RMPC},2}^{\text{II}(v)}$ [kg]
0	33.828	32.517	32.557	33.828	32.130	34.846
1	33.777	33.932	34.022	33.842	33.526	33.525
2	33.826	33.720	33.761	33.865	33.528	33.527
3	33.842	33.683	33.683	33.826	33.478	33.463
4	33.865	33.681	33.682	33.896	33.523	33.522
5	33.896	33.685	33.686	33.933	33.518	33.517
6	33.933	33.689	33.690	33.777	33.268	33.144
7	33.843	30.650	30.603	33.843	30.190	30.111
8	33.781	29.879	29.682	33.781	29.310	29.336

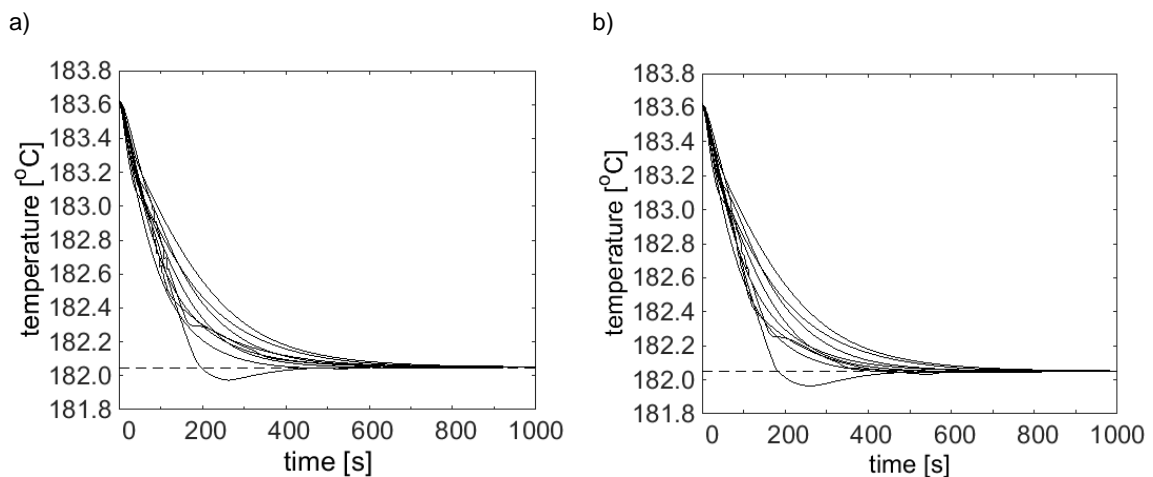


Figure 4: Control responses of the temperature of the output stream of the HEN in Case II: a)  $\text{RMPC}_1$ , b)  $\text{RMPC}_2$ , control trajectories for system vertices (solid) and reference (dashed)

On the other hand, the maximal saving of natural gas assured by  $\text{RMPC}_2$   $\Delta m_{\text{RMPC},1}^{(8)} = 13.2\%$  in Case II was the same as  $\text{RMPC}_1$  in Case II. In general, simulation results confirmed that the RMPC-based strategies improved the control performance and increased the energy savings compared to the LQR control.

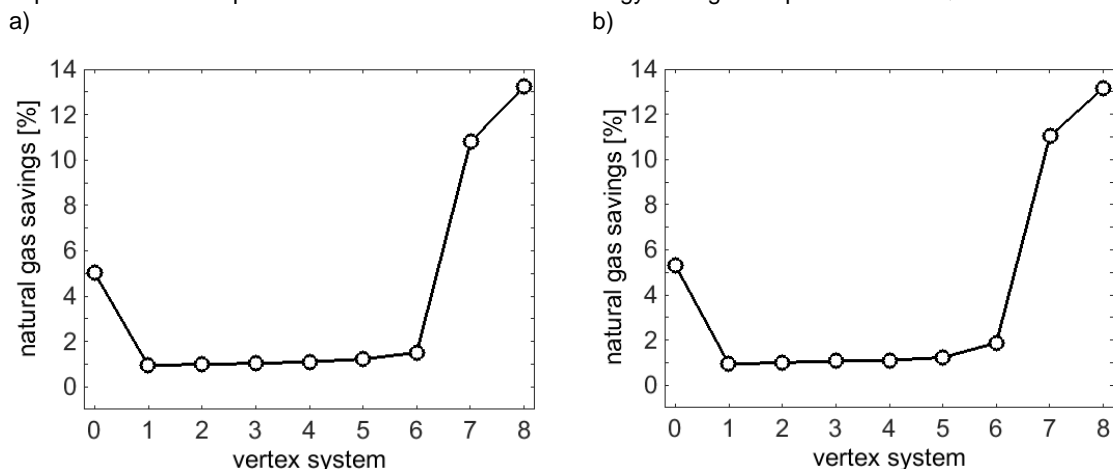


Figure 5: Relative savings of natural gas ensured by RMPC subject to LQR in Case II: a)  $\text{RMPC}_1$ , b)  $\text{RMPC}_2$

#### 4. Conclusions

This paper presents the advanced robust model predictive control design for the optimization of the heat exchangers in series with uncertain parameters. We investigated a case study with two various significant disturbances in the temperature of input stream in the heat exchanger network. Although the input temperature varied from 14 °C to 27 °C, RMPC-based methods ensured the more aggressive control action to keep the temperature at the required reference than the well-known LQ optimal control approach. Moreover, the total consumption of the natural gas used in the technology with HEN was reduced up to 13 % in comparison to the LQ optimal control strategy during operation lasting 1,000 s. In practice it may lead to the significant energy savings and reduction the overall input costs of HEN utilization.

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