CHEMICAL ENGINEERING TRANSACTIONS
VOL. 88, 2021

A publication of AIDIC
The Italian Association of Chemical Engineering
Online at www.cetjournal.it

Guest Editors: Petar S. Varbanov, Yee Van Fan, Jiří J. Klemeš

**ISBN** 978-88-95608-86-0; **ISSN** 2283-9216

DOI: 10.3303/CET2188006

# A Multi-Region Input-Output Model for Optimizing Trade Under Footprint Constraints

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Multi-region input-output models have been used extensively for describing and analysing the production and trade of goods. However, there have been no prior publications on the use of such models to determine optimal trade benchmarks considering footprint limits. Such benchmarks can provide valuable insights for economic planning, in the same manner that pinch analysis targets provide insights in process integration. In this work, a mixed-integer linear programming input-output model is developed to minimize environmental footprint by adjusting production and trade levels in a cluster of regions or countries with specified final demand constraints. A case study based on a simplified, low-resolution model of the Philippines using land footprint as objective function is solved to illustrate this new approach.

## 1. Introduction

The production of goods and provision of services in modern economies have environmental impacts that are often not directly observable. Such impacts can be quantified using different footprint metrics that measure different sustainability dimensions (Čuček et al., 2012). Environmentally extended input-output (EEIO) models provide a systematic means of estimating footprints using public statistics (Aguilar-Hernandez et al., 2018). These models are based on the standard input-output formulation first proposed by Leontief (1936) to describe the network structures of economic systems. Input-output models are based on public records of intersectoral transactions, which are usually measured in monetary units. Use of physical flows in these models augments the embedded economic principles, and allows more accurate representation of real systems (Merciai, 2019). A comprehensive description of the basic input-output model and common variants can be found in the book by Miller and Blair (2009). A brief tutorial can also be found in the chapter by Tan et al. (2017).

The EEIO model has been used as the mathematical framework for sustainability constructs such as Industrial Ecology (IE) (Duchin, 1992), Circular Economy (CE) (Aguilar-Hernandez et al., 2018), and the Water-Energy-Carbon Nexus (Wang et al., 2020c). EEIO models have been used to track virtual flows of footprints embedded in trade. For example, this approach has recently been used to analyze the economies of China (Wang et al., 2020a), the European Union (Wang et al., 2020b), and the Asia-Pacific Region (Yang et al., 2020). A review of applications is given by Liu et al. (2017). In addition, Mathematical Programming (MP) models based on the input-output framework have also been proposed. The excess degrees of freedom can arise from technology choice (Duchin and Levine, 2011) or structural flexibility (Cayamanda et al., 2017). Heijungs et al. (2014) used this approach in a model that maximizes economic welfare within the limits of the planetary boundaries. MP models based on the input-output framework have been used to assess decarbonization options in the Philippines (Cayamanda et al., 2017) and China (Su et al., 2021). Capital stock for renewable energy generation was included in an EEIO model by Kang et al. (2020). Rojas-Sanchez et al. (2019) developed a multi-objective optimization model using the EEIO framework to account for different sustainability dimensions. EEIO models have also been combined with Process Integration (PI) tools such as P-graph (Aviso et al., 2015) and Pinch Analysis (PA) (Tan et al., 2018). Models have also been developed to optimize trade of industrial (Aviso et al., 2011) and agricultural (Aviso et al., 2018) goods under water footprint constraints.

Despite the prevalence of multi-region EEIO models for quantifying virtual flows of footprints embodied in trade, these methods are usually descriptive, and are rarely applied to the problem of planning optimal production and

Paper Received: 28 June 2021; Revised: 19 September 2021; Accepted: 1 October 2021
Please cite this article as: Aviso K.B., Tan R.R., Yu K.D., 2021, A Multi-Region Input-Output Model for Optimizing Trade Under Footprint Constraints, Chemical Engineering Transactions, 88, 37-42 DOI:10.3303/CET2188006

trade. To address this research gap, this paper develops an optimization model based on a multi-region EEIO formulation. It can be used to provide benchmarks to guide decision-makers in economic planning, in the same manner that utility targets guide process design or retrofit in PI (Klemeš, 2013). The rest of this paper is organized as follows. Section 2 gives the formal problem statement, while Section 3 discusses the model formulation. In Section 4, a three-region case study based on the Philippine economy is solved to illustrate the model. Finally, Section 5 gives the conclusions and discusses prospects for future research.

## 2. Problem statement

The optimization problem may be formally stated as follows:

- Given an economic system which consists of R regions;
- Given that each region consists of S different economic sectors;
- Given fixed input ratios for each unit output of sector i in region l;
- Given the environmental intensity of each economic sector i for environmental impact category k;
- Given regional environmental impact limits and economic productivity targets;

The problem is to determine the optimal trading matrix between economic sectors within a region and between regions to minimize the total footprint of the system. Figure 1 shows an example of a system with two regions and two sectors.

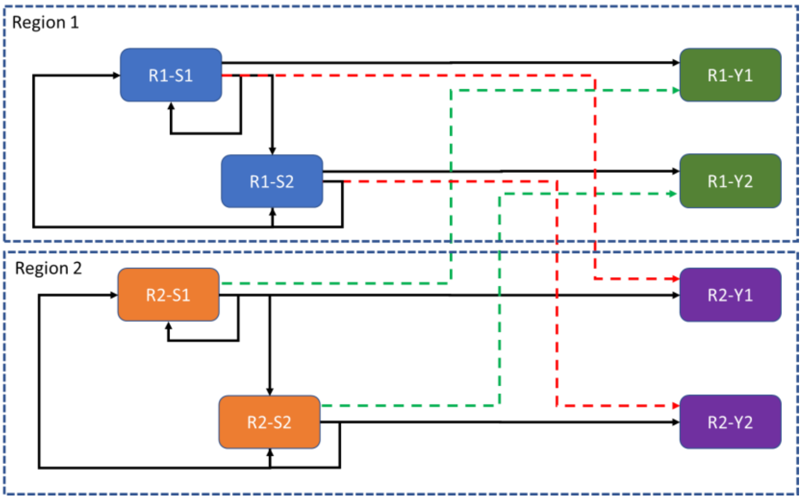


*Figure 1: Example of a two-region, two-sector economic system with interregional trade flows shown as broken lines*

## 3. Model formulation

The objective is to minimize the total environmental footprint of the system as indicated in Eq(1) where $z_{kl}$ is the total environmental footprint k in region l. Eq(2) indicates that the final demand of commodity i in region l, $y_{il}$, is the sum of the net internal production in region l and the difference between imported goods i from region m to region l, $u_{iml}$, and amount of exported goods i from region l to region m, $v_{ilm}$, for all regions; $Id_{ij}$ represents the elements of the identity matrix $\overrightarrow{\text{Id}}$, $a_{ijl}$ represents the internal transactions between sector i and sector j in region l and $x_{jl}$ is the size of sector j in region l. This formulation assumes that any product i exported from a region l to region m is used to satisfy the final demand in region m, while any product i imported by region l from region m is used to satisfy the final demand requirement in region l. The traded goods are not used as raw material in any of the sectors in the receiving region. The associated environmental footprint of the economic activities can be calculated using Eq(3) where $B_{jkl}$ is the associated environmental footprint k for each unit output of sector j in region l and $z_{kl}$ is the over-all environmental footprint k of region l when all sectors are considered. Variable $r_{iml}$ ($s_{ilm}$) is activated when there is an import (export) stream for commodity i from (to) region m to (from) region l as indicated in Eq(4) and Eq(5) where $r_{iml}$ and $s_{ilm}$ are binary variables which assume a value of 1 if trade of commodity i exists between regions m and l as indicated in Eq(6) and Eq(7), respectively. Each commodity i in region l can only be either exported or imported to or from region m, as indicated in Eq(8). Export and import of the same commodity between two regions cannot occur simultaneously. Note that Eq(4) to Eq(8) excludes self-trade. These conditions may be implemented automatically in some optimization software; otherwise, it is necessary to manually add equivalent constraints setting self-trade streams to zero. The total environmental footprint k in region l should not exceed the regional target environmental impact level, $Z_{kl}^{max}$ (Eq(9)) which can represent available resources in a region. In addition, the final demand for each sector in each region should meet a predefined minimum level, $Y_{il}^{min}$ (Eq(10)).

$$\min \sum_{k}^{P} \sum_{l}^{R} z_{kl} \tag{1}$$

$$\sum_{j}^{M} (Id_{ijl} - a_{ijl}) x_{jl} + \sum_{m}^{R'} (u_{iml} - v_{ilm}) = y_{il} \quad \forall\, i \in S, \forall\, l \in R \tag{2}$$

$$\sum_{j}^{S} B_{jkl} x_{jl} = z_{kl} \quad \forall\, k \in P, \forall\, l \in R \tag{3}$$

$$u_{iml} \leq M r_{iml} \quad \forall\, i \in S, \forall\, l, m, l \neq m \in R \tag{4}$$

$$v_{ilm} \leq M s_{ilm} \quad \forall\, i \in S, \forall\, l, m, l \neq m \in R \tag{5}$$

$$r_{iml} \in \{0,1\} \quad \forall\, i \in S, \forall\, l, m, l \neq m \in R \tag{6}$$

$$s_{ilm} \in \{0,1\} \quad \forall\, i \in S, \forall\, l, m, l \neq m \in R \tag{7}$$

$$r_{ilm} + s_{ilm} \leq 1 \quad \forall\, i \in S, \forall\, l, m, l \neq m \in R \tag{8}$$

$$z_{kl} \leq Z_{kl}^{max} \quad \forall\, k \in P, \forall\, l \in R \tag{9}$$

$$y_{il} \geq Y_{il}^{min} \quad \forall\, i \in S, \forall\, l \in R \tag{10}$$

Note that this model is a mixed integer linear program (MILP) and can be solved to global optimality without major computational issues. In the case study that follows, it is implemented and solved using Microsoft Excel.

## 4. Case study

This case study uses official data from the Philippines for 2012 (PSA, 2018) coupled with land use data for footprint constraints (CIA, 2021). Due to space constraints, only three aggregated economic sectors (i.e., agriculture, industry, and services) for the three major regions of the country (Luzon, Visayas, and Mindanao) are used. Each sector within each region is coded for ease of reference; for example, the agriculture sector of Luzon is denoted as R1-A, and a similar convention is used for the others. Each sector is assumed to produce a homogeneous set of goods that are fully interchangeable, regardless of the region of origin. The technical coefficients of the input-output system are shown in Table 1. Each column represents the inputs required per unit of output of a sector in a given region; the ratio is expressed in terms of economic value. For example, the entries in the R1-A column indicate that, on average, every PHP of output of the agriculture sector in Luzon requires as inputs of PHP 0.07 of other agricultural products, PHP 0.13 of industrial products, and PHP 0.04 of services (note that the market exchange rate as of July 2021 is about PHP 60 = € 1).

*Table 1: Technical coefficients for case study*

| | R1-A | R1-I | R1-S | R2-A | R2-I | R2-S | R3-A | R3-I | R3-S |
|---|---|---|---|---|---|---|---|---|---|
| R1-A | 0.07 | 0.08 | 0.01 | - | - | - | - | - | - |
| R1-I | 0.13 | 0.36 | 0.17 | - | - | - | - | - | - |
| R1-S | 0.04 | 0.10 | 0.16 | - | - | - | - | - | - |
| R2-A | - | - | - | 0.07 | 0.08 | 0.01 | - | - | - |
| R2-I | - | - | - | 0.13 | 0.33 | 0.17 | - | - | - |
| R2-S | - | - | - | 0.03 | 0.10 | 0.15 | - | - | - |
| R3-A | - | - | - | - | - | - | 0.07 | 0.08 | 0.01 |
| R3-I | - | - | - | - | - | - | 0.13 | 0.34 | 0.18 |
| R3-S | - | - | - | - | - | - | 0.03 | 0.09 | 0.14 |

This case study focuses on land footprint, which must be a small fraction of actual land area to allow for a buffer that will provide essential ecosystem services (Rockström et al., 2009). Table 2 presents the land footprint coefficient and minimum final demand of each sector in each region. The land footprint coefficient gives the area required per unit of annual economic output, which is notably larger in magnitude for agriculture than for the industry or services sectors. The final demand column gives the minimum amount of goods from each sector in each region that is purchased by households for final consumption, firm consumption, government consumption, exports, and imports. The sum of the entries in this column is known as the gross domestic product (GDP).

*Table 2: Land footprint coefficients and final demand for case study*

| | Land footprint coefficient ($km^2$·y/Billion PHP) (CIA, 2021) | Minimum final demand (Billion PHP) (PSA, 2018) |
|---|---|---|
| R1-A | 49.77 | 606 |
| R1-I | 0.15 | 2,992 |
| R1-S | 0.15 | 6,058 |
| R2-A | 50.16 | 220 |
| R2-I | 0.14 | 504 |
| R2-S | 0.14 | 899 |
| R3-A | 49.92 | 475 |
| R3-I | 0.15 | 573 |
| R3-S | 0.15 | 884 |

Solving the model gives the optimal trading matrix shown in Table 3 with a corresponding land footprint of 117,053 $km^2$. Luzon (R1) imports PHP 2,992 billion ($10^9$) worth of industry products from Mindanao (R3), but also exports PHP 477 billion worth of agricultural products to that region. Visayas (R2) imports PHP 220 billion worth of agricultural products from R3, and exports PHP 3,565 billion worth of industrial products there. It should be noted that alternative solutions may exist with the same optimal objective function value. Such solutions can be enumerated by progressively adding integer cut constraints to the MILP model.
The best result for the no-trade scenario gives a land footprint of 120,104 $km^2$, which is 3 % larger than the optimal trade footprint. The total outputs of the different sectors for both scenarios are shown in Figure 2. Note that, when trade is optimized, R1 industry output drops dramatically, while the output levels of its other two sectors remain similar to those of the no-trade scenario. On the other hand, R2 produces more output across all three sectors, while R3 produces less. The distribution of land footprints for both scenarios are shown in Figure 2. Trade increases the footprints in R1 and R2 and decreases the footprint in R3. Since the respective land areas of the three regions are 105,000 $km^2$, 100,000 $km^2$, and 95,000 $km^2$, the optimal trade solution uses up 70 %, 30 %, and 15 % of the available land resource.

*Table 3: Optimal trading matrix with flow of goods in billion PHP*

| | Import from R1 | Import from R2 | Import from R3 | Export to R1 | Export to R2 | Export to R3 |
|---|---|---|---|---|---|---|
| R1-A | - | - | - | - | - | 477 |
| R1-I | - | - | 2,992 | - | - | - |
| R1-S | - | - | - | - | - | - |
| R2-A | - | - | 220 | - | - | - |
| R2-I | - | - | - | - | - | 3,565 |
| R2-S | - | - | - | - | - | - |
| R3-A | 477 | - | - | - | 220 | - |
| R3-I | - | 3,565 | - | 2,992 | - | - |
| R3-S | - | - | - | - | - | - |

In practical terms, the optimal trading matrix (or matrices) can serve as a guide for resource-constrained economic planning. However, real economies cannot be controlled directly in the same manner as process plants. Knowledge of optimal trading patterns can serve as a benchmark that is useful in the same manner that PA targets guide designers in PI problems.

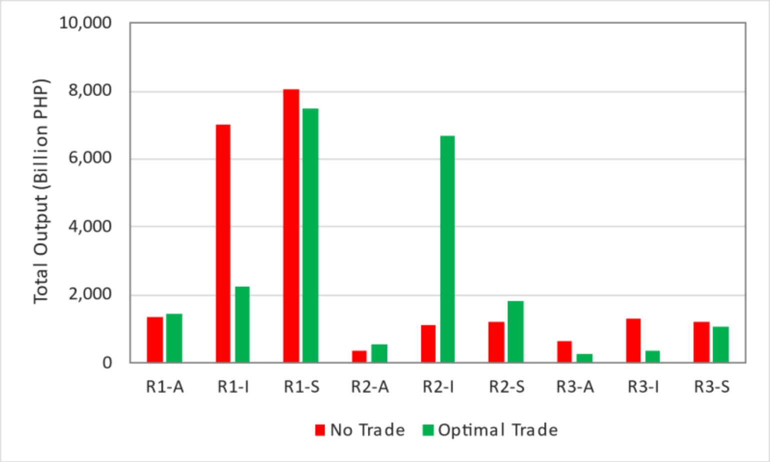


*Figure 2: Sector outputs without trade and with optimal trade*

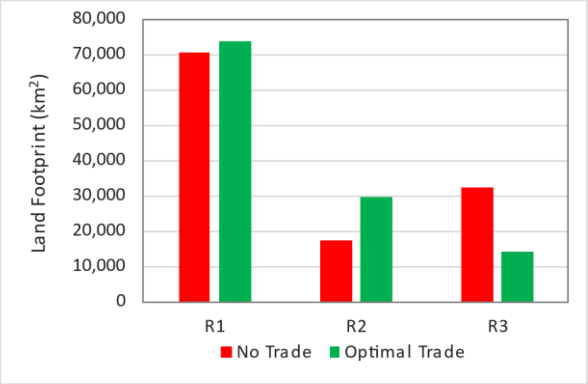


*Figure 3: Land footprints without trade and with optimal trade*

## 5. Conclusions

An input-output optimization model has been developed in this work for optimizing trade across multiple regions under footprint constraints. The model was illustrated using a case study of trade among the three major regions of the Philippines considering land footprint limits. Unlike conventional, descriptive multi-region input-output models, this formulation allows ideal system states to be determined; such solutions can be used like utility targets in PI to guide decision-makers, so that production levels of specific goods can be planned while accounting for footprint constraints. Future work should focus on extending this modelling framework to account for structural or technological change. Growth and expansion plans can also be included in future models. A multi-objective extension should also be developed to account for different sustainability dimensions.

### References

Aguilar-Hernandez G.A., Sigüenza-Sanchez C.P., Donati F., Rodrigues J.F.D., Tukker A., 2018, Assessing circularity interventions: A review of EEIOA-based studies, Journal of Economic Structures, 7, Article 14.

Aviso, K.B., Cayamanda, C.D., Solis, F.D.B., Danga, A.M.R., Promentilla, M.A.B., Yu, K.D.S., Santos, J.R., Tan, R.R., 2015, P-graph approach for GDP-optimal allocation of resources, commodities and capital in economic systems under climate change-induced crisis conditions, Journal of Cleaner Production, 92, 308–317.

Aviso, K.B., Holaysan, S.A.K., Promentilla, M.A.B., Yu, K.D.S., Tan, R.R., 2018, A multi-region input-output model for optimizing virtual water trade flows in agricultural crop production, Management of Environmental Quality, 29, 63–75.

Aviso, K.B., Tan, R.R. Culaba, A.B., Cruz Jr., J.B., 2011, Fuzzy input-output model for optimizing eco-industrial supply chains under water footprint constraints, Journal of Cleaner Production, 19, 187–196.

Cayamanda, C.D., Aviso, K.B., Biona, J.B.M., Culaba, A.B., Promentilla, M.A.B., Tan, R.R., Ubando, A.T., 2017, Mapping a low-carbon future for the Philippines: Scenario results from a fractional programming input-output model, Process Integration and Optimization for Sustainability, 1, 293–299.

CIA, 2021, The World Factbook, US Central Intelligence Agency <www.cia.gov/the-world-factbook/countries/philippines/> accessed 02.02.2021.

Čuček, L., Klemeš, J. J., Kravanja, Z., 2012, A review of footprint analysis tools for monitoring impacts on sustainability, Journal of Cleaner Production, 34, 9–20.

Duchin F., 1992, Industrial input-output analysis: implications for industrial ecology, Proceedings of the National Academy of Sciences of the United States of America, 89, 851–855.

Duchin F., Levine S., 2011, Sectors may use multiple technologies simultaneously: the rectangular choice-of-technology model with binding factor constraints, Economic Systems Research, 23, 281–302.

Heijungs, R., De Koning, A., Guinee, J.B., 2014, Maximizing affluence within the planetary boundaries, International Journal of Life Cycle Assessment, 19, 1331–1335.

Kang, J., Ng, T.S., Su, B., Yuan, R. 2020, Optimizing the Chinese electricity mix for $CO_2$ emission reduction: An input-output linear programming model with endogenous capital, Environmental Science and Technology, 54, 697–706.

Klemeš, J.J. (Ed.), 2013, Handbook of Process Integration (PI): Minimisation of Energy and Water Use, Waste and Emissions, Elsevier/Woodhead Publishing, Cambridge, UK.

Leontief W.W. 1936, Quantitative Input and Output Relations in the Economic Systems of the United States, Review of Economics and Statistics, 18, 105–125.

Liu, X., Klemeš, J.J., Čuček, L., Varbanov, P.S., Qian, Y., 2017, Virtual carbon and water flows embodied in international trade: a review on consumption-based analysis, Journal of Cleaner Production, 146, 20–28.

Merciai, S., 2019, An input-output model in a balanced multi-layer framework, Resources, Conservation and Recycling, 150, 104403.

Miller, R.E., Blair, P.D., 2009, Input-output Analysis: Foundations and Extensions, 2nd ed., University Press, Cambridge, UK.

Philippine Statistics Authority, 2018, 2012 Input-Output Tables, [data file], Philippine Statistics Authority <psa.gov.ph/sites/default/files/2012%20Input-Output%20Tables%20-%20Transaction%20Table.xlsx> accessed 02.02.2021.

Rockström, J., Steffen, W., Noone, K., Persson, A., Chapin, F.S., Lambin, E.F., Lenton, T.M., Scheffer, M., Folke, C., Schellnhuber, H.J., Niykvist, B., De Wit, C.A., Hughes, T., Van der Leeuw S., Rodhe, H., Sorlin, S., Snyder, P.K., Constanza, R., Svedin, U., Falkenmark, M., Karlberg, L., Corell, R.W., Fabry, V.J., Hansen, J., Walker, B., Liverman, D., Richardson, K., Crutzen, P., Foley, J.A., 2009, A safe operating space for humanity, Nature, 461, 472–475.

Rojas-Sanchez, D., Hoadley, A.F.A., Khalilpour, K.R., 2019, A multi-objective extended input–output model for a regional economy, Sustainable Production and Consumption, 20, 15–28.

Su, Y., Liu, X., Ji, J., Ma, X., 2021, Role of economic structural change in the peaking of China's $CO_2$ emissions: An input–output optimization model, Science of the Total Environment, 761, Article 143306.

Tan R.R., Yu K.D.S., Aviso K.B., Promentilla M.A.B., 2017, Input–output modeling approach to sustainable systems engineering, Chapter In: M. Abraham (Ed.), Encyclopedia of Sustainable Technology, Elsevier, Amsterdam, The Netherlands, 519–523.

Tan R.R., Aviso K.B., Foo D.C.Y., 2018, Carbon emissions pinch analysis of economic systems, Journal of Cleaner Production, 182, 863-871.

Wang, X.-C., Klemeš, J.J., Wang, Y., Dong, X., Wei, H., Xu, Z., Varbanov, P.S., 2020a, Water-Energy-Carbon Emissions nexus analysis of China: An environmental input-output model-based approach, Applied Energy, 261, Article 114431.

Wang, X.-C., Klemeš, J.J., Varbanov, P.S., 2020c, Water-energy-carbon nexus analysis of the EU27 and China, Chemical Engineering Transactions, 81, 469–474.

Yang, L., Wang, Y., Wang, R., Klemeš, J.J., Almeida, C.M.V.B., Jin, M., Zheng, X., Qiao, Y., 2020, Environmental-social-economic footprints of consumption and trade in the Asia-Pacific region, Nature Communications, 11, Article 4490.