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Mathematical Modeling the Relaxation Impact of Water Pollutions in the System of Reservoirs under the One-time Emissions through a Broken Dam

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The paper deals with modelling the process both of peak catastrophic emissions of contaminated liquids or wastewater during dams break and describing the spread of pollution caused by these incidents in adjacent reservoirs. In the first part of the paper the methods and algorithms for numerical solution of problems of water filtration through broken dams. The novelty of this part is that models and algorithms for calculating pollutions in ground materials both with isotropic and with orthotropic properties have been developed. In the next part of the paper the simple model describing the relaxation impact of the pollutions in the system of interconnecting water reservoirs after wastewater emissions through the broken dams has been submitted. The appropriate code and soft for the numerical experiments at different pollution discharge durations and various relations of the filtration coefficients have been developed and tested. The pollution relaxation times in the system of water reservoirs have been studied.

1. Introduction

Problems of mathematical modelling in ecology nowadays acquire the more significance the more empirical material is accumulated (Taozhen Huang and Wei Zheng, 2018). The understanding of interaction essence between technogenic processes and natural phenomena becomes deeper (Vojtesek and Dostal, 2014). This interaction is characterized by extreme variety and complexity of dynamic processes (Chiu-Sung Lin et al., 2015). At the same time it is very difficult, and sometimes impossible even in the simplest cases, to create complete, precise mathematical models of ecological situations (Pereira et al., 2012). And also it is very difficult to carry out a comprehensive analysis of such models (Alam and Ptathak, 2010).

Therefore, along with development of large complex models and their numerical investigation, the elaboration of simplified models, based on heuristic considerations and reflecting at the same time main influencing factors and qualitative regularities, remains very relevant (Kobelyev, 2000). Nowaday, the study of pollutions in the water reservoir systems is of great practical interest (Yang et al., 2007). The results of these investigations find an extensive use in resolving different environmental problems (Zhen-Gang, 2008).

In the submitted work the simplified models for evaluating the dynamics of pollution in running communicating water reservoir systems under the impact of industrial wastewater discharges both with consideration, and without consideration of water filtration in the soil with various properties have been developed and tested (Shakirov and Kurakbayeva, 2007).

The problems of monitoring fluid filtration through hydraulic constructions are very important for determining the design and size of these structures (Brebbia et al., 2012). Dams can be built from various materials, and they can be waterproof or porous in order to allow water to penetrate after the flow energy has been dissipated. Deficiencies in the design lead to great costs associated with ongoing maintenance. Water is filtered under the bases of these constructions, through embedded details and with bypassing their junctions on the banks. The filtration flow exerts pressure on the structures, washes the soil under them and promotes the spread of harmful impurities (Kurakbayeva et al., 2013). The study of accidents of hydraulic structures

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leads to the conclusion that a lot of them occur due to the destructive effect of filtration. Therefore, it is extremely important to have methods that allow making the right forecast for the filtration intensity and determining the optimal actions combating to these problems.

The main goal of this paper is to submit mathematical models and results of numerical simulation for describing the relaxation impact of water pollutions under the one-time emissions through a broken dams. The paper consists of the two sections. The first section deals with modelling the process of filtering water through a dam taking into account the possible breakthroughs in the cases of ground materials with isotropic or with orthotropic properties. The second section is devoted to the dynamic model of the pollution of interconnected water reservoirs by the indecomposable impurity after the breakthrough.

2. Water filtration through the soil dam with damaged pools and breakthroughs

The statistic investigations show that more than half of all accidents of the soil dams occur owing to the water filtration problems. Therefore, in the process of designing the dams it is necessary to carry out filtration calculations. In the course of these calculations the depression curve position in the dam body, the filtration flow gradients and the rate of filtration, the filtration flow consumption through the dam and through its base must be determined. This section is devoted to the method for calculate the change in the position of the filtration flow free boundary in the cases of isotropic and orthotropic ground materials, i.e. when the filtration coefficients depend on the direction of flow in a porous medium. Moreover, the situations when the upper and the lower dam pools could be damaged have also been considered. Such cases are quite often common in real conditions (Kurakbayeva and Shakirov, 2007).

2.1 Methods and results of the calculations

The boundary element method (BEM) (Brebbia, 2012), which demonstrates great possibilities for solving problems with free surfaces, has been used. The appropriate boundary element arrangement schemes for the soil unit at the flow with the free surface through the dam with the damaged upper and lower pools are shown in Figures 1 and 2.

The governing equation for velocity potential u in the coordinate axes associated with the orthotropic directions can be written in the two-dimensional case in the form

$$k_1 \frac{\partial^2 u}{\partial x_1^2} + k_2 \frac{\partial^2 u}{\partial x_2^2} = 0 \qquad (1)$$

with the boundary conditions

$q = 0$ at the impenetrable boundary (line AF in Figure 1), (2)

$u = const$ at the surfaces ABC and EF in porous medium, (3)

$u = x_2$ at the filter surface DE, (4)

$u = x_2$ and *q=0* at the free boundary CD. (5)

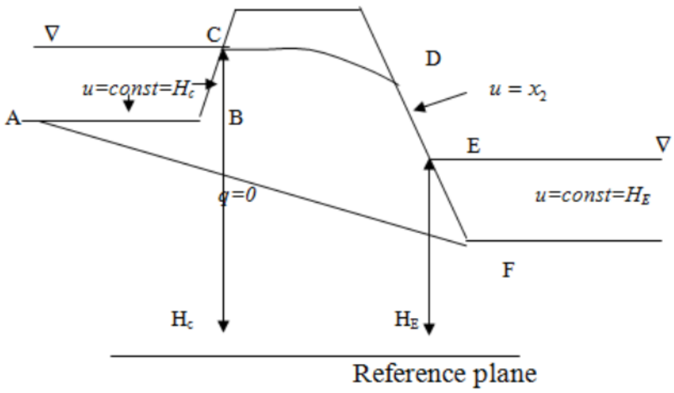


Figure 1: Scheme and the boundary conditions for the problem on water flow through the dam

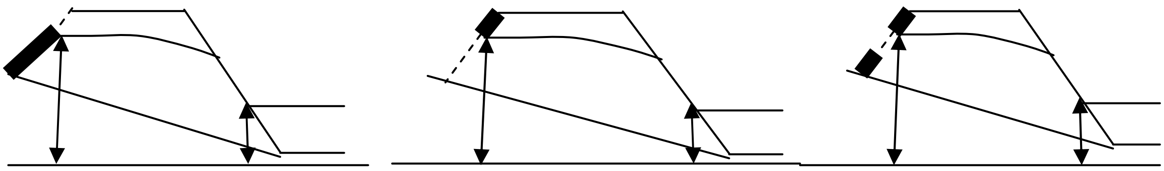
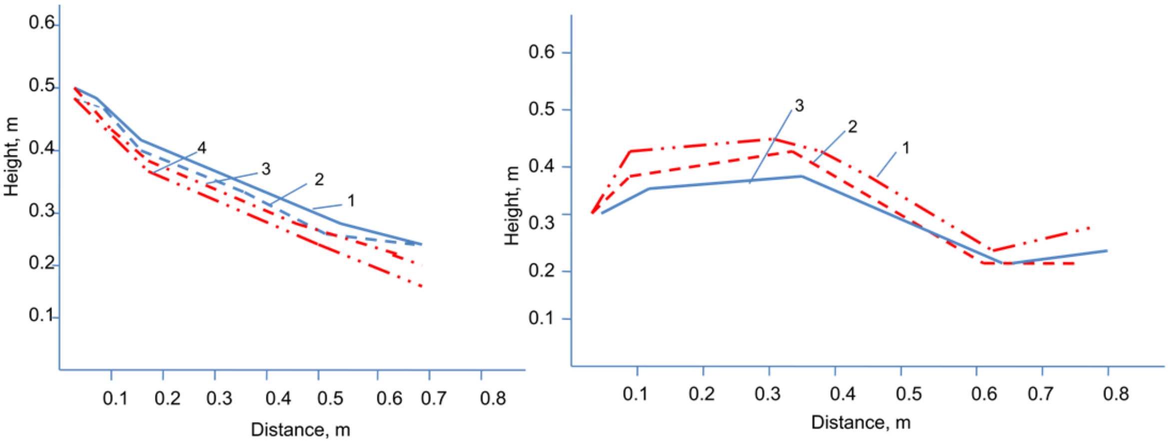


Figure 2: Various combinations of partial dam breaks



A) B)

A): *1- water filtration through a dam for an isotropic medium; 2- breakthrough at the upper pool for an isotropic medium; 3- breakthrough at the upper pool for an orthotropic medium for k_1 =0.075, k_2 =0.25; 4- breakthrough at the upper pool for an orthotropic medium for k_1 =0.4, k_2 =0.8.*
B): *1- breakthrough at the central part of the partition (k_1 =0.075, k_2 =0.25); 2- breakthrough at the central part of the partition (k_1 =0.4, k_2 =0.8); 3- breakthrough at the central part of the partition for an isotropic medium.*

Figure 3: Comparison between the calculated results for the flow potentials. A) *-different types of breaks of the upper and lower dam pools;* B)*- breakthrough at the central part of the partition*

At the numerical calculation in this problem the free surface initial position is defined in an arbitrary way, and, besides, in all points of this surface q=0 convention is accepted. The calculated potential value for each nodal point of the free surface is compared with the water surface height. If the difference among them turns out to be greater than the maximal permissible error, than this difference is algebraically summed up with the surface height in the corresponding nodal point, and a new iteration is carried out.
Some calculation results are shown in Figure 3.
The computer simulation showed that in the case of a dam breaking both at the upper and low parts of the partition (Figure 3 (A) and at its centre (Figure 3 (B), the least wetting was observed for isotropic materials.
The results of modelling and numerical experiments confirm the possibility of adapting the boundary element method for the calculations of filtering through continuous dams and dams with breakthroughs for cases of the complex, heterogeneous media. This opens up prospects for an adequate description of the filtration processes through dams in conditions close to real ones (Kurakbayeva et al., 2013).

3. Relaxation impact of water pollutions in the system of interconnected reservoirs

In general, industrial wastes propagated over the network of interconnected water reservoirs and form a rather complicated picture of pollutions (Yang et al., 2007). A subject of the investigation in this section is the system of three reservoirs. It is supposed that a source of pollutions of the given intensity is located on a bank of one of these reservoirs (Figure 4). The work on the model was focused on the analysis of impurity concentration in the system of communicating reservoirs with consideration of the impurity diffusion in the reservoirs and filtration in the soil. The characteristic times of the impurity propagation within each reservoir were introduced (Kurakbayeva, 2008). The problems related to dependence of such characteristics as a relaxation period in each of the reservoirs on the duration of the discharges and water filtration coefficients in the soil were studied. The objective of the research was to reveal dependencies of the impurity concentrations in each of the reservoirs on filtration intensities in channels, connecting these water basins during specified time (Kurakbayeva, 2008). The submitted model as a whole is generally consistent with the concept of network-connected CSTRs (Hurtado et al., 2015). However, unlike this concept, in the model under consideration, the influence of filtration flows from each of the reservoirs into the soil can be taken into account.

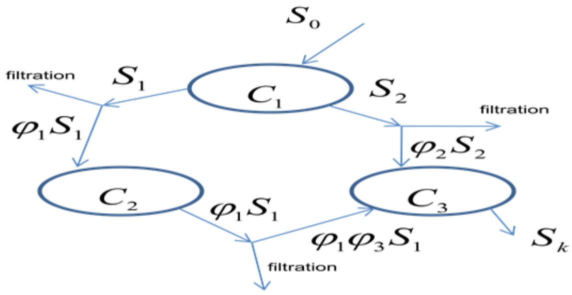


Figure 4: Scheme of flows in the system of three communicating water reservoirs with consideration of the filtration

The simplified dynamic model of the water reservoirs pollutions by the indecomposable impurity taking into account the filtration process can be written as follows:

$$\begin{cases} V_1 \dfrac{dc_1}{dt} = I(t) - s_0 c_1 = I(t) - (s_1 + s_2) c_1, \\ V_2 \dfrac{dc_2}{dt} = \varphi_1 s_1 c_1 - \varphi_1 s_1 c_2, \\ V_3 \dfrac{dc_3}{dt} = \varphi_3 s_2 c_1 + \varphi_3 \varphi_1 s_1 c_2 - (\varphi_1 \varphi_3 s_1 + \varphi_2 s_2) c_3. \end{cases} \tag{6}$$

Here c_1, c_2, c_3 are the mean impurity concentrations in the first, second and third reservoirs respectively, kg/m^3; s_0, s_1, s_2, s_k are water flows rates, m^3/s; V_1, V_2, V_3 are the water reservoirs volumes, m^3; I is the intensity of discharges, kg/s; t is time, s; $\varphi_1, \varphi_2, \varphi_3$ are the filtration coefficients.

The following conditions were accepted for the system (6):

$$s_1 + s_2 = s_0; \quad s_k = \varphi_1 \varphi_3 s_1 + \varphi_2 s_2 \tag{7}$$

The concentrations of impurities in the initial discharge moment in the water reservoirs were respectively:

$$c_1(0) = c_{1(0)}, \; c_2(0) = c_{2(0)}, \; c_3(0) = c_{3(0)} . \tag{8}$$

Figures 5 and 6 depict temporal dynamical diagrams of the impurity concentration both during and after the peak pollution. These qualitative diagrams testify for the mean impurities concentrations in each of the reservoirs begin to increase after start of the discharge, and then they stabilize after reaching the defined values.

It is clear that the constant common concentration is established in each of the reservoirs with a sufficiently long duration of the constant intensity discharge. However, in the short discharge duration, the concentrations dynamics in each of the reservoirs can significantly vary (Kurakbayeva et al., 2015). The impurity concentration in the first reservoir after start of the discharge begins to increase more rapidly in comparison with concentrations in the second and third reservoirs. For example, the next three temporal periods were chosen: 4,65·105 s, 4·105 s and 1·105 s. So, in the course of experiment at T=4,65·105 the impurity concentrations in the water reservoirs during the discharge with in the first, second and third reservoirs were respectively 99,991 kg/m3, 99,653 kg/m3, 99,413 kg/m3. Calculations of the impurity concentrations curves in each of the reservoirs (C_1, C_2, C_3) in a some specified set of filtration coefficients were also carried out. Particularly, below there is example for the next filtration coefficients: **1.** $\varphi_1 = 0.2, \varphi_2 = 0.6, \varphi_3 = 0.8$; **2.** $\varphi_1 = 1$,

$\varphi_2 = 1, \varphi_3 = 1$; **3.** $\varphi_1 = 0.2, \varphi_2 = 0.2, \varphi_3 = 0.2$; **4.** $\varphi_1 = 0.8, \varphi_2 = 0.8, \varphi_3 = 0.8$; **5.** $\varphi_1 = 0.2, \varphi_2 = 0.8, \varphi_3 =$ 0.6; **6**. $\varphi_1 = 0.8, \varphi_2 = 0.6, \varphi_3 = 0.2$.

The discharge duration has been varied from 200000s to 800000s. Below, some results obtained in the course of a numerical experiment are submitted.

Figures 5 and 6 show diagrams for qualitative dependence of concentration on time in the reservoirs at $\varphi_1 =$ 0,2, $\varphi_2 =$ 0,8, $\varphi_3 =$ 0,6 in different T_1, T_2, T_3, T_4.

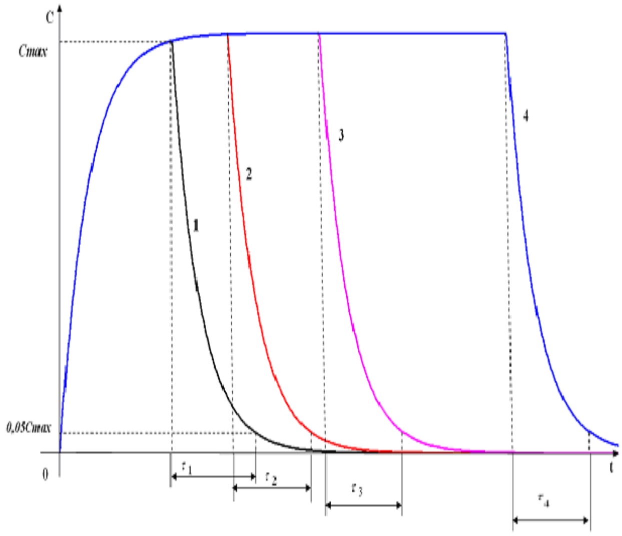


Figure 5: Qualitative dependences of the impurity concentrations on time in the first reservoir in different discharge durations T_1, T_2, T_3, T_4 ($\varphi_1 = 0.2$, $\varphi_2 = 0.8$, $\varphi_3 = 0.6$)

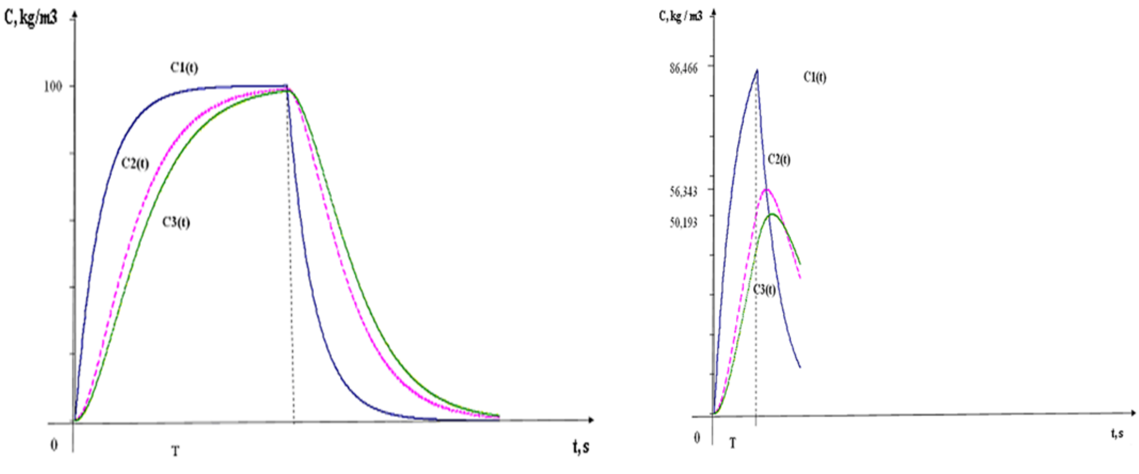


Figure 6: Qualitative diagrams of the impurity concentration temporal dynamics in three water reservoirs (corresponding curves C1, C2, C3) during and after the discharge in: A - *T=4·10^5 s ;* B- *T=1·10^5 s.*

Then, the relaxation periods τ, i.e. periods during which the concentration drops after the discharge completion from $C_{\max}$ to $0.05C_{\max}$, were determined at each discharge duration. The numerical experiments showed that in different discharge durations the relaxation periods in the first reservoir are equal and consist approximately 150000 s in the given discharge amplitude. The second reservoir was considered similarly with the same filtration coefficients and the same discharge duration values. Investigating the third reservoir in the similar way, we obtained another picture. The relaxation periods τ during which the concentration significantly drops after the discharge completion from C_{max} to $0.05C_{max}$, were determined at each discharge duration. The numerical experiments showed that in different discharge durations the relaxation periods in the first reservoir remain the same and consist approximately 150000 s in the given discharge amplitude. The similar behaviour has been observed for the second reservoir with the same filtration coefficients and the same discharge duration values. The concentrations dynamics in the third reservoir was another. The relaxation period in the third reservoir at $\varphi_1 = 0.2$, $\varphi_2 = 0.8$, $\varphi_3 = 0.6$ achieved the largest value at the discharge duration in 800000 s, and it consisted 641000 s. The relaxation period here increased with increase in the discharge duration.

4. Conclusions

A combined simplified model for describing the spread of harmful impurities and wastewater during one-time breakthroughs of industrial dams has been developed. The novelty of the model lies in the possibility of taking into account the influence of the location of the breakthrough in the body of the dam, as well as the orthotropy of the medium in which the effluent is filtered, on the dynamics of relaxation of the impurities concentration in the system of water reservoirs after breaking the dam. The numerical experiment confirmed good adaptation possibilities of the developed model, allowing observe the temporal dynamic and relaxation of pollution in the system of reservoirs during the wastewater discharge. The appropriate code and software have been developed too. The submitted model can be recommended for analysis of pollutions dynamic and substantiated preliminary forecasting of environmental state in the area of industrial enterprises.

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