Dynamic Programming and Greedy Algorithm Strategy for Solving Several Classes of Graph Optimization Problems

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Abstract

This article is devoted to the development of the scientific methods of dynamic programming and greedy algorithms to expose the software of innovation methods of mathematical and computational approaches in advancing modern science and teaching. The tools applied in this development are based on the graph theory, generalized approximate dynamical programming method and greedy algorithms. The results are presented on both the fundamental and applied level.

Keywords: Dynamic Programming; Greedy Algorithm; Graph; Optimization.

1. Introduction

The optimization problem in modern business belongs to the most general task of mathematical optimization problems. In our investigation solving this problem with general mathematical methodology and powerful algorithms as a way of furthering our interdisciplinary approach in research and teaching proceedings is considered.

In this work, optimization problems for financial investment and supply management of a company, as a particular example, are considered. Planning Software to solve complex conservation planning problems in financial investment is delivered. Investigations include the following stages of sub-problems:

- Statement optimization problems for financial investment of business-company (FIC)
- Description how graph theory can be utilized as a representation language of building information models and implementation of basic concepts in a wide range of models of business processing.
- Construct the corresponding graphs of FIC
- Investigation of basic solve strategies of FIC
- Investigation of greedy algorithm strategy with respect to problems FIC
- Justify of numerical approximation
- Implementation of program code for the optimization problem.

2. Notations and Definitive Concepts

This paper deals with the problem of optimal allocation of financial resources of the company, modeled with the use of directed graph-diagrams.

One of the features of the modern economy is dynamism on each level of management. It defines the realization of different flows of goods and services, involves the movement and storage of raw materials during planning.

The dynamic process must be balanced and optimal. The research of this problem makes the construction of a corresponding mathematical model necessary, because only through computer simulation we can determine optimal variant of the management of relevant flows of finance or goods in proper time so that they are reflected in the business plans of the company.

We know many papers on the issues of optimal allocation of financial resources, using appropriate optimization methods, including (Jeremy, 2012) But little work is devoted to the construction of optimal management system in the graphs.

The problem of optimal management of investments relates to the problems of dynamic programming (Sniedovich, 2010). In the methods of dynamic programming, the initial Information is based on the consideration that in the model the real challenges are: Objective; Constraint condition; Parameters of management.

Solutions of the problems occur by the method of dividing the investment operations into the multi-stage recursive procedures. Profit during the entire operation is achieved from a sequence of profit at each stage separately.

The problem of optimal resources management is from a class of the additional tasks. In this paper, we address several constrained optimization problems (financial investment, supply management) by using the multistage graph modeling method (Berge ,1962), because the graph is a particular way for visualization of the storing and organizing dates of the investments and corresponding profits.

We have constructed the Software Planning (SP) used to manage all types of resources and find the algorithm of greedy strategy use tool- optimizer to obtain the best results of solutions.

3. Statement Problem

In this work, we have considered practical problems of optimal management of resources of a company to find the tour of investing projects to obtain the best available benefit given in the defined financial domain.

Let us consider the mathematical formulation of the problem, thus we introduce the following notations:

$C = (C_{ij})_{nxm}$ - the matrix of initial values of projects investment in the company, $R = (R_{ij})_{nxm}$ - the matrix of the value of corresponding benefits (i=1,..,n; j=1,..,m), S- total sum invests of projects, n- number of the company, m-number of projects, $X = (X_{ji})_{mxn}$- the matrix of selected projects

where

$$X_{ji} = \begin{cases} 1, & if\ project\ is\ selected \\ 0\ , & if\ project\ is'nt\ selected \end{cases}$$

Problem. It is required to construct – the matrix of optimal plan of investment in the company - $X = (X_{ji})_{mxn}$ satisfy the following conditions:

Constraint conditions:

$$\sum_{i=1}^{n} \sum_{j=1}^{m} (C_{ij} * X_{ji}) \leq S \qquad (1)$$

Criteria of optimization:

$$\sum_{i=1}^{n} \sum_{j=1}^{m} (R_{ij} * X_{ji}) \rightarrow max \quad (2)$$

Let us consider modeling proceedings of this problem.

For the simplest case, we have considered the solution of the problem for the following particular case when n=3, m=4 and S=5. Data value of initial investment of projects are presented in table 1.

Table 1**.** Data value of initial investment

Project	**Company I**		**Company II**		**Company III**	
#	Invest- C_{1j}	Profit-R_{1j}	Invest-C_{2j}	Profit-R_{2j}	Invest-C_{2j}	Profit-R_{2j}
1	0	0	0	0	0	0
2	1	5	2	8	1	3
3	2	6	3	9	1	3
4	2	6	4	12	1	3

The multi-stage graph of management investment of projects in the company, corresponding the dates of the table are constructed and has the following form (look pic.1):

SP program maintenance enables us to define the effective version of management of proper financial flows by using a visual diagram on each level of investment and find the optimal solution. The software performs the following procedures by using the corresponding tools:

- To construct the table of date, for the particular value (table 1.)
- To construct the multistage graph, corresponding the dates of table1. (look figure 1.)

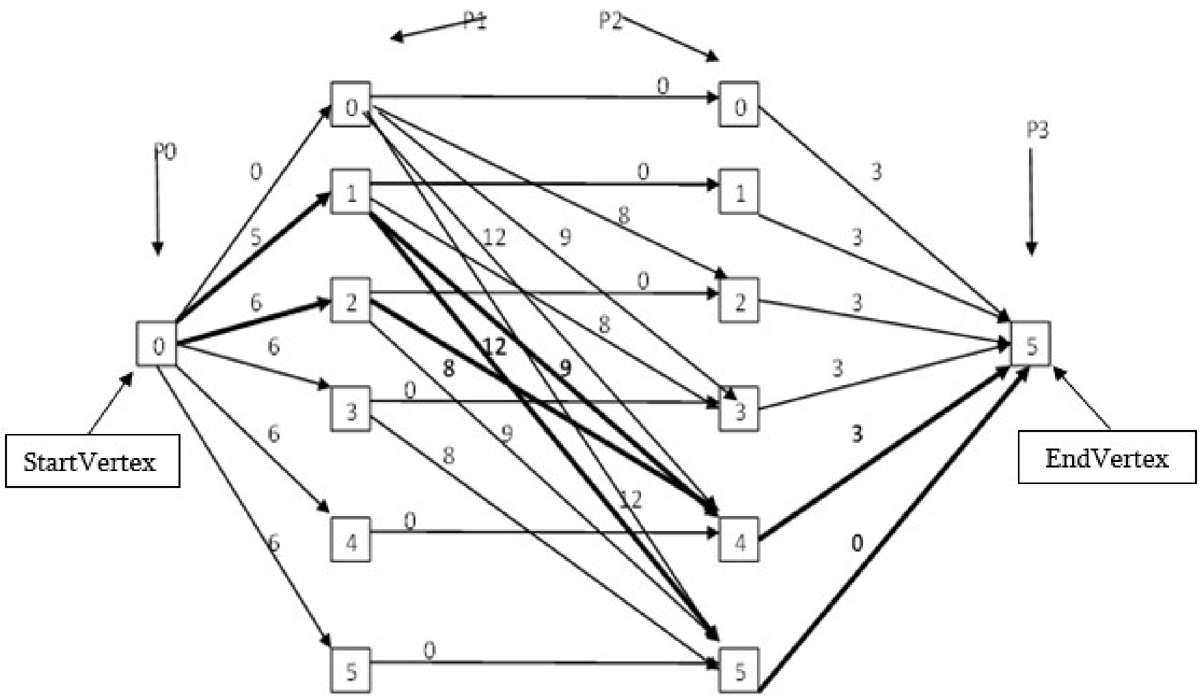

Figure 1. Directed graph of investment

Where P_j (j=0, 1, 2, 3) designates the stage financial process for given projects of the company.

The vertex (V_{jk} , where j is the number of the stage) of the graph are used for storing the sum invests of the given stage (from the beginning to the end):

$$V_{2k}= V_{1k}+ C_{2k}\,;\ V_{31}=5= V_{2k}+ C_{3k}\,;\ k=0..5;$$

The nodes 0 and 5 are terminal points: 0 (only variant) defines the initial position of the system, 5 (only variant) are used for stored constraint condition.

The edges of graphs are labeled by the benefits of the corresponding stage of investing plan accordingly to table 1.

For example, the label of vertex, which connects V_{24} of P_2 column to connect V_{31} of P_3 column is labeled by the benefit -3mln corresponding the investment-1mln of the third stage.

4. Problem Solution

We will consider the dynamical process of investment. The solution of complex problems is based on breaking them down into simpler substructures:

- In the first stage the financial resources for the first company is invested, there are the six possible ways of investment between 0 flow-of- investment and 5 flow-of- investment (look at vertex of the column P_1 of graph), in the nodes the column P_1(first stage) the date of initial investment of first stages is stored.
- In the second stage, the rest of the financial resource of the company is invested. There are the six possible ways of investment, in the nodes of column P_2 (second stage) the sum invested together with the first and second stages are stored.
- In the third stage, the rest of the financial resources for the company is invested. There is the only variant of investment, according to the methods of dynamic programming, in the terminal node of column P_3 (third – final stage) the total sum of investment (from the beginning to the end) is stored and receives a value of financial constraints-5mln.

The solution model according to the method of the dynamic programming is considered from the terminal node of column P_3 before the initial node the column P_0 of the graph.

There are six possible ways of conditional optimal profit between P_2 and P_3 (look graph-diagram), for example, one of them the way 1 - 5 defined in P_2 put in 1 million (mln) investment and in P_3 put in 4mln investment, then 3mln. conditional optimal profit is given. Analogically may be considered the conditional optimal profit between P_1 and P_2, there are six possible ways too (look graph-diagram), for example:

$$\mathbf{0} \rightarrow 0+15=15;$$
$$\mathbf{1} \rightarrow 5+12=17;$$
$$\mathbf{2} \rightarrow 6+11=17;$$
$$\mathbf{3} \rightarrow 6+8=14;$$
$$\mathbf{4} \rightarrow 6+3=9;$$
$$\mathbf{5} \rightarrow 6+0=6.$$

One of them: the way 1 – 4 defined in P_1 put in 1mln investment and in P_2 put in 3mln investment, then 3mln conditional optimal profit are given (9+3=12). For column P0-initial node (only variant) there are six possible ways of conditional optimal profit between P_0 and P_1.

After considering all tour investing projects, the best profit -17mln is chosen. There are 3 tours of optimal plan investment:

- TOUR(T_1): **0→1→ 4 →5** defined, that in P_1 put 1mln., in P_2 put 3mln. and in P_3 put 1 mln., accordingly to (1) and (2) elements of the matrix selected projects get the values:

$$X_{21} = 1 \; X_{32} = X_{23} = 1;$$
$$X_{11} = X_{31} = X_{41} = X_{12} = X_{22} = X_{42} = X_{13} = X_{33} = X_{43} = 0.$$
$$\text{COST } (T_1) = \mathbf{5+9+3=17}$$

- TOUR(T_2): **0→1→ 5→5** defined, that in P_1 put 1mln., in P_2 put 4mln. and in P_3 put 0 mln., accordingly to (1) and (2) elements of the matrix selected projects get the values:

$$X_{21} = X_{42} = 1;$$
$$X_{11} = X_{31} = X_{41} = X_{12} = X_{22} = X_{32} = X_{13} = X_{23} = X_{33} = X_{43} = 0.$$
$$\text{COST } (T_2) = \mathbf{5+12+0=17}$$

- TOUR(T_3): **0 → 2 → 4 →5** defined, that in P_1 put 2mln., in P_2 put 2mln. and in P_3 put 1mln., accordingly to (1) and (2) elements of the matrix selected projects get the values:

$$X_{31} = X_{22} = X_{23} = 1;$$
$$X_{11} = X_{21} = X_{41} = X_{12} = X_{32} = X_{42} = X_{13} = X_{33} = X_{43} = 0.$$
$$\text{COST } (T_3) = \mathbf{6+8+3=17}$$

This method is based on considering all the ways to invest all alternative projects for choosing the best profit and is called “consider all plans for selecting optimal value " but the obtained result is not achieved in reasonable time. The “dark color “indicates the tour of an approximate optimal plan of the investment project.

The pseudo code of dynamic method has the form:

```
Pseudo code - dynamic method ( )
{
   List paths;
   List Profit;
   for each path in GraphPaths (StartVertex, EndVertex)
   {
       PathProfit =0;
       for (i=0; i<path.Verterxs.count-1; i++)
       {
           PathProfit = PathProfit +
                        (path[i], path[i+1]).Weight -
                        (path.Vertexs[i+1].Label- path.Vertexs[i].Label)
       }
       Profit.add(PathProfit)
       paths.add(path)
   }
   MaxProfit= Profit.item[0];
   pathindex=0;
   for (j=0;j< Profit.item.count; j++)
       {
          if(MaxProfit< Profit.item[j])
          {
             MaxProfit= Profit.item[j];
             pathindex=j;
          }
       }
   print(path[pathindex], “›” ,MaxProfit )
}
```

Greedy algorithms (*Bang-Jensen, 2004*) look for simple, easy-to-implement solutions to complex, multi-step problems by deciding which next step will provide the most obvious benefit. A greedy strategy does not produce an optimal solution in general, but nonetheless, a greedy heuristic may locally yield to the optimal solutions that approximate a global optimal solution in a reasonable time.

Greedy answer: **5+12+0=17**

Accordingly of (1) and (2) elements of the matrix selected projects get the values:

$$X_{21} = X_{42} = 1;$$
$$X_{11} = X_{31} = X_{41} = X_{12} = X_{22} = X_{32} = X_{13} = X_{23} = X_{33} = X_{43} = 0$$

The pseudo code for greedy algorithm has the form:

```
Pseudo code- greedy ()
{
    O[m], I[m]
    CurentVertex=StartVertex
    for (i=1; i<m;i++)
    {
       for each v in Adj[CurentVertex]
       {
         if ( IsMax( (CurentVertex, v ).Weight-(v.label-
              CurentVertex.label)) in (CurentVertex, Adj [CurentVertex])
)
            {
               I[i]=v.label-I[i-1];
               O[i]=(CurentVertex, v).Wight;
               CurentVertex=v;
            }
      }
   }
   for (i=1; i<m;i++)
   {
      print(I[i]," >",O[i])
   }
}
```

5. Conclusion

In this work, dynamic programming and greedy algorithm strategy for solving optimization problems of FIC models are investigated. The particular example of financial investment companies is considered and a corresponding multistage graph has been constructed. Approximation algorithms, which solved multistage problems, have been delivered.

It is important to mention that similarly to investment management, this program is able to address supply management and other logistic problems.

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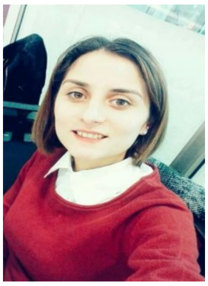

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