

# Vulnerability Analysis for Park-Wide Water Management Using Dynamic Inoperability Input-Output Model

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Park-wide water management is an important way to conserve water resources. However, the enhancement of inter-plant connection will weaken the system stability and strengthen the cascade effect of disaster losses. In this work, a dynamic inoperability input-output model is applied to park-wide water network to predict the impact of disturbances. The inoperability is expressed as the degree of water loss. The dependence of different water networks on freshwater is presented. This work analyzes the impact of freshwater supply disturbance on the water volume. By simulating the time-varying trajectory of inoperability, the indicators (i.e., resistance to disturbance, adaptability, and recoverability) are determined to evaluate the vulnerability of the water network in each scenario. The results show that under the same freshwater disturbance, water networks that are less dependent on freshwater are more robust but weaker in recoverability. In the case of freshwater disturbance, the possibility of disruptions in the participating plants of the without integrated optimization water network is about 1.5 times that of the integrated optimization water network.

## 1. Introduction

Interdependent Infrastructure System is a system formed by several interrelated and interacting infrastructures, which provide the most basic and important safeguards for human society (Barker and Haines, 2009). In recent years, various types of disasters which result in emergencies (i.e., earthquakes, fires, floods, tornadoes, terrorist attacks) have been occurring more frequently. This has resulted in huge impacts on the infrastructure systems. Supply-driven disturbances can affect the services provided by critical infrastructure. Snowstorms for example can disrupt the function of power supply systems, transportation systems, and prevent these systems from continuing to function properly (Ghosh, 1958).

Vulnerability studies include the assessment of system disturbances, sensitivity analysis of system response to disturbances, the potential impact of disturbances on the system, and system adaptation analysis. Its main purpose is to identify the vulnerable points in the system and then to defend or protect them so that economic losses due to emergency disturbances are minimized. Based on the Leontief Input-output Model, Haines and Jiang (2001) introduced the concept of 'inoperability', and developed the first generation of the Inoperability Input-output Model (IIM) and introduced resilience coefficients to explain the Dynamic Inoperability Input-output Model (DIIM). DIIM can be utilized in examining the capability of systems to recover after a disruption.

The water network of an industrial park is one of the key infrastructures. If the freshwater supply is reduced due to the impact of emergencies, and no other emergency water management measures are taken, the park-wide water supply will be reduced and the desired level of production may no longer be guaranteed. Therefore, in the context of network complexity caused by park-wide water integration optimization, there is an urgent demand for system vulnerability studies. This paper proposes an improved DIIM to account for the vulnerability of the designed park-wide water network to emergency event disturbance as an effective tool for assessing system stability. The work proposes relevant time indicators to represent the vulnerability performance: robustness, adaptability, recoverability, in order to evaluate the vulnerability of the park-wide water network. Then, the

vulnerable points of each scenario are identified to provide guidance to the park-wide management in taking emergency measures to minimize the inoperability caused by deepening connections between plants.

## 2. Method

IIM was originally developed to study economic equilibrium behavior. In recent years, with the deepening of research, it has been extended to the study of population, energy, environment and other fields. Because the material flow exchange of each plant satisfies the conservation of energy and material, this work applies the DIIM to specific water networks example, analyzes the inoperability change trajectory of water plants under particular initial disturbance, and provides data support for the vulnerability analysis of water network.

### 2.1 Dynamic Inoperability Input-output Model

Haimes (2005) discusses the theory and methodology supporting the development of the Inoperability Input-output Model (IIM). Dynamic Inoperability Input-output Model (DIIM) as an extension that supplements and complements the static IIM. The normalized DIIM equation proposed in that paper is as follows.

$$\frac{dp(t)}{dt} = -K^{(S)} [p(t) - A^{(S)*} p(t) - z^*(t)] \quad (1)$$

Where,  $p(t)$  is a vector that denotes the level of system inoperability at time  $t$ ;  $K^{(S)}$  denotes the resilience coefficient matrix that express the recoverability after disturbance;  $z^*$  denotes supply disturbance vector caused by emergency events;  $A^{(S)*}$  denotes deriving interdependence matrix, converted from Eq (2) – Eq (5):

$$A^{(S)*} = \text{diag}(\hat{x})^{-1} A^{(S)} \text{diag}(\hat{x}) \quad (2)$$

$$A^{(S)} = (A^*)^T \quad (3)$$

$$A^* = [(\text{diag}(\hat{x})^{-1}) A (\text{diag}(\hat{x}))] \quad (4)$$

$$A = a_{ij} = \frac{x_{ij}}{x_i} \quad (5)$$

Where,  $\hat{x}$  denotes planned production degree;  $A$  denotes technical coefficient matrix (direct consumption coefficient matrix),  $x_j$  denotes the output of sector  $j$ ,  $x_{ij}$  denotes the amount of input from sector  $i$  to sector  $j$ .

Eq (1) is the general expression for the DIIM, which is a standard first order linear differential equation. When the initial value is  $p(0)$ , its continuous solution can be obtained:

$$p(t) = e^{-K^{(S)}(I-A^{(S)*})t} p(0) + \int_0^t K^{(S)} e^{-K^{(S)}(I-A^{(S)*})(t-T)} z^*(T) dT \quad (6)$$

When  $z^*$  is fixed, then:

$$p(t) = (I - A^{(S)*}) z^* + e^{-K^{(S)}(I-A^{(S)*})t} [p(0) - (I - A^{(S)*}) z^*] \quad (7)$$

The discrete solution is as follows:

$$p(t+1) - p(t) = -K^{(S)} [p(t) - A^{(S)*} p(t) - z^*(t)] \quad (8)$$

### 2.2 Resilience

DIIM can also be used to describe the dynamic recovery process of the correlation infrastructure sector, assuming that:

$$p_i(0) > 0 \quad p_j(0) = 0 \quad i \neq j, z^* = 0 \quad (9)$$

then the recovery trajectory of sector  $i$  is obtained by solving the continuous equation:

$$p_i(t) = e^{-K_i^{(S)} [1 - a_{ii}^{(S)*} t]} p_i(0) \quad (10)$$

Assuming that the time  $T_i$  of sector  $i$  is estimated by experts or obtained from field investigations, from the initial disturbance  $p_i(0) > 0$  was back to  $p_i(T_i)$ , the resilience coefficient of sector  $i$  can be obtained using Eq (11):



### 3.2 Scenario setting

In order to compare the vulnerability with and without integrated optimization water networks, two scenarios were set up to obtain relevant comparative indicators.

Scenario 1: Assuming that freshwater is affected by an emergency event that generates an initial disturbance, Within the time  $\tau$ :

$$\int_0^{\tau} z_F^*(t) dt = 20\%, p_i(0) = 0, i = M, O, P, A, H, F \quad (12)$$

Scenario 2: Based on the actual investigation, it is obtained that when the inoperability level of P is larger than 10 %, it will cause the system to be unable to guarantee the minimum production level, considering the following emergency measures before  $T(p(T)=10 \%)$ : restoring F or finding a new alternative water source. Simulations are performed to obtain the inoperability recovery trajectory.

MATLAB simulation is applied to iteratively obtain the trajectory of inoperability over time for each plant in each scenario. In this study, inoperability indicates the ratio of water loss and planned water supply. It is a dimensionless scalar in the range of 0 to 1.

### 3.3 supply-driven DIIM

Assuming that each plant has a resilience coefficient of 0.1, the DIIM equation describing each plant can be derived as follows (in the scenario 1):

$$\begin{cases} p_{M1}(t+1) = p_{M1}(t) - 0.1[p_{M1}(t) - p_{F1}(t) - z_{M1}^*(t)] \\ p_{O1}(t+1) = p_{O1}(t) - 0.1[p_{O1}(t) - p_{F1}(t) - z_{O1}^*(t)] \\ p_{P1}(t+1) = p_{P1}(t) - 0.1[p_{P1}(t) - p_{F1}(t) - z_{P1}^*(t)] \\ p_{A1}(t+1) = p_{A1}(t) - 0.1[p_{A1}(t) - 0.0115p_{M1}(t) - 0.9885p_{O1}(t) - z_{A1}^*(t)] \\ p_{H1}(t+1) = p_{H1}(t) - 0.1[p_{H1}(t) - 0.5757p_{P1}(t) - 0.4243p_{A1}(t) - z_{H1}^*(t)] \\ p_F(t+1) = p_F(t) - 0.1[p_F(t) - z_F^*(t)] \end{cases} \quad (13)$$

### 3.4 Simulation results

From Figure 1 and Figure 3, it shows the propagation curve of the failure of each plant when a disturbance occurs in freshwater, it can be seen that reaching the new equilibrium, the level of inoperability of each plant is 0.2, same as freshwater initial disturbance. Due to the different order of water transfer and dependence on F, the time to reach the new equilibrium varies among plants. According to Figure 1, the time used to reach equilibrium for each plant:  $F < M = O = P < A < H$ , the longer the time, the stronger the resistance to damage, so the resistance to damage:  $H > A > P = O = M > F$ ; The first-class water plant for F in scenario 2 is only O, by comparing the data in Table 1 and Table 2, the dependence of the water plants on F is drastically reduced in scenario 2 compared to the scenario1. From Figure 3, the time used to reach the new equilibrium:  $F < O < A < H < B < P < M$ , so the resistance to damage:  $M > P > B > H > A > O > F$ . The difference between the two scenarios is due to the increase in the regeneration plant of scenario 2 and the internal water use of the park resulting in a change in the dependence on F and the sequence of water transfer.

System robustness measured by average time to new equilibrium, so the robustness of scenario 2 is greater than that of scenario 1. It shows that the integrated optimization scenario increases the resistance of the water network to freshwater disturbances. Although the final inoperability is the same for each scenario, the time experienced varies.

From Figure 1, when  $t=17$  h, the inoperability of P reaches 10.36 %. Therefore, emergency management measures are required to restore freshwater or activate the backup water source of P before  $t=17$  h. From Figure 2, the time used to return the ideal state:  $H > A > M = O = P$ , so the recoverability:  $F > M = O = P > A > H$ . Similarly, from Figure 4, the time used to return the ideal state:  $M > P > B = H > A > O > F$ , so the recoverability:  $F > O > A > H = B > P > M$ ; System recoverability is measured by the average time it takes for each plant to recover from maximum inoperability to the ideal state. So, the recoverability of scenario 1 is greater than that of scenario 2. The adaptability is measured by the maximum inoperability that the system can achieve throughout the disturbance and recovery process. So, the adaptability of scenario 2 is greater than that of scenario 1. It shows that the fluctuations of the optimization scenario are smaller than the without optimization scenario for the same disturbance conditions.

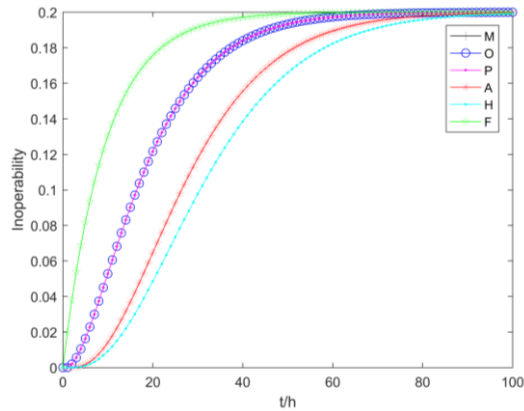


Figure 1: scenario 1 disturbance propagation

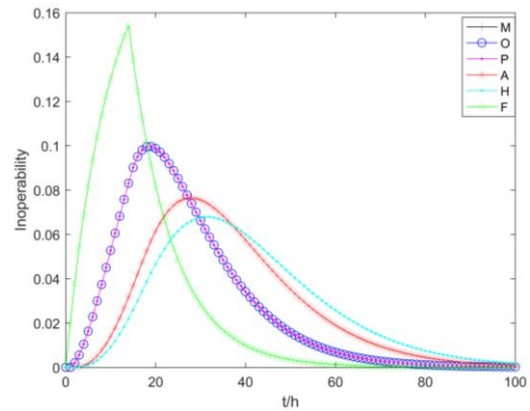


Figure 2: scenario 1 restored curve

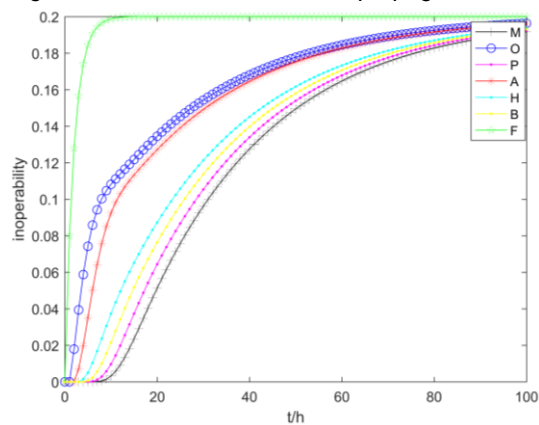


Figure 3: scenario 2 disturbance propagation

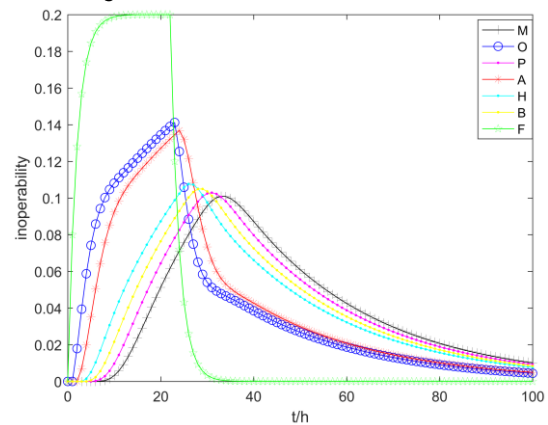


Figure 4: scenario 2 restored curve

### 3.5 Emergency management for scenario 2

Vulnerability point analysis. Scenario 2 (with integrated optimization), which satisfies the maximum economic benefits of park, has the stronger resistance to freshwater disturbance, but the weaker recoverability; Scenario 1 (without integrated optimization) has the stronger recoverability, but the weaker resistance to freshwater disturbance. Therefore, it is necessary to perform management measures for the scenario 2 to increase the stability of the water network structure.

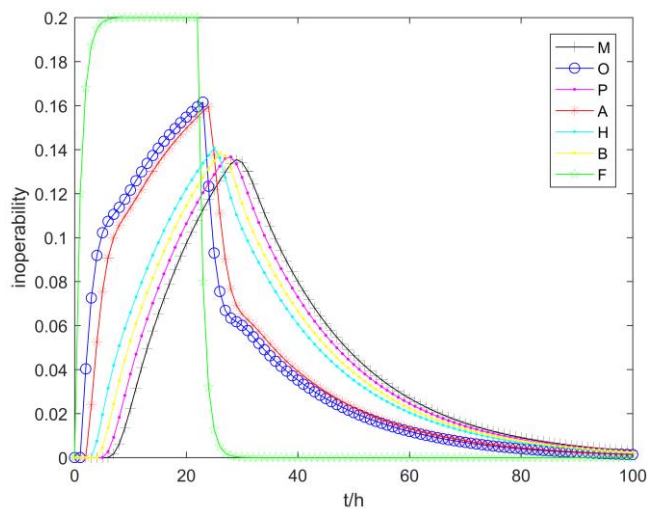


Figure 5: Inoperability change for the resilience coefficient of 0.6 for scenario 2

Scenario 2 is based on the maximum water recycling rate obtained between water plants, which has little dependence on freshwater. When the freshwater suddenly decreases, the spread speed of disasters between plants is slow, the resistance is the strongest, but the recoverability is the weakest. In order to increase the recoverability, one of the measures is to increase the resilience coefficient of the plant itself are taken to shorten the recovery time. According to the simulation, the resistance to damage and recoverability of the scenario were improved, as shown in Figure 5.

#### 4. Conclusions

When the freshwater supply is reduced due to an emergency, the water balance of plants is broken due to the ripple effect. This paper uses the Dynamic Inoperability Input-output Model to investigate the vulnerability of different water networks. Concepts related to vulnerability including robustness, adaptability and recoverability are introduced to characterize the corresponding performance from the perspective of time. Integrated optimization scenarios enhanced inter-plant recycling and weakened the dependence on freshwater. When the freshwater is reduced, some extension of the time to reach the new equilibrium or restore the ideal state. The stronger the resistance to disturbance is, the more time the water management department in the park can take emergency measures. The model is generic and provides vulnerability analysis support for integrated programming.

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#### Reference

- Aviso K.B., Tan R.R., Culaba A.B., 2010. Designing eco-industrial water exchange networks using fuzzy mathematical programming, *Clean Technologies and Environmental Policy*, 12, 353-363.
- Barker K., Haines Y.Y., 2009. Uncertainty Analysis of Interdependencies in Dynamic Infrastructure Recovery: Applications in Risk-Based Decision Making. *Journal of Infrastructure Systems*, 15, 394-405.
- Geng Y., 2014. Resilience measure of supply network based on DIIM, *Computer Integrated Manufacturing Systems*, 20, 1211-1219.
- Ghosh A., 1958. Input-output approach in an allocation system, *Economica New Series*, 25(97), 58-64, doi: 10.2307/2550694.
- Haines Y.Y., Jiang P., 2001. Leontief-based model of risk in complex interconnected infrastructures. *Journal of Infrastructure Systems*, 7, 1-12.
- Haines, Y.Y., Horowitz, B.M., Lambert, J.H., Santos, J.R., Lian, C., Crowther, K.G., 2005, Inoperability input-output model for interdependent infrastructure sectors. I: Theory and methodology, *Journal of Infrastructure Systems*, 11, 67-79.
- Leontief W.W., 1986. *Input-output economics*, 2nd ed. Oxford University Press, New York, USA.
- Santos J.R., Haines Y.Y., 2004. Modelling the demand reduction Input - Output (I - O) inoperability due to terrorism of interconnected infrastructures. *Risk Analysis: An International Journal*, 24, 1437-1451.
- Zhang C., Kong J.J., Simonovic S.P., 2018. Restoration resource allocation model for enhancing resilience of interdependent infrastructure systems. *Safety Science*, 102, 169-177.
- Zhang J., 2020. Infrastructure risk management of industrial parks based on inoperability input-output model, Master thesis, Qingdao University of Science and Technology, Qingdao, China (in Chinese).