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Gray System Theory-based Thermal Error Modeling and Application of Precision Machine Tool

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In this paper, the author applies the gray system theory to build the thermal error model and analyze thermal error of precision machine tools. This paper mainly applies the gray system theory, cutting processing experiment and analysis of guide rail, unloading beam modal, and spindle and so on to study the thermal error of precision machine tools. The experiment shows that the accuracy of GM (0,4) and that of GM (1,4) models is relatively high. By referring to the cutting parameters, the analysis data of the guide rails, unloading beam modal and the spindle, we learn that the prediction accuracy of the model GM (X, N) can be improved by 40% at the most.

1. Introduction

Among the error sources of the machine tool, thermal error takes a large proportion.In order to reduce the thermal error of machine tools, the processing precision of machine toolmust be improved, and then effectively guarantees the processing quality of machine tool. To optimize structure design of the machine tool and to improve the processing environment are effective ways to compensate thermal error of machine tools. The software compensation method is not that high.Under normal circumstances, software compensation is only used to auxiliary hardware compensation method.

Nowadays, multivariate linear analysis, the gray theory and timing analysis are the main contents of mathematical theory. The gray theory mainly refers to judging the future development trend through the past-future gray model. In this paper, model GM (0,4) and model GM (1,4) are mainly built on the base of the gray system theory to analyze the thermal error of precision machine tool.

2. Literature review

Abdulshahed and others used thermocouples to measure the influence of the heat source of machine tools on the surface temperature of the machine tools, such as the distance between the thermocouple position and the heat source of the machine tool, the contact performance of the thermocouple and the surface of the machine tool. They arranged 23 thermocouples on the machine tool to measure the temperature rise of the machine tool. At the same time, they deployed the 5 capacitance sensors near the spindle to measure the thermal deformation of the spindle, and collected the data by the analog to digital converter (Abdulshahed et al., 2015). Cheng et al. installed 80 thermocouples in the turning center to collect the temperature of the machine tool. They applied 8 laser interferometer and 3 non-contact capacitance sensors to collect 11 deformation errors (Cheng et al., 2018). Miao et al. proposed a method of using the ball bar system to simultaneously measure the spindle thermal error and geometric error of the machine tool (Miao et al., 2015). This method replaces the traditional capacitance displacement sensor measurement method (Miao et al., 2015). Hongyao et al. put forward a new error measurement method. By analyzing the structure of the machine tool, the displacement sensors are arranged in the spindle, column, spindle box and tool of the machine tool. The thermal deformation of each part of the machine can be obtained by calculating the collected displacement data (Hongyao et al., 2017).

In the process of machining, heat expansion occurs due to a large amount of heat produced during the process of machining, which affects the relative position between the machining tool and the work-piece to be machined, and reduces the machining precision. Thermal error modeling technology is to establish a

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mathematical model between temperature variable and thermal error, and to conduct accurate and real-time prediction in the process.

Ma et al. proposed the energy identification method of the motion dimension chain to establish the thermal error model. The method is based on a certain premise that the energy lost on the kinematic dimension chain is a function of machine tool processing conditions. After determining the heat source and working environment of the machine tool, the energy loss of the components on the machine tool and the temperature field of the machine tool can be determined. Firstly, the finite element analysis method is used to establish the thermodynamic model of the machine tool, and then the regression analysis method is used to establish the thermal error model. The geometric error and thermal error of the three-axis machining center are analyzed, and 32 machine tool errors are obtained. Then, the overall error model of the machine tool is established to compensate for the machining error. The machining error is reduced from 196μm to 8μm (Ma et al., 2017). Feng et al. used artificial neural network to establish thermal error models for machine tools. The neural network is a three-layer pre-posed artificial neural network, which takes the key point temperature as the input of the neural network. The thermal error of the machine tool is regarded as the output of the neural network, and then the neural network is trained to get the thermal error model (Feng et al., 2015). Du and so on proposed an online correction model to improve the robustness of RBF (Radial Basis Function) neural network. According to the residual error value of RBF neural network, the actual compensation error value is deduced and transmitted to the error compensator for compensation. The experimental results show that the improved thermal error model has higher prediction accuracy and a greater robustness. They also proposed a RBF neural network based on the human immune system for the difficult determination of the number of hidden nodes and the central location of the RBF neural network. The model adjusted the number of hidden nodes and determined the center and width of the Gauss function by using the advantages of adaptive regulation of human immunity. The results show that the prediction accuracy of the model is better than that of the traditional RBF neural network, and it has the tracking property to the abrupt data (Du et al., 2015). There are still some scholars exploring thermal error modeling method of least squares support vector machine based on the dynamic adaptive weighted. The method adds the square of the error to the target function of the standard support vector machine, and uses the weighting method to perform the operation, which well solves the problem of robustness and sparsity of the model. Some scholars use Bayesian network to establish thermal error models. Some scholars use Bayesian network to establish thermal error models. This method uses graph theory to describe the causality between factors that produce thermal errors, then analyses the correlation between the factors according to the probability theory, and finally produces the modeling result by the regional probability distribution of the thermal error.

The above research work mainly establishes the mathematical model between temperature variable and thermal error in the process of measuring the temperature and thermal deformation in the thermal error compensation technology. In the process of working, accurate and real-time prediction is profoundly studied, but there are few researches on thermal error modeling and application of precision machine tools based on grey system theory. Therefore, based on the above research situation, the thermal error modeling and application of precision machine tools based on grey system theory are focused on. The basic frame structure of thermal error of precision machine tools based on grey theory is put forward.

3. Experimental principle and experimental method

Effective thermal error compensation mainly relies on reliable measurement devices, efficient measurement methods and statistical models that accurately reflect the inherent relationship between temperature data at critical temperature points and thermal error data of the machine tool. Currently, the widely used mathematical theory includes multiple linear regression, timing analysis, neural network and gray system theory. Among them, the gray system theory establishes a gray model from the past to the future based on the past and present known or uncertain information to determine the trend of the future development of the system. With differential equation as a tool, the gray system theory can reflect the nature of the development of things though without any clear understanding of the forecasting system. The research data can be randomly generated. Therefore, compared with the traditional statistical methods, the gray system theory has the advantage of studying small samples, poor information and arbitrary data distribution. Due to the processing conditions differ greatly brought from the different shapes of the machined parts, it is more time-consuming to obtain as much modeling data as possible by simply increasing the experimental data acquisition time, and the cost is not high. The gray system theory is more suitable for the study of thermal error modeling because the data collected by the actual machining experiment is consistent with the characteristics of small samples and poor information.

In this paper, the author applies the experimental data of the produced parts from a CNC lathe, builds model GM (1, 4) and model GM (0, 4) respectively to reflect the relationship between the four key temperature measuring points distributed on the machine tool and the machining thermal error. With the actual processing data applied, the established model is more suitable for industrial processing.

4. Experimental process

4.1 Modeling

As the establishment of the gray system model needs processing the modeling data sequence to weaken its randomness and highlight its changing trend, various transformations need to be defined first.
Set X(0)=(x(0)(1),x(0)(2),,,x(0)(n)) as the original sequence, set its first-order accumulatively generated (1-AG0) sequence as X(1)= (x(1)(1),x(1)(2),,,x(1)(n)). Among them,

$$X^{(1)} = \sum_{i=1}^{n} x^{(0)}(i)$$

The immediate mean value sequence Z (1) of the sequence X (1) is defined as Z(1)=(z(1)(2),z(1)(3),...,z(1)(n)), among them, z(1)(n) =(x(1)(n)+x(1)(n-1))/2,x(1)(n) is the corresponding accumulatively generated value of the original sequence X(0).
Set X (0) 1 as a thermal error element sequence of the machine tool studied, X (0) i as the sequence of temperature values related to the thermal error element sequence, i=2,3,4,5, then GM 4), thenthe model GM (0, 4) is defined as follows:

$$\hat{X}_1^{(1)}(n) = \sum_{i=2}^{5} b_i x_i^{(1)} + a$$

^ a = [a b2b3b4b5] T
Among them, parameters ^a can be calculated by the following formula:

$$B = \begin{cases} x_2^{(1)}(2)\cdots & x_5^{(1)}(2) & 1 \\ x_2^{(1)}(3)\cdots & x_5^{(1)}(3) & 1 \\ x_2^{(1)}(n)\cdots & x_5^{(1)}(n) & 1 \end{cases}, Y = \begin{cases} x_1^{(1)}(2) \\ x_1^{(1)}(3) \\ x_1^{(1)}(n) \end{cases}$$

This model does not contain the derivation operation and it is a static model. Although its calculation is similar to multivariate linear regression, it is possible to reduce the long-term tendency of random factor interference and to reflect changes in the data sequence since the model is built based on the 1-AGO sequence of the original data sequence,.
Set X (0) 1 as a thermal error element sequence of the machine tool studied, X (0) i as the sequence of temperature values related to the thermal error sequence, i = 2, 3, 4, and 5,then the dynamic differential equation built is as follows:

$$\frac{dx_1^{(1)}}{dt} + a x_1^{(1)} = \sum_{\mathrm{i}=2}^{5} \mathrm{b_i} x_i^{(1)}$$

The response at approximate time is as follows:

$$\hat{x}_1^{(1)}(n+1) = \left[x_1^{(0)}(1) - \sum_{\mathrm{i}=2}^{5} \mathrm{b_i} x_i^{(1)}(n+1) \right] e^{-an} + \sum_{i=2}^{5} b_i x_i (n+1)$$

The estimate value of thermal errors is obtained by subtracting and reducing the resulting ^ x (0) 1 sequences:
^x1(0)(n+1) = ^x(1)1(n+1) -^x(1)1(n)
In the above formula, the parameter ^ a = [a b2 b3 b4 b5] T, and it can be calculated by the following formula:
^ a = (BTB) -1BTY

$$B = \begin{bmatrix} -z_1^{(1)}(2) & x_2^{(1)}(2) & \cdots & x_5^{(1)}(2) \\ -z_1^{(1)}(3) & x_2^{(1)}(3) & \cdots & x_5^{(1)}(3) \\ \vdots & \vdots & & \vdots \\ -z_1^{(1)}(n) & x_2^{(1)}(n) & \cdots & x_5^{(1)}(n) \end{bmatrix}, \ Y = \begin{bmatrix} x_1^{(0)}(2) \\ x_1^{(0)}(3) \\ \vdots \\ x_1^{(0)}(n) \end{bmatrix}$$

This model is a dynamic model that contains derivative operations. The analytical solution of the equation in the classical differential equation theory is complicated, but the approximate numerical solution can be obtained with the above method. Compared with multivariate linear regression algorithm, only the corresponding sequence transformation before and after solving ^ a is increased and the computational complexity is further reduced; therefore, the computational complexity is moderate.

4.2 Cutting Experiment

As the model based on null cutting machining data without actual machining loads or numerical simulation with FEM calculation software ignores some of the factors that affect thermal errors and the effects of cutting forces during actual machining of the machine, the model based on these dada will have a larger deviationin the actual application process. Therefore, this experiment applies actual cutting data from a CNC machine tool.

The machined workpiece is short stub bar with light cutting depth, so we can ignore the angular offset along the axis due to cutting force of the workpiece, that is, the thermal offset along axis direction of the workpiece is consistent. For lathe machining, since the direction x is the direction of error-sensitivity, here we only build a model on thermal errors in this direction. As the machine temperature changes very slowly, the sampling period can be relatively long.Sinceit takes 5minutes to process a bar, the sampling period of experiment is 5minutes. In order to collect as much relevant data of the machine tool as possible under as processing conditions, the experiment conducted a complete sampling of the three shifts processed during awhole day.

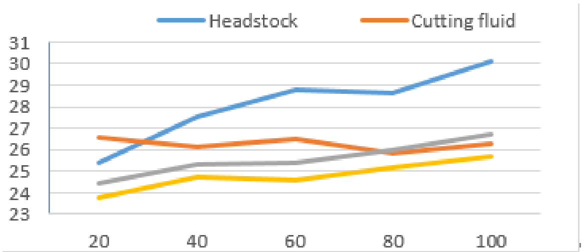

Figure1: schematic diagram of temperature change

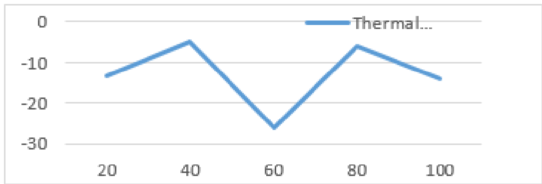

Figure2: schematic diagram of thermal error

Table 1: model residual statistics

	Minimum	Maximum	Mean value	The variance
GM(1,1)	-0.508	5.639	2.016	1.588
GM(0,4)	-2.268	1.189	-0.587	0.994
GM(1,4)	-1.587	1.923	-0.0006	0.965

As can be seen from Figure 1 and Figure 2, in the morning shift 30 parts were processed in totalin 215h; in the afternoon shift 45 parts were processedin 3175h; and 26 parts were processed in total in 2125h.

From Figure 4, it can be observed that in the morning hours, the machine tool starts to run from the cold state, so the thermal error increases greatly. Due to the stop of the machine tool at lunch break, the thermal error is greatly reduced. In the afternoon shift, the thermal error further increases as the machine operates, and the thermal error value fluctuates only within a certain range after the machine tool reaches the thermal equilibrium. The thermal error during the late break decreases again as the machine tool stops processing. As the machine processesin the night shift and it temporarily shutdownsfor special reasons, thermal error changes correspondingly but it fails to reach the heat balance during the night shift.

It can be seen from Figure 2 that in the three production shifts of the day the thermal error in the x-axis direction of the machine has three obvious changes. The author divides the measured 101 sets of data, builds a model with the 71 sets of data in the afternoon shift andthe night shift, verifies the model with the 30 sets of data in the morning shift. The fitting accuracy of the model to the actual thermal error data is higher than that of the traditional model. Being a dynamic model, it possesses a higher modeling accuracy than that of the model GM (0,4) model. Related statistics are shown in Table 1.

4.3 Analysis of Rail, Unloading Bam Mode and Spindle

When the horizontalrail vibrates in the vertical axis, it mainly deforms. This is known as the horizontal vibration of the rail. As the influence of inertia and shear deformation during the movement of the guide rail is very small, we only consider the lateral vibration of the guide rail in the symmetry plane of XY in the cross section, y = y (x, t), and all the loads act in the plane. The rail is regarded as a beam fixed at both ends.According to the beam lateral vibration equation:

$$\frac{\partial^4 y}{\partial x^4} = -\frac{1}{c^2} \frac{\partial^2 y}{c^2 \partial t^2}$$

In the formula $c = \sqrt{EI/m'}$, EI refers to the bending stiffness of the beam and m refers to the mass per unit length of the beam.
In order to verify the correctness of the above method, we use ANSYS again to solve. In the ANSYS software, the material properties of the rails given to the beam elements are analyzed. The natural frequencies of the guide rail system obtained by the above two methods are shown in Table 2, and the table shows that the natural frequencies of the guide rail obtained by the two methods are in agreement.

Table 2: rail and unloading beams before the fifth natural frequency

Order		1	2	3	4	5
X rail	Theory (Hz)	932.68	2570.8	5040.1	8331.6	12446
	ANSYS (Hz)	932.68	2571	5040	8330	12440
Unloading beam	Theory (Hz)	596.6	1644.5	3222.7	5327	7958.6
	ANSYS (Hz)	596.33	1644	3222	5326	7954

The heat load generated in the machining process will make the temperature distribution of the spindle uneven, which will cause the deformation of the spindle structure, thereby affecting the machining accuracy of the machine tool. From the above analysis we can seethe thickness of bearing film, the bearing capacity and the stiffness change with the thermal error generated during the process. However, in the thermo-mechanical model, the deformation of the main axis mainly depends on the bearing stiffness. Therefore, the analysis of the two related processes of the thermal model and the mechanical model of the main axis system can predict the deformation of the main axis well during the actual machining. This can provide a good design theory for the machine tool design and helps the performance of the entire machine tool in the process of achieving be best.

Figure 3: spindle axial displacement detection

Figure 3 shows the experimental device of the spindle to analyze thermal error. Inductance micrometer is used to detect changes in the displacement of the spindle table. The test bench is removed, the micrometer base is fixed at the end, and the probe hits the thrust plate. First, open the oil pump to transmit oil in the spindle, the spindle is static at this moment.Theinductance micrometer shows v1 = 0.8μm after achieving thermal balance. Then check the deformation of the entire spindle system at the maximum rotation speed after thermal stability, the probe amplitude v2 = 1.6μm. The detected thermal drift Et = v2-v1 = 0.8 μm.
The environment temperature of the machine is at constant 20oC.If the maximum temperature of the oil film in the bearing in the process is consistent with the room temperature, the corresponding thermal deformation of the spindle system reaches a minimum value. In order to reduce the thermal error of the main spindle, the optimum temperature of the oil film flowing out from the oil pump is calculated when the equilibrium temperature of the entire spindle system is 20oC. When the initial temperature of the oil film is 17.5oC, the corresponding temperature distribution of the main spindle is shown in FIG4. Figure 5 shows the axial deformation of the spindle system at a maximum speed of ω = 15.7 rad / s is 9.8 μm, which is also smaller than the deformation at an initial temperature of 20°C.

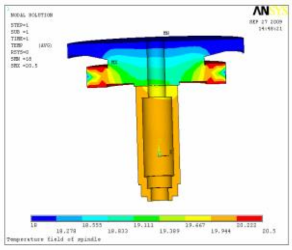

Figure 4: Temperature field atω = 15.7 rad / s

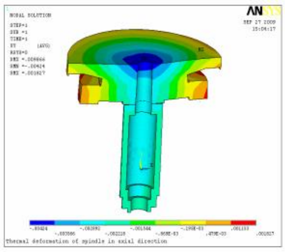

Figure 5: Thermal deformation atω = 15.7 rad / s

5. Conclusion

From the experiments, we learn that the model GM (0, 4) and the model GM (1,4) are more accurate than the model GM (1,1). At the same time, we can see that the prediction precision of model GM (X, N) can be improved by 40% with reference to the cutting parameters, the analysis of guide rail and unloading beam and the spindle. In this paper, model GM (0, 4) and model GM (1, 4) take the temperature of the machine tool and the change of the temperature value into account. As the temperature value changes with time, the modeling precision of thermal error for the machine tool is higher.

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