

# Optimization of Cascade Cooling System with Absorption Refrigeration Cycle using Thermodynamic Models

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Cooling systems have been widely studied over the past few years, but there are little researches on cascade cooling system containing different cooling methods. Cascade cooling system is suitable for the hot stream cooling process with large temperature variation. A new cascade cooling system containing waste heat recovery, air cooling, water cooling, absorption refrigeration and compression refrigeration is proposed. In this system, hot water is used to recycle waste heat from the hot stream, then drive the absorption refrigeration cycle (ARC), providing refrigerant water for hot stream cooling process. The mass flowrate of hot water determines the amount of waste heat recovered, and the final hot water temperature affects the thermal efficiency of ARC. Both the flowrate and the final temperature of hot water influence the cooling capacity generated by ARC, and further affect the heat load distribution of cascade cooling system. The hot water mass flowrate is a critical decision variable for the optimal design of the system. This study develops a model for the techno-economic optimization of cascade cooling system with ARC. ARC is modelled using thermodynamics with the concept of state points. The proposed model determines the optimal heat load distribution of cascade cooling system and the optimal design of ARC, with the minimum total annual cost (TAC) simultaneously. The effectiveness of this method is demonstrated with a case study in a polysilicon enterprise.

## 1. Introduction

A large amount of waste heat, mostly low-grade waste heat, is discharged into atmosphere during the production processes. The waste heat occupies about 17-67 % of the industrial thermal energy consumption (Liu et al., 2018) and 60% of the waste heat can be recovered in theory (Yang et al., 2016). In China, it is pointed out that at least 50 % of all energy consumption is wasted in industry, mainly in the form of low-grade waste heat. It is also estimated that the loss of low-grade waste heat was as high as  $7.6 \times 10^6$  TJ in northern China each year (Wang et al., 2019), which is nearly 260 Mt of coal equivalent. As concerns about the global energy crisis and global warming have increased, it is becoming increasingly important to recover and reuse industrial low-grade waste heat.

Absorption refrigeration cycle (ARC) is an effective method to recover and reuse low-grade waste heat. Yang et al. (2019) proposed a cascade system to recover 90-150 °C low-grade waste heat. In this system, the low-grade waste heat is utilized by LiBr/H<sub>2</sub>O absorption refrigeration cycle and transcritical CO<sub>2</sub> cycle in a cascade approach. It provides a potential way to generate electricity and refrigeration capacity using low-grade waste heat. Salmi et al. (2017) proposed a steady-state thermodynamic model of ARCs with water-LiBr and ammonia-water working pairs for ships. The absorption refrigeration systems were examined with exhaust gases, jacket water and scavenge air as energy sources. By using different waste heat sources and ARC, the optimal generator temperatures under different refrigerant temperatures were found. Ebrahimi et al. (2015) discussed the technical and economic issues involved in the recovery of waste heat from data centres through the use of absorption cooling machines. The theoretical possibility of using heat dissipated from servers to power an absorption system is considered for providing cooling for other servers in the data centre.

Three main cooling methods are usually applied in industry, e.g. air cooling, water cooling and refrigeration. Among them, air cooling/water cooling is used to take away waste heat or cool down streams to above ambient temperature, while absorption/compression refrigeration is applied to cool downstream to a sub-ambient target

temperature. There have been many studies on the four cooling methods. However, very few researches have been done on the cascade cooling system, which combined the three or four cooling methods.

In a polysilicon enterprise, some hot streams have a large cooling temperature span from over 100 °C to sub-ambient temperature. In a conventional design, air/water cooling is used to cool down hot streams to near ambient temperature, and compression refrigeration is used to cool down streams to target temperature. In this situation, ARC can be used to firstly recover low-grade waste heat from hot streams, and then the generated cooling duty can be used to reduce the duty of compression refrigeration. In this work, a cascade cooling system containing waste heat recovery, air cooling, water cooling, absorption refrigeration and compression refrigeration is proposed. Compared with predecessors' works, this work considers the heat load distribution between the four cooling methods. As an important factor, the flow rate of hot water for recovering heat is optimized to balance the performance of ARC and piping and pumping cost.

## 2. Methodology

### 2.1 Objective function

The proposed cascade cooling system in this paper mainly consists of the cooling process of hot streams and the ARC, which is shown in Figure 1. E1-E5 represent waste heat recovery heat exchanger, air cooler, water cooler, absorption refrigeration cooler and compression refrigeration cooler. This work aims to obtain an optimal design of the cascade cooling system with minimum TAC. The objective function of the system is Eq(1).

$$\min TAC = Af \times TCC + TOC$$

$$\min TAC = Af(CC_{HEX} + CC_{PUMP} + CC_{ARC} + CC_{CRC} + CC_{CT}) + (OC_{PUMP} + OC_{AC} + OC_{ARC} + OC_{COMP} + OC_{CT}) \quad (1)$$

In Eq(1),  $Af$  is the annualized factor,  $TCC$  is the total capital cost (TCC) and  $TOC$  is total operation cost (TOC).  $CC_{HEX}$ ,  $CC_{PUMP}$ ,  $CC_{ARC}$ ,  $CC_{CRC}$  and  $CC_{CT}$  denote the capital cost of heat exchangers, pumps, ARC, compression refrigeration cycle (CRC) and cooling tower.  $OC_{PUMP}$ ,  $OC_{AC}$ ,  $OC_{ARC}$ ,  $OC_{COMP}$ , and  $OC_{CT}$  are the operation cost of pumps, air coolers, ARC, the compressor for CRC and cooling tower.

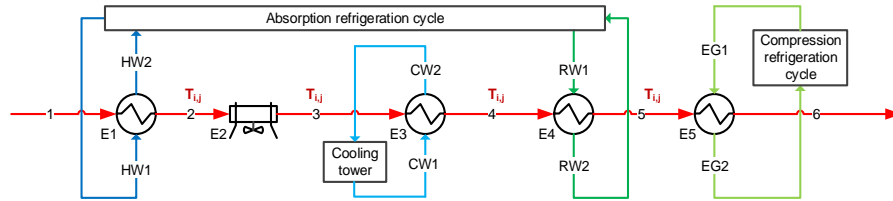


Figure 1: Schematic diagram of new cascade cooling system (HW = hot water, CW = cooling water, RW = refrigerant water, EG = ethylene glycol solution)

The heat load of heat exchangers in the cascade cooling system is described as Eq(2), in which  $i$  and  $j$  are hot and cold streams, and  $j = 1\sim 5$  represent HW, air, CW, RW, and EG.  $M$  and  $m$  are the mass flowrate of hot and cold stream,  $hin$  and  $hout$  are the inlet and outlet specific enthalpy of hot stream,  $Cp$  is the specific heat capacity of a cold stream,  $tin$  and  $tout$  are the inlet/outlet temperature of the cold stream.

$$Q_{i,j} = M_{i,j}(hin_{i,j} - hout_{i,j}) = m_{i,j}Cp_{i,j}(tout_{i,j} - tin_{i,j}) \quad (2)$$

In this work,  $T_{i,j}$  is the outlet temperature of hot stream  $i$  in heat exchanger  $j$ , and it is an important variable to optimize. It influences the heat load distribution of cascade cooling system and the area of heat exchangers. It also affects the mass flowrate of cold streams required, and further influences the capital cost of pumps, the operation cost of pumps and air coolers, and the capital and operation cost of the cooling tower. Another important variable is the mass flowrate of hot water used in the waste heat recovery. The hot water mass flowrate determines the hot water outlet temperature of waste heat recovery and influences the performance of ARC.

### 2.2 Thermodynamic model of ARC

The ARC used in this work is a traditional single-effect LiBr/H<sub>2</sub>O absorption refrigeration cycle, it consists of generator, condenser, evaporator, absorber and solution heat exchanger (SHE). The schematic diagram of LiBr/H<sub>2</sub>O ARC is shown in Figure 2 (Wang et al., 2017).

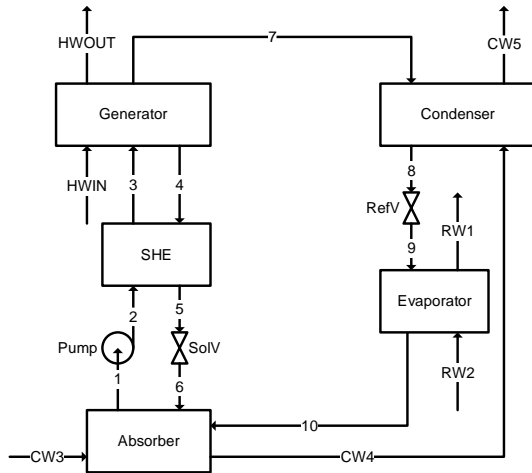


Figure 2: Schematic diagram of single-effect LiBr/H<sub>2</sub>O ARC

To facilitate the calculation of the thermodynamic parameters of streams, state points are selected to represent the streams. These system state points (1-10) are indicated in Figure 2, 1 - 3 are the weak solution, 4 - 6 represent the strong solution, and 7 - 10 are refrigerant (pure water). Some reasonable assumptions on the partial physical properties of each point according to the design experience of ARC are made in Table 1.

Table 1: ARC system state points and assumptions

State point	Assumption	Temperature (°C)	Pressure (Pa)
1	Saturated liquid	$T_1=36.62$	872
2	Determined by solution Pump	$T_2=36.62$	9,589.78
3	Determined by SHE	$T_3$	9,589.78
4	Saturated liquid	$T_4=T_{HWIN}-10$	9,589.78
5	Determined by SHE	$T_5=T_4-0.64(T_4- T_2)$	9,589.78
6	Determined by SolV	$T_6$	872
7	Saturated vapour under phase equilibrium with state 4	$T_7=T_4$	9,589.78
8	Saturated liquid	$T_8=45$	9,589.78
9	Determined by RefV	$T_9=5$	872
10	Saturated vapour	$T_{10}=5$	872

Given the temperature and pressure, the specific enthalpy of streams and the mass fraction of LiBr/H<sub>2</sub>O solution can be calculated, and vice versa. Through mass and energy balance, the heat load calculation formulas of generator, condenser, evaporator, absorber and SHE can be obtained, which are shown as follows:

$$Q_{GEN} = M_{EVAP}(h_7 + (a - 1)h_4 - ah_3) \quad (3)$$

$$Q_{COND} = M_{EVAP}(h_7 - h_8) \quad (4)$$

$$Q_{EVAP} = M_{EVAP}(h_{10} - h_9) \approx M_{EVAP}(h_{10} - h_8) \quad (5)$$

$$Q_{ABS} = M_{EVAP}(h_{10} + (a - 1)h_6 - ah_1) \approx M_{EVAP}(h_{10} + (a - 1)h_5 - ah_1) \quad (6)$$

$$Q_{SHE} = M_{EVAP}a(h_3 - h_2) = M_{EVAP}a(h_3 - h_1) \quad (7)$$

$$h_3 = \frac{a - 1}{a}(h_4 - h_5) + h_1 \quad (8)$$

where  $Q_{GEN}$ ,  $Q_{COND}$ ,  $Q_{EVAP}$ ,  $Q_{ABS}$  and  $Q_{SHE}$  are the heat load of generator, condenser, evaporator, absorber and SHE,  $M_{EVAP}$  is the mass flowrate of the refrigerant vapour vaporized from the generator,  $a$  represents the circulation ratio of ARC, which means the mass flowrate of weak solution required to produce 1 kg of refrigerant vapour in the generator.  $M_{EVAP}$  and  $a$  are calculated by Eq(9) and Eq(10). In Eq(10),  $X_{strong}$  and  $X_{weak}$  are the mass fraction of strong and weak LiBr/H<sub>2</sub>O solution.

$$M_{EVAP} = \frac{Q_{EVAP}}{h_{10} - h_9} \approx \frac{Q_{EVAP}}{h_{10} - h_8} = \frac{\sum Q_{i,A}}{h_{10} - h_8} \quad (9)$$

$$a = \frac{X_{strong}}{X_{strong} - X_{weak}} \quad (10)$$

### 3. Case study

The hot streams are the intermediate product of hydrochlorination plant in a polysilicon enterprise, which are composed of HCl (hydrogen chloride), DCS (dichlorosilane), TCS (trichlorosilane), STC (silicon tetrachloride) and H<sub>2</sub> (hydrogen). The specific enthalpy of hot streams can be calculated by Eq(11).

$$h_{i,j} = 9 \times 10^{-5} T_{i,j}^3 - 5.9 \times 10^{-3} T_{i,j}^2 + 1.2221 T_{i,j} - 3997.6 \quad (11)$$

The inlet/outlet temperature of air, CW, RW and EG are set as constant for simplicity of calculation. Some economic parameters of the case study are presented in Table 2 (Ma et al., 2018).

Table 2: Economic parameters of case study

Items	Data	Remarks
Air cooler capital cost (\$)	4,778 $A^{0.525}$	$A$ in $m^2$
Heat exchanger capital cost (\$)	11,000 + 260 $A$	$A$ in $m^2$
Pump capital cost (\$)	14,104 + 11,988.4 $Q_V H_T$	$Q_V$ in $m^3/s$ , $H_T$ in $m$
Pump operation cost (\$)	$m g H_T / \eta Pe Hy$	$m$ in $kg/s$
Air cooler fan efficiency	70 %	
Pump efficiency	70 %	
Compressor efficiency	70 %	
COP of the compression refrigeration cycle	3.4	
Price of electricity ( $Pe$ )	0.15 \$/kWh	
Price of freshwater ( $Pw$ )	0.5 \$/t	
Plant operation time ( $Hy$ )	$2.88 \times 10^7$ s/y	
Interest rate	15 %	
Annualized factor	0.298	

#### 3.1 Results and discussions

The original cooling system containing air cooling, water cooling and compression refrigeration is calculated as a base case. Then the new cascade cooling system is optimized in MATLAB with genetic algorithm (GA). Figure 3a and 3b are used to indicate the heat load distribution of the original and new cascade cooling system.

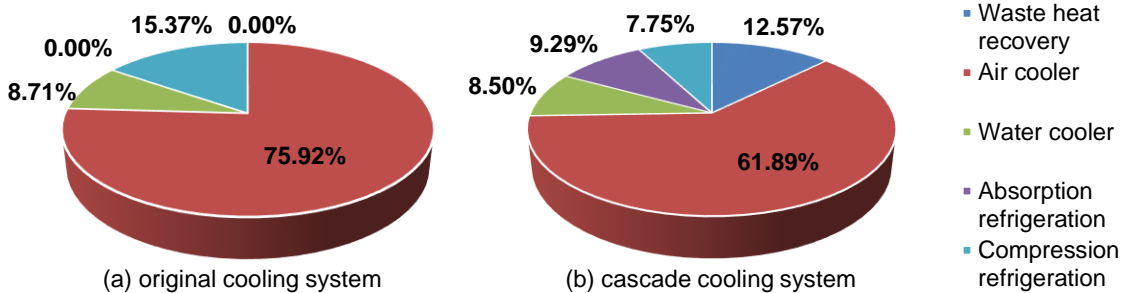


Figure 3: The heat load distribution of heat exchangers

It can be seen from Figure 3a that large amounts of low-grade waste heat are released to the environment from air coolers because of higher inlet temperature of hot streams. By using the new cascade cooling system, about 14.03 % of waste heat is recovered, and 9.29 % of cooling duty is produced by ARC. The cooling duty of water cooling reduced by 0.21 %, and cooling duty of compression refrigeration reduced by 7.62 %, which means less consumption of freshwater and electricity.

Table 3: Comparison of results between the original and new cascade cooling system

Items	Original cooling system	New cascade cooling system
TAC (\$/y)	2,579,000	1,680,200
TCC (\$)	490,373	835,634
TOC (\$/y)	2,432,854	1,431,145
OC <sub>COMP</sub> (\$/y)	2,138,800	1,016,420

From Table 3, the TCC of the new cascade cooling system increased by 345,261 \$ compared to the original cooling system, but the TOC decreased by 1,001,709 \$/y and the TAC decreased by 898,800 \$/y, mainly because the operation cost of compressor reduced by 52.48 %. However, the operation cost of compressor still accounts for more than 60 % of TAC and 70 % of TOC, indicating that the compressor is still the main part of energy consumption. The optimal design of new cascade cooling system for hot streams is shown in Figure 4. For the optimal design of ARC in the cascade cooling system, the mass flowrate of LiBr/H<sub>2</sub>O solution is 5.79 kg/s, and the COP is 0.7389.

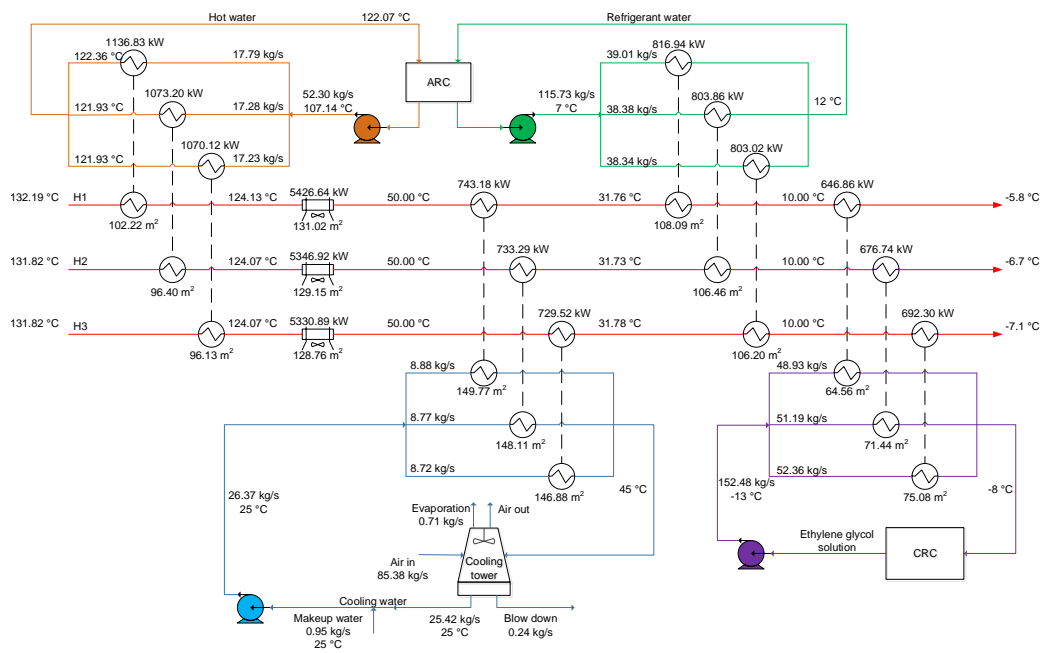


Figure 4: The optimal design of new cascade cooling system for hot streams

It can be seen from Figure 4, the inlet temperature of compression refrigeration cooler is 10 °C, which is the minimum outlet temperature of absorption refrigeration cooler because operation cost of compressor accounts for more than 60 % of the TAC, indicating that the smaller inlet temperature for compression refrigeration cooler, the smaller the heat load of compression refrigeration, and the smaller energy consumption by the compressor. Water cooling consumes both electricity and freshwater, it is a little expensive than air cooling, so the inlet temperature of the water cooler is 50 °C, the minimum outlet temperature of air cooler.

### 3.2 Sensitivity analysis for the new cascade cooling system

In this section, sensitivity analyses of electricity and freshwater charges are carried out to investigate their influences on the new cascade cooling system. As can be seen from Figure 5a and 5b, both electricity charge and freshwater charge have little influence on TCC. TOC and TAC increase with electricity charge, while they have little increase with freshwater charge. That is because electricity charge influences all operation cost of the system, but freshwater charge only affects the cost of water cooling and cooling tower. The heat load of water cooling and absorption refrigeration change in exactly the opposite direction, which also means that the temperature break-point between water cooling and absorption refrigeration decreases with electricity charge and increases with freshwater charge. This shows that electricity charge has a greater impact on absorption refrigeration, while freshwater charge influence water cooling more. To minimize the TAC, more water cooling and less absorption refrigeration are used with the increase of electricity charge, and that is opposite to the increase of freshwater charge.

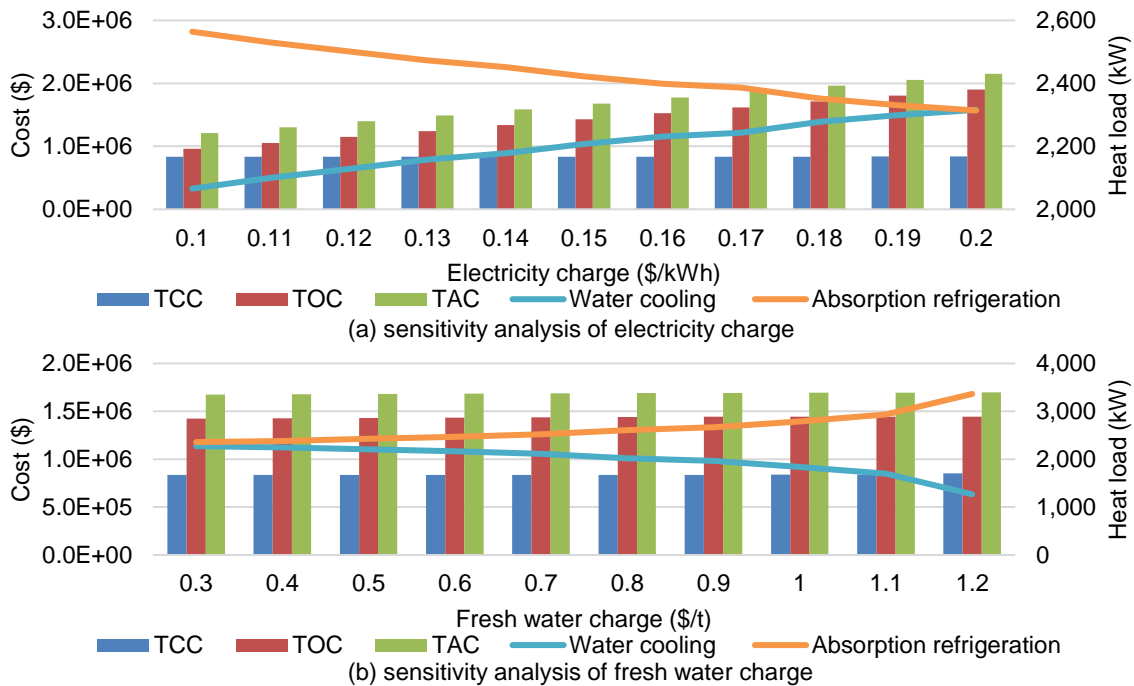


Figure 5: Sensitivity analysis for the new cascade cooling system

#### 4. Conclusions

In this paper, a new cascade cooling system containing waste heat recovery, air cooling, water cooling, absorption refrigeration and compression refrigeration is proposed. The mathematical models including hot streams cooling process and ARC are formulated and solved. Through optimization, the optimal design of the new cascade cooling system is obtained. 3, 280.15 kW of waste heat is recovered and 2,423.82 kW of cooling capacity is supplied by ARC, and the optimal COP of ARC is 0.7389. The TAC of the new cascade cooling system is 1,680,200 \$/y when electricity charge is 0.15 \$/kWh and freshwater charge is 0.5 \$/t, which saves 898,800 \$/y than the base case. Through sensitivity analysis, it is found that electricity charge has a big impact on the TAC of the system, with the electricity charge increases 0.01 \$/kWh, the TAC increases by about 95,000 \$/y, while freshwater charge has little influence. The temperature break-point between water cooling and absorption refrigeration is a major factor affecting the system, it influences the heat load distribution of water cooling and absorption refrigeration, further influences the amount of waste heat recovered. The new cascade cooling system can realize waste heat recovery and energy conservation with a good economic benefit.

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