[1]Michele Emmer
emmer@mat.uniroma1.it

[1]Dip. Mat. "G. Castelnuovo"
Univ. Roma "La Sapienza",
Roma, Italy

Colours and Soap Bubbles

It's because I don't do anything, I chatter a lot, you see, it's already a month that I've got into the habit of talking a lot, sitting for days on end in a corner with my brain chasing after fancies. It is perhaps something serious? No, it's nothing serious. They are soap bubbles, pure chimeras that attract my imagination.

Fedor Dostoevsky, Crime and Punishment

1. INTRODUCTION

It is very interesting to study the parallel story of soap bubbles and soap films in art and science. Noting that mathematicians in particular have been intrigued by their complex geometry, the interest, both scientific and artistic, was first on the colors on the surface of soap films. Probably motivated by the large diffusion of paintings of children and puttos playing with soap bubbles.

On December 9, 1992 the French physicist Pierre-Gilles de Gennes, professor at Collège de France, after being awarded the Nobel Prize for physics concluded his conference in Stockholm with a poem on soap bubbles, adding that no conclusion seemed most appropriate. The poem appears as a closure to an engraving of 1758 by Daullé from François Boucher's lost painting *La souffleuse de savon*. De Gennes did not want to allude to the allegorical meanings that soap bubbles had had for many centuries: symbol of vanity, fragility of human ambition and of human life itself.

Soap bubbles and soap films were one of the matters of his conference, which was entirely devoted to the *Soft Matter.* Bubbles that *"are the delight of our children",* he wrote. A reproduction of the engraving was included in the article. [1] Soap bubbles are one of the most interesting matters in many sectors of scientific research: from mathematics to chemistry, from physics to biology. But not only, also in architecture and in visual arts, not to speak of design and even of advertising. A story that began so many centuries ago and it is still present.

Figure 1 - F. Boucher, L*a souffleuse de savon,* 1758 [2]

2. ART AND SCIENCE: A PARALLEL HISTORY

It is natural that among the first ones to be attracted by the iridescent soap films were the artists, in particular the painters. While for mathematicians soap films are models of a geometry of very stable forms, for the artists soap bubbles have been of great interest not just for their playfull aspect but as symbol, as allegory of the brittleness, of the frailty of the human things, of life. They are an aerial and light symbol, always fascinating for their endless variety of colors and forms.

A series of engravings realized by Hendrik Goltzius (1558-1617) is the starting point of the fortune of soap bubbles in the XVI and XVII century Dutch art. The best known is entitled *Quis evadet* (Who escapes) and is dated 1594. The history of the relationships between soap bubbles and visual art has been told, including numerous reproductions, in a book published in 2009 [2]. One of the most famous painting, also remembered in his writings by de Gennes, was realized in the first part of the 1700s by the famous French artist Jean Baptiste Siméon Chardin (1699-1779), in different versions, under the title *Les Bulles de savon*. It is very suggestive and of a rare beauty.

Soap bubbles attracted Chardin because of his interest in children and young people, in their world, their games. He was also attracted by the colors of soap bubbles, as the painting (it exists in three different versions) shows very clearly. It is very likely that playing with soap bubbles was a very popular children's game at that time. And it is natural that also scientists became interested in the phenomena surrounding the formation of soap bubbles and the colors on their surfaces.

3. SCIENTISTS START STUDYING SOAP BUBBLES

In 1672 the English scientist Hook presented a note to the Royal Society, later published by Birch in the *History of the Royal Society* in 1756. Hook wrote that with a solution of soap numerous small bubbles were blown through a small tube of glass. He noted that it could easily be observed that at the beginning the liquid film forms a spherical surface that imprisons a globe of air. It is a liquid film white and clear without the least coloration; but after a few moments, while the film is gradually thinning, all varieties of colours can be observed to appear on its surface as in a rainbow.

If Hook is among the first ones to attract the attention of the scientists on the problem of the formation of colours on the thin soap bubbles, it is Isaac Newton in *Opticks*, [3] whose first edition is published in 1704, to describe in detail the phenomena that are observed on the surface of the soap films. In volume II, Newton describes his observations on soap bubbles. In particular he observes that if a soap bubble is formed with some water made more viscous using soap, it is very easy to observe that after a while a great variety of colours will appear on its surface. Newton noted that in this way colours were disposed according to a very regular order, like many concentric rings beginning from the highest part of the soap bubble. He also observed that as the soap film became thinner due to the continuous diminution of the contained water, such rings slowly dilated and finally covered the whole film, moving down to the low part of the bubble and then disappeared.

The phenomenon that Newton observed is known as interference: it happens when the thickness of the soap film is comparable to the wavelength of visible light.[a] An easy experiment can be performed with a rectangular loom that is vertically extracted from a solution of soapy water; the light reflected by the soap film produces a system of horizontal lines, due essentially to the fact that the soap film is constituted by the two non parallel faces of the same film.

In Optics Newton wrote:

"The precedent Observations were made with a rarer thin Medium, terminated by a denser, such as was Air or Water compress'd between two Glasses. In those that follow are set down the Appearances of a denser Medium thin'd within a rarer, such as are Plates of Muscovy Glass, Bubbles of Water, and some other thin Substances terminated on all sides with air.

Obs. 17. If a Bubble be blown with Water first made tenacious by dissolving a little Soap in it, 'tis a common Observation, that after a while it will appear tinged with a great variety of Colours. To defend these Bubbles from being agitated by the external Air (whereby their Colours are irregularly moved one among another, so that no accurate Observation can be made of them,) as soon as I had blown any of them I cover'd it with a clear Glass, and by that means its Colours emerged in a very regular order, like so many concentrick Rings encompassing the top of the Bubble. And as the Bubble grew thinner by the continual

Figure 2 - Anonimous, Dutch School, XVII century

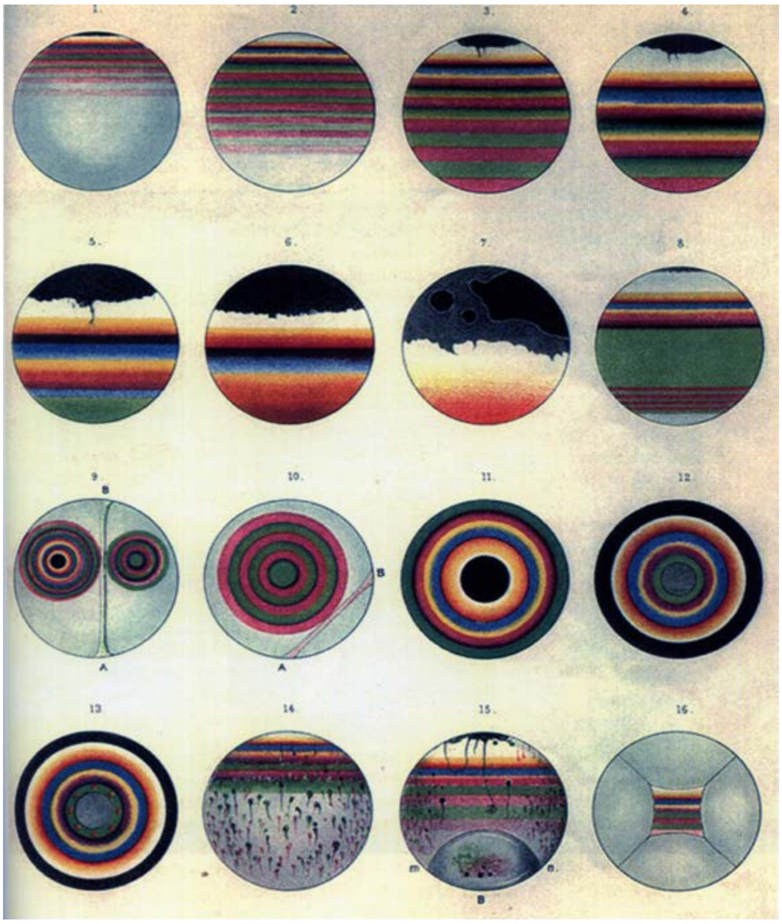

Figure 3 - D. Brewster [4]

subsiding of the Water, these Rings dilated slowly and overspread the whole Bubble, descending in order to the bottom of it, where they vanish'd successively. In the mean while, after all the Colours were emerged at the top, there grew in the center of the Rings a small round black Spot, like that in the first Observation, which continually dilated it self till it became sometimes more than 1/2 or 3/4 of an Inch in breadth before the Bubble broke Besides the aforesaid colour'd Rings there would often appear small Spots of Colours, ascending and descending up and down the sides of the Bubble, by reason of some Inequalities in the subsiding of the Water. And sometimes small black Spots generated at the sides would ascend up to the larger black Spot at the top of the Bubble, and unite with it.

Obs. 18. Because the Colours of these Bubbles were more extended and lively than those of the Air thinn'd between two Glasses, and so more easy to be distinguish'd, I shall here give you a farther description of their order, as they were observ'd in viewing them by Reflexion of the Skies when of a white Colour, whilst a black substance was placed behind the Bubble. And they were these, red, blue; red, blue; red, blue; red, green; red, yellow, green, blue, purple; red, yellow, green, blue, violet; red, yellow, white, blue, black…
In the mean while at the top, which was of a darker blue than the bottom, and appear'd also full of many round blue Spots, something darker than the rest, there would emerge one or more very black Spots, and within those, other Spots of an intenser blackness, which I mention'd in the former Observation; and these continually dilated themselves until the Bubble broke."

A century later after Newton, David Brewster wrote a paper by the title *"On the Colours of the Soap-Bubble".* [4]

"The colours of the soap-bubble have been the subject of frequent observation since the time of Boyle, Hook, and Newton, and they have been invariably ascribed not to any colour in the medium itself in which they are formed, or on whose surfaces they appear, but solely to its greater or less thickness."

The author of this paper had been led to doubt the correctness of this opinion, and while repeating the beautiful experiments of Professor Plateau *"On the equilibrium of liquid films,"* he discovered the true cause of these colours, whether they are observed on the soap bubble or on plane, convex, and concave films stretched across the mouths of closed or open vessels.

The paper, which is illustrated with numerous coloured drawings, is divided into five parts.

1. On the phenomena of colour in a vertical plane film.
2. On the production of revolving systems of coloured rings on the soap film.
3. On the form and movements of the bands and rings on convex and concave films.
4. On the phenomena produced by different solutions.
5. On the origin and development of the colours on the soap bubble.

In these sections the author has shown that the colouring matter of the soap-bubble is secreted from the soap solution when reduced to the state of a film;—that it rises to the highest point of the film in colourless portions, in the form of a tadpole, which pass into molecules of every possible order of colour, and then take their proper place in the coloured bands;—that these bands move over the surface of the film under the influence of gravity, and may be blown into fragments or into molecules of all colours, or even recombined with the film.

"It is impossible," the author adds, *"to convey in language an adequate idea of the molecular movements, and the brilliant chromatic phenomena exhibited on the soap films, and it is equally impossible for art to delineate them."*

For the scientists of the XVIII century the connection among the soap bubbles and some natural phenomena that follow schemes of maximum and minimum was not at all clear; only in the XIX century it will be understood that soap bubbles furnish an experimental model for problems of mathematics and physics, inserting soap films to full title in the sector of mathematics called Calculation of the Variations.

4. A BLIND MATHEMATICIAN

One of the most important problems for which soap bubbles and soap films furnish an experimental model of the solution is called the Plateau problem, from the name of a Belgian physicist. The general mathematical solution to the problem of Plateau was difficult to obtain.
Antoine Ferdinand Plateau (1801-1883) began his scientific career in the field of astronomy. In 1829 during an experiment he exposed too long his eyes to sunlight, causing irreversible damage to his sight. Since 1843 he was completely blind. He took an interest in the nature of forces in molecular fluids, to discover the forms that generate soap films contained in metal wires immersed into soapy water. In

1873 he published the result of fifteen years of research in two volumes: *'Statique expérimentale et théorique des liquides soumis aux seules forces moléculaires'* [5]

Plateau himself introduced the general principle that is the basis of his work. The idea is to draw a closed contour with the only condition that it contains a limited portion of the surface and that it is compatible with the surface itself; if then a wire identical to the previous contour is constructed, plunged entirely in the soapy liquid and then pulled out, a set of soapy films is generated representing the portion of area under consideration. Plateau cannot do without noting that these surfaces are obtained *'almost by magic.'*

And here is the great discovery of Plateau, incredible at first sight: however high the number of soap films that come into contact with each other, there can be only two types of configurations. Precisely the three experimental rules that Plateau discovers about soap films are:

> 1) a system of bubbles or a system of soap films attached to a supporting metallic wire consists of surfaces flat or curved that intersect with each other along lines with very regular curvature.
> 2) surfaces can meet only in two ways: either three surfaces meeting along a line or six surfaces that give rise to four curves that meet in a vertex.
> 3) the angles of intersection of three surfaces along a line or of the curves generated by six surfaces in a vertex are always equal in the first case to 120°, in the second to 109 ° 28 '.

Another question remained still open: were the laws discovered experimentally by Plateau for the geometry of soap films correct or not?

> *"In this work we provide a complete classification of the local structure of singularities in the three-dimensional space, and the results are that the singular set of the minimal set consists of fairly regular curves along which meet three films of the surface with angles equal of 120 degrees and isolated points where meet four of these curves giving rise to six films also with equal angles.*
> *The results apply to many real surfaces that are generated by surface tension, as to any aggregate of soap films, and so provide a proof of experimental results obtained from Plateau over a hundred years ago."*

Thus begins one of the best known work of mathematics of last century. Written by Jean E. Taylor, entitled The Structure of Singularities in Soap Bubble-and-Like Soap-Film-Like minimal Surfaces. [6] Fred Almgren and Jean Taylor wrote a well - known article on their research published in Scientific American in 1976. [7]

In 1979 I realized the film Soap Bubbles, in the series Art and Mathematics, starring Fred Almgren and Jean Taylor. [8] The film was made at Princeton University, using real models with soapy water, while in the new film on minimal surfaces produced by A. Arnez, K. Polthier, M. Steffens and C. Teitzel at University of Bonn and at Technical University of Berlin in 1995 all models are made with computerized animation. [9] In both films the brilliant colours of soap films and bubbles have a fundamental role.

Of course the new computerized soap does not alter the charm of playing with real soap bubbles! Mark Twain was right when he wrote:

> *'A soap bubble is the most beautiful and most elegant object, that there is in nature ..... I wonder what would be required to buy a soap bubble if in the world there existed only one.'*

5. SOAP BUBBLES IN ART AND ARHITECTURE

The famous French artist Manet painted Les bulles des savon in the same years of the publication of the research of Plateau Some years after D'Arcy Thompson in the book On Growth and Form, [10] a classic book dedicated to the study of animal forms using mathematical models, devotes a chapter to the discoveries of Plateau and the use of his laws on soap films to explain the shape of some of the Radiolaria.

> *"The peculiar beauty of soap bubbles, the resulting forms, are so pure and so simple that we come to look on them as a mathematical abstraction"*, wrote D'Arcy Thompson.

Since the publication of the book by Thompson some of the images have been always linked to the geometry of soap films. Images that have influenced many designers, artists and architects. [1]

When in 1976 the mathematician Jean Taylor proved that the laws of Plateau were correct, a student of art, Bradley Miller, went to Princeton University where Taylor worked together with her husband Fred Almgren. Miller had the idea of using photography to fix the structure of soap films. These images were printed in an art book in 2007 [11] and were on show in a gallery in Venice during the annual congress on Mathematic and Culture.

In 1890 Boys completed his book Soap Bubbles, [12] in which he summarized his own experience in explaining to a large public the geometry of soap bubbles and soap films:

"I do not suppose that there is anyone who has not occasionally blown a common soap bubble, and while admiring the perfection of its form, and the marvelous brilliancy of its colour, wondered how such a magnificent object can be easily produced.
I hope that none of you are yet tired of playing with bubbles, because as I hope we shall see, there is more in a common bubble that those who have only played with them generally imagine."

In his book Boys published a colour plate on the colour on the surface of soap bubbles and soap films.

In the XVIth Century colour in soap bubbles became fascinating to children and artists, like Chardin and Manet. In the XVIIth Century those colours attracted scientists like Newton. The interest they aroused has never come to an end. As foreseen by Lord Kelvin, artists, architects, mathematicians, physicists and biologists among others continued to study colour and shape in soap films throughout the centuries. The story of soap bubbles in art and science is a never ending story...

NOTES

a) for more details http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/soapfilm.html

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