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Calculating the Efficiency of Complex Shaped Fins

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In the calculation of heat exchangers with individual finned tubes or continuous fin heat exchangers, i.e., plate-fin and tube heat exchangers (PFTHEs), it is necessary to determine the efficiency of complex fins. Usually, these are circular or rectangular fins mounted on circular, elliptical, or oval tubes as well as flattened tubes. In case of PFTHEs, the continuous fin is divided into virtual fins, which are rectangular for an inline pipe layout or hexagonal for a staggered pipe arrangement. This paper presents a procedure based on the finite element method for determining the efficiency of fins of any shape placed on tubes of any shape. The article shows examples of calculating the efficiency of virtual fins in most commonly used PFTHEs. The research also assesses the accuracy of determining the efficiency of complex-shaped fins using approximated methods such as the equivalent circular fin method (Schmidt's method) and the sector method. The efficiency of a fin as a function of heat transfer coefficient for four different, most common geometries in PFTHEs was depicted. An example of determining the efficiency of a hexagonal fin using various methods for air velocities measured at the exchanger inlet was presented.

1. Introduction

Straight and circular fins have wide applications, i.e., in electronic components, ventilation systems (Ganesh et al., 2020), cooling towers (Ma et al., 2020), hot water systems (Rahmati and Gheibi, 2020). Continuous plate fins are commonly used as a part of fin and tube heat exchangers, which have several applications, i.e., in heating and cooling systems, power plants, car radiators, etc. Finned surfaces are widely used in gas side heat exchangers (Pendyala et al., 2015). An algorithm has been created in MATLAB to analyze the shape and performance of fins. It is an useful tool for engineers and students to design fins (Obregon et. al., 2018). Fin efficiency is defined as a ratio of heat transferred through the real fin to the heat flow rate transferred through the isothermal fin at the temperature of a fin's base. In the case of the simple geometrical fin, it is possible to determine the precise analytical formula for these calculations (Taler and Taler, 2014). However, for complex geometrical fin, approximate formulas need to be used, such as sector method (Shah and Sekulić, 2003), Schmidt method (Schmidt, 1949) or various numerical methods: the finite element method (FEM) or finite volume method (FVM) (Taler and Taler, 2014). The sector method is more accurate than the Schmidt method. However, the Schmidt method is less complicated. Numerical methods can be used to determine fin temperature distribution or fin efficiency for both simple and complex fin geometries (Taler and Taler, 2014). Several fin geometries (triangular, rectangular, hexagonal, and trapezoidal) have been analyzed so far in steady-state using FEM or FVM (Osorio et al., 2017).
In this paper, the comparison of several methods is presented: sector, Schmidt, and numerical methods for standard geometrical fins (such as straight or circular) and complex geometrical fins (rectangular, hexagonal, and elongated hexagonal). Mesh independent study has been done for all cases of numerical simulations. The minimum temperature on the fin end, dependent on the number of elements was calculated. Subsequently, mesh element size has been selected with the assumption that the next few temperature values are at a constant level. Numerical simulations were carried out for the following mesh element size values: 3×10^{-4} m, 2×10^{-4} m, 1.5×10^{-4} m, 1×10^{-4} m, and 5×10^{-5} m for all fin shapes. Almost all meshes consist of triangular, rectangular or trapezoidal elements. Mesh element size is a maximum length of the element's side. Each fin surface area is similar for different geometries. Therefore mesh elements are almost exactly the same for various mesh sizes.
This article presents the following issues:

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- Analytical and numerical methods for simple fin geometries - straight and circular,
- Approximate and numerical methods for complex fin geometries - rectangular and hexagonal,
- Numerical simulations for complex fin geometries - elongated hexagonal.

2. Problem statement

This article presents two-dimensional steady state problem of fin efficiency for straight, circular and continuous fins with following conditions:

- Heat transfer from the fin tip to the air can be omitted (Figure 1)
- Heat transfer on the side surfaces can be omitted, if assumption $w >> L_c$ (the fin is very thin) is satisfied (Figure 1a)
- Bound of imaginary fin appointed from continuous fin holds adiabatic boundary condition, because of the symmetry
- Temperature of fin base is constant
- Temperature difference over the thickness of the fin can be neglected
- Uniform thermal conductivity (λ) and heat transfer coefficient (α)
- Negligible radiation effects

Taking into account the above assumptions, the heat conduction Eq(1) for the fin has the following form:

$$\frac{\partial T^2}{\partial x^2} + \frac{\partial T^2}{\partial y^2} = \frac{2\alpha}{\lambda\delta}(T - T_{cz}) \tag{1}$$

3. Results and discussion about comparing numerical simulation results with exact analytical and approximate solutions for determining fin efficiency

This paper is to present various methods of determining the temperature field and the fin efficiency of most commonly used types of fins, such as straight, circular, imaginary rectangular, and imaginary hexagonal. Additionally, all of the presented methods have been applied for more complex geometries. The efficiency of the fins was calculated for a uniform coefficient of heat transfer on the fin surface using the following formula:

$$\eta = \frac{\bar{T}_{fin} - T_{cz}}{T_b - T_{cz}} \tag{2}$$

The symbol $\bar{T}_{fin}$ [K] denotes the average temperature of the fin surface at which heat exchange with the environment occurs. Relative differences between various solutions have been calculated as follows:

$$e = \frac{\eta_{CFD} - \eta_{sec\,tor/Schmidt}}{\eta_{CFD}} \tag{3}$$

However, in the case of elongated hexagonal fin is different:

$$e = \frac{\eta_{CFD(0.00005\,m)} - \eta_{CFD(0.0003\,m - 0.0001\,m)}}{\eta_{CFD(0.00005\,m)}} \tag{4}$$

Analytical methods for calculating a simple fin efficiency are efficient and easy to use (Taler and Duda, 2006). However, numerical simulations have greater possibilities.

3.1 Simple straight and circular fin on a round tube

Straight and circular fins of constant thickness are depicted in Figure 1. It also shows straight fin width (w), circular fin outer radius (r_{in}), fin length (L_c), and thermal parameters: T_{cz}, T_b, α, λ.

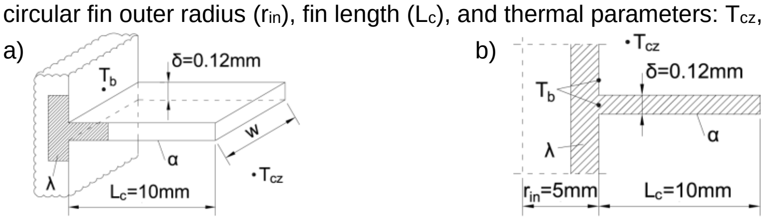

Figure 1: Schemas of the fins of regular geometry a) straight fin of constant thickness, b) circular fin of constant thickness

The following data were used for the calculation: T_b = 373.15 K, T_{cz} = 273.15 K, and λ = 204 W/(m×K). The values of the minimum temperature (Figure 2) and the fin efficiency obtained by the analytical method are very similar to those calculated by the numerical method (Figure 3).

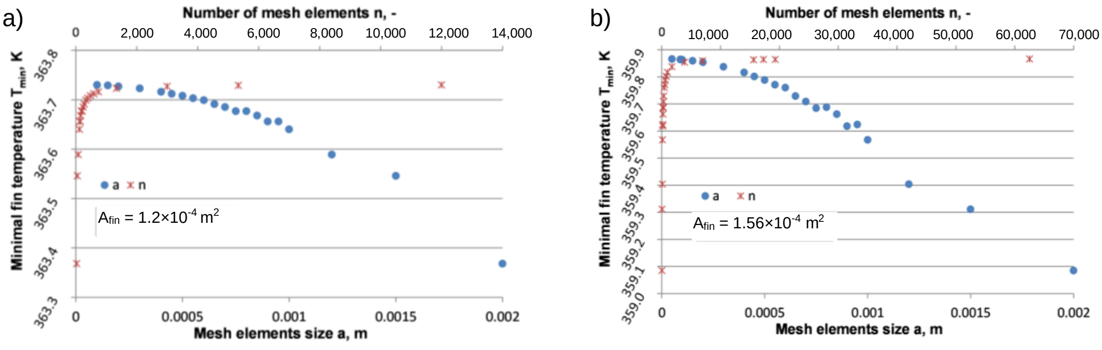

Figure 2: Fin's surface minimum temperature (for heat transfer coefficient: 25 W/(m^2×K)) as a function of number 'n' of mesh elements and element size 'a'; a) straight fin, b) circular fin

Figure 3 presents the comparison of relative differences between analytical and numerical simulation results for five different mesh sizes. We can observe that results are very accurate, even for a mesh element size of 3×10^{-4} m (which means that straight fin's mesh contains 1,333 elements and circular fin's mesh – 1,736 elements). Relative differences are smaller than 0.1 % for a mesh element size ranging from 3×10^{-4} m to 5×10^{-4} m (meshes ranging from 1,333 to 48,000 elements for straight fin and from 1,736 to 62,512 elements for circular fin). The results are very similar, even for the biggest considered mesh element, which is 3×10^{-4} m. It can be observed that the precision rises with the number of mesh elements.

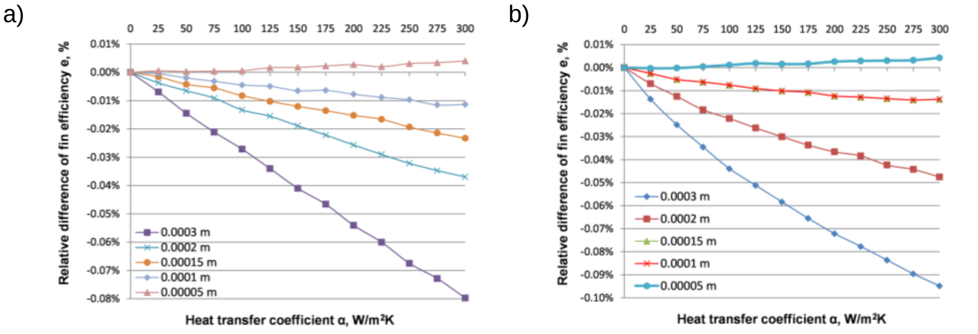

Figure 3: Relative differences of fin efficiency between analytical method and numerical simulation for several different mesh sizes as a function of heat transfer coefficient: a) straight fin, b) circular fin

3.2 Complex rectangular and hexagonal - not equilateral fin on a round tube

In the design and performance calculations of plate-fin and tube heat exchangers (PFTHEs), the continuous fin is divided into smaller imaginary fins attached to individual pipes. The designation of imaginary fins simplifies the calculation of their performance (Figure 4).

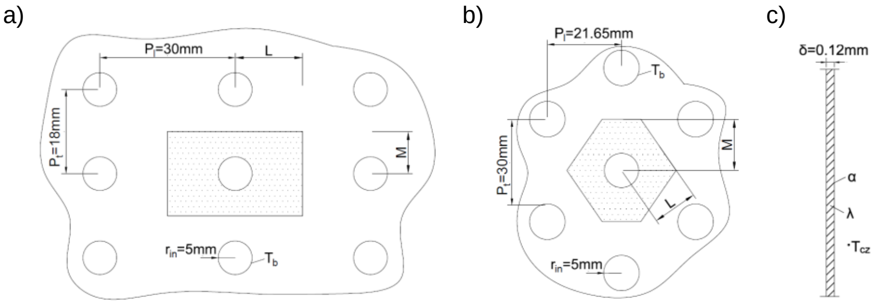

Figure 4: Division of the continuous fin into apparent fins; a) rectangular imaginary fin for the parallel pipe arrangement, b) hexagonal imaginary fin for the staggered pipe arrangement, c) cross-section of fins a) and b)

The division of the imaginary fins in finite elements was done. Fin's surface temperature stabilizes at a constant level for mesh element size of 3×10^{-4} m (Figure 5). The same situation was observed for straight and circular fins (Figure 2). Both approximate methods, sector, and Schmidt's methods have already given satisfactory results compared to the FEM for mesh element size of 3×10^{-4} m.

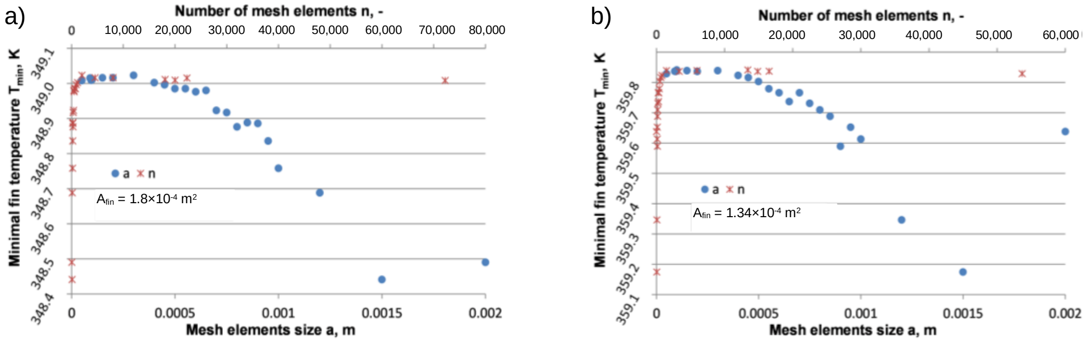

Figure 5: Fin's surface minimum temperature (for heat transfer coefficient: 25 W/(m²·K)) as a function of number 'n' of mesh elements and element size 'a'; a) rectangular fin, b) not equilateral hexagonal fin

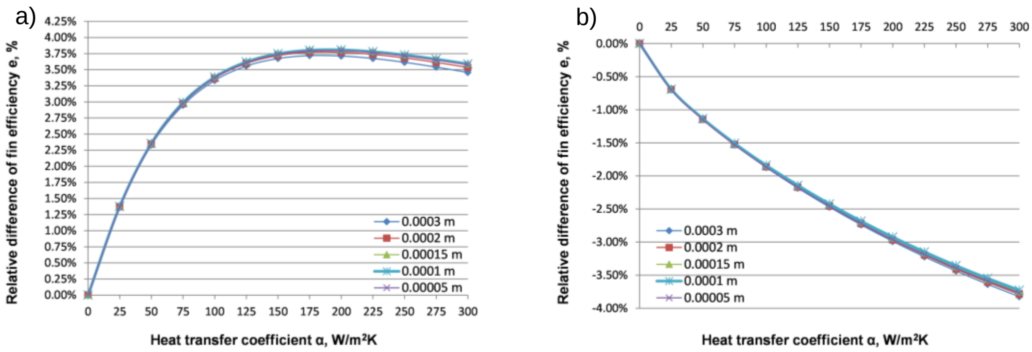

Figure 6: Relative differences of fin efficiency between approximate sector method and the numerical simulation for several different mesh sizes as a function of heat on heat transfer coefficient; a) rectangular imaginary fin, b) not equilateral hexagonal imaginary fin

The comparison relative differences of a fin efficiency between approximate methods and the numerical simulation for five different mesh sizes are presented in Figure 6 and Figure 7. Relative difference is smaller than 3.25 % for sector method (Figure 6) and less than 3.4% for Schmidt's method (Figure 7) for mesh elements size ranging from 3×10^{-4} m to 5×10^{-5} m (meshes ranging from 2,005 to 72,184 elements for rectangular fin and from 1,491 to 53,672 elements for hexagonal fin). The results obtained by both methods are very similar even for the biggest considered mesh element size of 3×10^{-4} m. It can be observed that the accuracy rises with the number of mesh elements (Figure 6 and Figure 7).

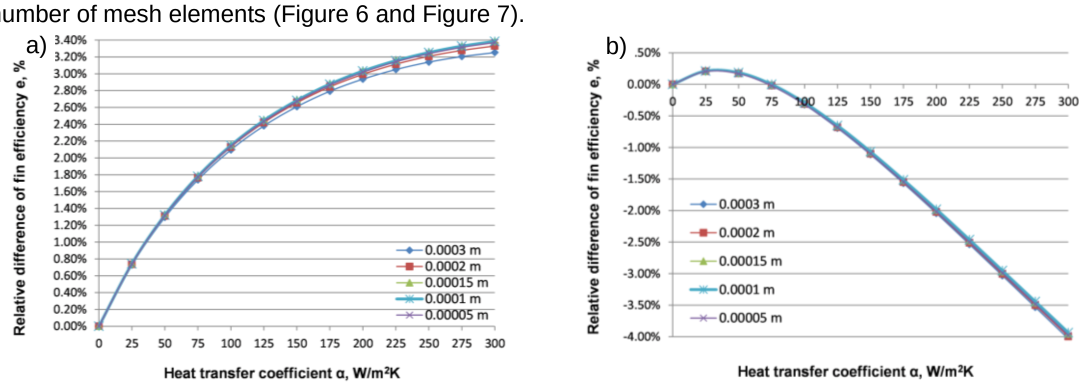

Figure 7: Relative differences of fin efficiency between approximate Schmidt's method and the numerical simulation for several different mesh sizes as a function of heat transfer coefficient; a) rectangular imaginary fin, b) not equilateral hexagonal imaginary fin

3.3 The complex elongated hexagonal fin on a flat tube

The method proposed in the article to calculate the fin temperature distribution and its efficiency may be applied to the fins of any shape attached to pipes of any shape of the cross-section. The application of the numerical modelling for determination of fin efficiency on flattened pipes (Figure 8), which cannot be calculated by sector or Schmidt's method is presented.

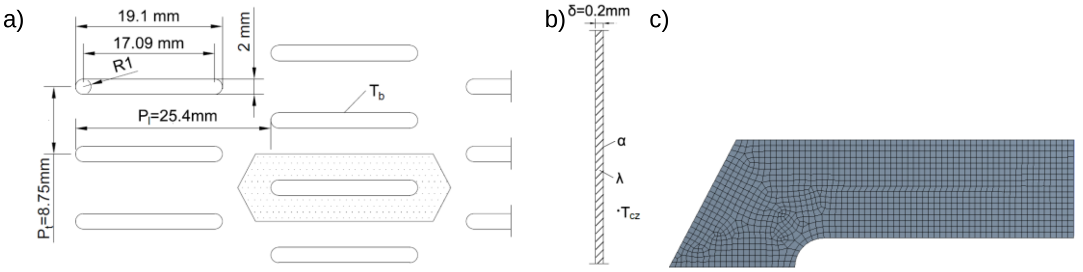

Figure 8: Plate finned tube heat exchanger made of flattened pipes with staggered pipe arrangement; a) elongated hexagonal imaginary fin for the inline pipe arrangement analyzed in (Thulukkanam, 2013), b) cross-section of the fin, c) exemplary division into finite elements for mesh element size 3×10^{-4} m (514 mesh elements)

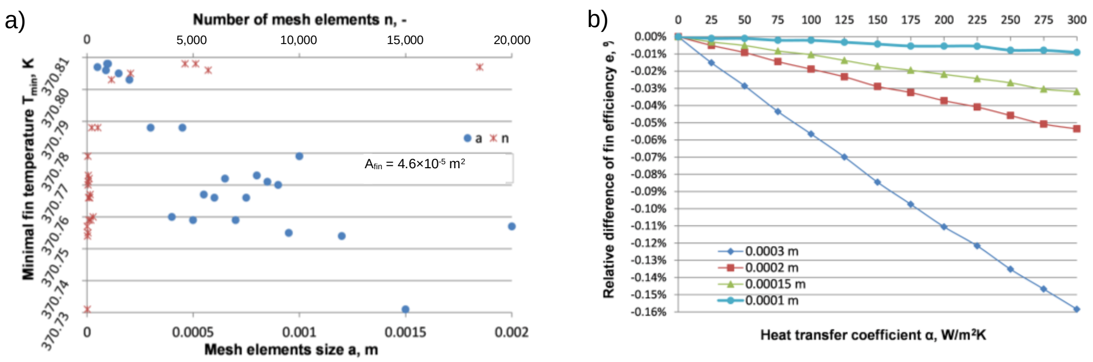

Figure 9: Effects of finite element size and number of finite elements on minimum temperature and efficiency of the fin attached to the flattened pipe; a) minimum fin temperature (for heat transfer coefficient: 25 W/(m^2·K)) as a function of the number of mesh elements, b) relative differences of fin efficiency between mesh element sizes: 3×10^{-4} m, 2×10^{-4} m, 1.5×10^{-4} m, and 1×10^{-4} m) and mesh element size of 5×10^{-5} m

Continuous fins can have different geometries from simple to complex (Thulukkanam, 2013). There are no analytical methods to calculate such fins exactly. Approximate methods are precise, but often only for a particular range of the heat transfer coefficient α. However, they are becoming more and more accurate (Suárez et al., 2019). All the above examples for straight, circular, rectangular, hexagonal fins presented that CFD simulation is a very precise and reliable method. The simulation was validated by performing it independently on several types of meshes with different mesh element sizes. The minimum temperature on the fin surface as a function of the number of elements was calculated (Figure 9a). Minimum fin temperature was set as 370.81 K while the fin base temperature was 373.15 K. Relative differences are less than 0.16 % for mesh element size ranging from 3×10^{-4} m to 5×10^{-5} m (meshes ranging from 514 to 18,494 elements). The five results of independent simulations showed slightly different results. Comparability of results is satisfactory (Figure 9b).

4. Conclusions

The comparison of analytical, approximate, and numerical methods was conducted in the paper. Presented examples demonstrated that numerical simulations are reliable. However, fine element mesh, and suitable validation is needed. In the comparison of the various calculation methods, the heat transfer coefficient α varied between 0 to 300 W/m^{2}K. The following conclusions can be drawn from the analyses and calculations carried out:

- Relative differences between analytical, approximate and numerical simulation results for mesh element size ranging from 3×10^{-4} m to 5×10^{-5} m (meshes ranging from 514 to 72,184 elements), are smaller than 4.0 % for $0 \leq \alpha \leq 300$ W/(m^2×K). However in case of the most common range in thermal applications: $0 \leq \alpha \leq 100$ W/(m^2×K) relative differences are smaller than 3.5 %,

- Relative differences between results of approximate methods and numerical simulation are smaller than relative differences between analytical method and numerical simulation. This is due to the fact that approximate methods are not very accurate (Taler and Taler, 2014),
- In the mesh independent study, a close relationship was noticed. For all simulations that were carried out minimum temperature and constant fin, efficiency occurs for the maximum mesh element size of 3×10^{-4} m (ranging from 514 to 2,005 mesh elements). Stable values of the fin efficiency changed slightly for mesh element size between 3×10^{-4} m – 5×10^{-5} m (ranging from 514 to 72,184 mesh elements),
- It can be observed that relative difference doesn't always increase with the heat transfer coefficient α.

Nomenclature

- a – mesh size, m
- A_{fin} – fin surface area, m^2
- L_{ex} – fin extended length, m
- L_c – fin length, m
- n – number of mesh elements, -
- P_l – longitudinal fin pitch, m
- P_t – transversal fin pitch, m
- r_{in} – outer radius of a plain tube, m
- T – fin surface local temperature, K
- T_b – fin base temperature, K
- T_{cz} – ambient temperature, K
- w – fin width, m
- x, y – cartesian coordinates
- α - heat transfer coefficient, W/(m^2·K)
- λ – thermal conductivity, W/(m·K)
- δ – fin thickness, m
- η – fin efficiency,

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