A multiregional sources of growth model for school enrolment projections

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Abstract

Background

Education planning requires accurate and efficient projection models. Current projection models either do not make use of all available information and are reliant on idiosyncratic expert judgement, or are too complex to be maintained and explained.

Aims

To test whether a multiregional projection model performs better than current methodology in explaining and projecting school enrolments in a school system with student mobility.

Data and methods

A multiregional cohort model was developed for projecting enrolments for multiple schools or districts simultaneously. For illustration, data were obtained for all government schools in the Australian Capital Territory (ACT) for the years 2008–2016. Multiregional projections were compared with a cohort transition model and the ACT Education Directorate's own projections.

Results

(i) There is great diversity in the sources of school enrolment growth that need to be accommodated in enrolment projections; and (ii) multiregional projections perform slightly better than traditional methods with less effort and more transparency.

Conclusion

A sources of growth approach guides the understanding of enrolment change, which is critical for making informed projections.

Key words

Enrolment projections; cohort transition; multiregional demography; Australian Capital Territory; Australia.

1. Introduction

School districts require information about the expectation of future school enrolments so that they can plan the supply of classrooms, teachers, materials and facilities. School enrolments are also important for understanding the education system within a larger social and economic context (Stone 1965). Without accurate projections, education may be disrupted because of inadequate facilities or a lack of teachers or other resources. Alternatively, education may be over-budgeted in the provision of teachers, infrastructure and resources, thus causing a misallocation of public funds.

While projections are never entirely certain due to unobserved factors influencing student mobility decisions (Morrison 2000), there are a variety of demographic tools that can be applied to increase projection certainty and help better judgements to be made. Surprisingly, in our review we find that most education jurisdictions neglect investments in testing or improving projection methodology and instead rely on relatively crude projection models with ad-hoc assumptions built in.

In this paper, we assess a method to project future enrolments for multiple schools or school districts simultaneously. Our method is illustrated with data obtained for the Australian Capital Territory (ACT) from the ACT Education Directorate for the period 2008–2016. Our aim is to provide better understanding of the demographic drivers enrolment change, and to develop more efficient projection systems. The multiregional projections are designed to study transitional changes that have occurred over time across school levels, as well as between schools and school regions.

2. Review of methods used to project school enrolments

Sweeney and Middleton (2005, p. 366) list several distinct approaches used to project school enrolments. These include *cohort survival* and *regression* methods that use past enrolment data and birth records, as well as extended versions of these methods which incorporate other information such as migration and student drop-out rates. They also list the modifiable spatial filter method (Rushton, Armstrong and Lolonis 1995), which combines cohort survival methods with residential address data to determine the size of future school attendance areas. These methods can incorporate birth records, past and future housing unit data and the number of students each housing type yields. Webster (1971) provides a similar list of projection approaches but also includes those based directly on age-specific population counts. Johnstone (1974) further distinguishes between models that are deterministic (including Markov Chain models), stochastic and constrainable.

One key issue is whether projections should be focused at the school level or at the broader district level. This also applies to projections for subgroups in heterogeneous populations (Grip 2009). A bottom-up (school level) approach can be tailored to fit each school's composition and needs; however, this approach may not consider wider demographic changes in the region around the school. A top-down (district level) approach, on the other hand, is often considered pragmatic because projections for large areas tend to be more accurate and less prone to rapid population changes (Berk & Associates 2008; Grip 2009; Schellenberg and Stephens 1987, p. 13; Stronge and Schultz 1981; Swanson and Tayman 2012, p. 281). It can also incorporate wider demographic changes which are occurring in the region of interest. The main disadvantage is that a top-down approach may produce unrealistic results for schools which have trends or student profiles that are substantially different from other schools within their district.

In order for projections to be understood and utilised, the methodology and assumptions need to be transparent and replicable. The projection approach must also be flexible and able to be adapted to a variety of situations and changes that occur to enrolment patterns if the time-consuming process of developing specific projection models for each school or school district is to be avoided (Gould 1993; Johnstone 1974; Stone 1965). This includes schools in established neighbourhoods, as well as those in areas that are currently being developed or planned.

At times, it may be necessary to incorporate expert judgements or scenarios to account for future change possibilities (Morrison 2000). Furthermore, the projection methodology should be regularly assessed in terms of accuracy (how well it predicts the truth) and efficiency (amount of time and effort required to produce results) (Swanson and Tayman 2012).

In most developed countries, including Australia, there tends to be very high rates of grade progression (close to 100 per cent) in most schools. This implies the expectation that, for example, nine years from now nearly all students enrolled in academic level 1 today will be enrolled somewhere in level 10. Moreover, there tends to be high retention of students at particular schools. The exception, of course, is for students transitioning between primary and secondary school or, in jurisdictions such as the ACT, from a government high school (years 7 to 10) to a secondary college (years 11 and 12). Current bulges or dips in student numbers can be expected to move through the academic levels over time. If primary school enrolments are lower in one year, secondary or high school enrolments will follow at predictable times in the future.

Projections for new schools require a relational model where auxiliary information such as births, population projections, household composition and anticipated migration is used to estimate the long-term stable size and composition of future student enrolments. There are other situations where auxiliary information is also needed. For example, newly developed areas in middle or outer growth precincts may be expected to first attract young families with preschool and primary school aged children due to the relative affordability of land or housing for newly established household units. Over time, these children will progress through the school system, with the school eventually reaching a steady state in respect to the number of new and continuing enrolments (Herrick 1952).

One approach which has not been considered widely in projecting future education requirements is a multiregional or multistate population model, which is an extension of the cohort transition model. Multiregional population models provide a general and flexible platform for modelling and analysing subnational population change over time (Rogers 1975, 1995; Land and Rogers 1982; Schoen 1988). They enable the combination of all the main components of population change by age with various transitions that each population group may experience over the life course. These transitions may be between academic levels (including primary to secondary or high school or high school to a secondary college) or between government and non-government schools.

Despite the many theoretical and analytical advantages, multiregional models have been relatively unexplored because of the large amount of input data needed and requirement for matrix calculations to perform the projections and analyses. One exception is Sweeney and Middleton (2005), who applied a multiregional cohort enrolment projection method to better understand intra-district school mobility and evaluate existing enrolment projections in Santa Barbara, California. Their aim was to understand intra-district school transfers in an open enrolment system for a heterogeneous population. In this paper, we build on their ideas to improve school enrolment projections in the ACT.

3. Analysing the sources of enrolment change

The population of the ACT has grown substantially over the past decade and a half from 322,000 in 2001 to 398,000 in 2016 (ACT Government 2017). This growth has been driven both by natural increase and net internal migration. Net internal migration contributed only a small amount despite large flows as levels of in-migration tended to offset out-migration. The ACT's average annual population growth rate between 2001 and 2014 was 1.4 per cent with a peak of 2.2 per cent occurring in the 2006–2007 period.

The ACT has seven population regions: Belconnen, Gungahlin, North Canberra, South Canberra, Weston Creek, Woden Valley and Tuggeranong.[1] The relatively new region of Gungahlin, located in Canberra's north, experienced the most rapid growth during this time, increasing from 25,000 in 2001 to 62,000 in 2014 with an average annual population growth rate of 7.4 per cent. In Australia, like in many other federal systems, school planning is undertaken at the state or territory level. The ACT is considered a geographically small system by Australian standards with four main levels of public school-based education:

- preschool – the level before full-time schooling, denoted as level P
- primary school – the first seven years of full-time schooling, referred to as level K (Kindergarten) to level 6
- high school – the next four years of full-time schooling, or levels 7 to 10
- secondary college – the last two years of full-time schooling, or levels 11 and 12.

The percentage of students in the ACT attending a government school increased from 57.5 per cent in 2008[2] to 59.6 per cent in 2016[3]. The remainder were in the non-government school sector, with transitions between both systems as documented in this paper.

The ACT Education Directorate is responsible for 88 public schools in the ACT comprising six early childhood schools (i.e., preschool to level 2), 51 primary schools (some with preschools attached), 10 high schools, nine combined primary/high schools, eight secondary colleges and four specialist schools. [4] Record-level data from 2008 to 2016 taken at each year's census in February were provided for this study. The data represents 611,674 student observations and contains the following variables: student ID (anonymised); census date; school; academic level (levels P–12); suburb; state. From these data, the transitions between academic levels were calculated, as well as movements amongst schools for all students in the ACT.

Following Rees and Willekens (1986), data were obtained that could be used to identify the main sources of enrolment change. This accounting framework is outlined in Table 1. The variable g_{ij} represents internal migration events from one school region i to another school region j. Internal migration events are excluded when $i = j$. Instead, terms r_i are entered, which are accounting balances that capture the result of subtracting all possible exit events from the starting enrolment population.

[1] Note, for our analyses, the Canberra Central region is divided into North Canberra and South Canberra. A map of the schools and regions is available from http://www.education.act.gov.au/__data/assets/pdf_file/0005/73319/160210-ACT-Public-Schools-Map-2016.pdf

[2] http://www.abs.gov.au/AUSSTATS/abs@.nsf/allprimarymainfeatures/FFBE2CE6D8296D21CA2576EA0011F617?opendocument

[3] http://www.abs.gov.au/AUSSTATS/abs@.nsf/DetailsPage/4221.02016?OpenDocument

[4] http://www.education.act.gov.au/publications_and_policies/publications_a-z/annual_report/annual-report-2015-2016/section-b/b.1-organisational-overview. Specialist schools are excluded from this study.

This number represents students who transition to the next academic level while remaining in the school region. Total internal out-migration from each school region are denoted by g_{i+} and total internal in-migration to each school region are denoted by g_{+j}. The i_i variable signifies the number of migration events from outside the system of interest and o_i tabulates the corresponding number of persons leaving the system of interest. m_i and n_i denote transfers from and to non-government schools, respectively. The graduation events, x_i, and preschool, b_i, complete the flows in the table. Note, in this table, persons who leave the school system without graduating are included in o_i.

Table 1: A sources of growth accounting framework for government school enrolments across regions

Region	Destination										
	Region (t+1)										
(t)	1	2	3	4	5	6	7	NG	MIG	GRAD	Total
1	r_1	g_{12}	g_{13}	g_{14}	g_{15}	g_{16}	g_{17}	m_1	o_1	x_1	e_1^t
2	g_{21}	r_2	g_{23}	g_{24}	g_{25}	g_{26}	g_{27}	m_2	o_2	x_2	e_2^t
3	g_{31}	g_{32}	r_3	g_{34}	g_{35}	g_{36}	g_{37}	m_3	o_3	x_3	e_3^t
4	g_{41}	g_{42}	g_{43}	r_4	g_{45}	g_{46}	g_{47}	m_4	o_4	x_4	e_4^t
5	g_{51}	g_{52}	g_{53}	g_{54}	r_5	g_{56}	g_{57}	m_5	o_5	x_5	e_5^t
6	g_{61}	g_{62}	g_{63}	g_{64}	g_{65}	r_6	g_{67}	m_6	o_6	x_6	e_6^t
7	g_{71}	g_{72}	g_{73}	g_{74}	g_{75}	g_{76}	r_7	m_7	o_7	x_7	e_7^t
P	b_1	b_2	b_3	b_4	b_5	b_6	b_7	0	0	0	b_+
NG	n_1	n_2	n_3	n_4	n_5	n_6	n_7	0	0	0	n_+
MIG	i_1	i_2	i_3	i_4	i_5	i_6	i_7	0	0	0	i_+
Total	e_1^{t+1}	e_2^{t+1}	e_3^{t+1}	e_4^{t+1}	e_5^{t+1}	e_6^{t+1}	e_7^{t+1}	m_+	o_+	x_+	

Notes: (1) Definitions of variables and subscripts: e = school enrolment population; r = balancing terms; g = internal migrations (within the ACT); P (b) = preschool enrolments; NG (m, n) = transfer to/from non-government schools; MIG (o, i) = migration to/from other states within Australia or overseas; GRAD (x) = exits due to graduation from government schools; 0 = structural zeroes; t = time; i = subscript for region; and + = summation over regions. (2) Assumes zero deaths.

The sum of the numbers in the rows of Table 1 add up to the enrolments at the beginning of the time interval, e_i^t. The balancing term is obtained by subtracting the total number of migrations to other regions within the ACT, transfers to non-government schools, migrations to places outside the ACT, and graduations from the population at the beginning of the time interval, i.e.,

$$r_i = e_i^t - g_{i+} - o_i - m_i - x_i$$

Similarly, the variables in the columns of Table 1 add up to the enrolments at the end of the time interval. We can compute these by adding to the balancing term the total in-migrations, migrations from outside the ACT, transfers from non-government schools and preschool entrants, i.e.,

$$e_i^{t+1} = r_i + g_{+i} + i_i + n_i + b_i$$

If we combine these two equations, the balancing term cancels out and we obtain the components of school enrolment change:

$$e_i^{t+1} = e_i^t - g_{i+} - o_i - m_i - x_i + g_{+i} + i_i + n_i + b_i$$

Thus, the information described in Table 1 provides the basis for understanding the sources of enrolment change from one year to the next at the region level.

Consider Table 2, which contains the sources of growth and cohort transitions occurring across school regions in the ACT for levels P–11 in 2015 to levels K–12 in 2016. The row sums equal the P–11 enrolments for schools in each of the seven ACT regions. The column sums equal the corresponding K–12 enrolments one year later.

Between 2015 and 2016, 72,694 students made a transition. This included 5,393 students who moved out of the ACT and 6,387 students who moved to the ACT. Over 80 per cent of the transitions occurred within the seven regions, with non-government schools having the highest retention. The proportion of students leaving government schools between 2015 and 2016 in the ACT ranged from 5.6 per cent in Weston Creek to 10.8 per cent in Woden Valley. In-migration was greatest to non-government schools (n=2,142), followed by Gungahlin (n=823), Belconnen (n=788) and North Canberra (n=696).

Table 2: Sources of enrolment change and cohort transition proportions for school regions in the ACT: levels P–11 (2015) to levels K–12 (2016)

Region 2015	**Enrolment Region 2016**									
	GUN	BEL	SCAN	TUGG	WOD	WEST	NCAN	NG	MIG	Total
	Enrolment change									
GUN	5,635	84	21	8	9	6	55	332	628	6,778
BEL	124	8,582	27	18	20	12	124	331	722	9,960
SCAN	12	17	3,455	32	145	3	14	129	392	4,199
TUGG	33	32	22	6,010	146	38	32	346	706	7,365
WOD	14	11	47	98	3,177	21	132	152	441	4,093
WEST	8	4	10	33	121	1,612	22	77	112	1,999
NCAN	56	88	89	21	19	2	4,319	144	558	5,296
NG	190	133	140	214	125	30	128	23,823	1,834	26,617
MIG	823	788	576	526	609	169	754	2,142	0	6,387
Total	6,895	9,739	4,387	6,960	4,371	1,893	5,580	27,476	5,393	72,694
	Transition proportions									
GUN	0.831	0.012	0.003	0.001	0.001	0.001	0.008	0.049	0.093	1.000
BEL	0.012	0.862	0.003	0.002	0.002	0.001	0.012	0.033	0.072	1.000
SCAN	0.003	0.004	0.823	0.008	0.035	0.001	0.003	0.031	0.093	1.000
TUGG	0.004	0.004	0.003	0.816	0.020	0.005	0.004	0.047	0.096	1.000
WOD	0.003	0.003	0.011	0.024	0.776	0.005	0.032	0.037	0.108	1.000
WEST	0.004	0.002	0.005	0.017	0.061	0.806	0.011	0.039	0.056	1.000
NCAN	0.011	0.017	0.017	0.004	0.004	0.000	0.816	0.027	0.105	1.000
NG	0.007	0.005	0.005	0.008	0.005	0.001	0.005	0.895	0.069	1.000
MIG	823	788	576	526	609	169	754	2,142	0	6,387

Notes: GUN = Gungahlin; BEL = Belconnen; SCAN = South Canberra; TUGG = Tuggeranong; WOD = Woden Valley; WEST = Weston Creek; NCAN = North Canberra; NG = ACT non-government schools; MIG = migration to/from other states within Australia or overseas.

Next, consider the cohort transitions presented in Table 3 (next page) from level 10 to level 11. The diagonal elements represent those who remained in schools within the region and progressed from level 10 to level 11. Most regions retained the majority of their students in the transition from high school to secondary college, except Weston Creek (0 per cent, where no secondary college is available) and Woden Valley (46%). Tuggeranong had the highest retention (83%). The data clearly show that substantial movements occurred across school regions in the transition from high school to secondary college.

Table 3: Cohort transition proportions for school regions in the ACT: level 10 (2015) to level 11 (2016)

Region	**Enrolment Region 2016**									
2015	GUN	BEL	SCAN	TUGG	WOD	WEST	NCAN	NG	MIG	Total
GUN	**0.807**	0.048	0.013	0.000	0.003	0.000	0.033	0.008	0.090	**1.000**
BEL	0.056	**0.789**	0.016	0.004	0.007	0.000	0.047	0.005	0.074	**1.000**
SCAN	0.010	0.005	**0.829**	0.025	0.005	0.000	0.005	0.020	0.101	**1.000**
TUGG	0.009	0.009	0.018	**0.826**	0.055	0.000	0.000	0.005	0.078	**1.000**
WOD	0.006	0.006	0.091	0.360	**0.457**	0.000	0.000	0.000	0.080	**1.000**
WEST	0.013	0.000	0.046	0.119	0.722	**0.000**	0.013	0.007	0.079	**1.000**
NCAN	0.038	0.077	0.162	0.031	0.198	0.000	**0.396**	0.018	0.080	**1.000**
NG	0.048	0.023	0.043	0.042	0.026	0.000	0.016	**0.747**	0.055	**1.000**
MIG	67	99	78	95	161	0	72	87	**0**	**659**

Notes: GUN = Gungahlin; BEL = Belconnen; SCAN = South Canberra; TUGG = Tuggeranong; WOD = Woden Valley; WEST = Weston Creek; NCAN = North Canberra; NG = ACT non-government schools; MIG = Migration to/from Canberra.

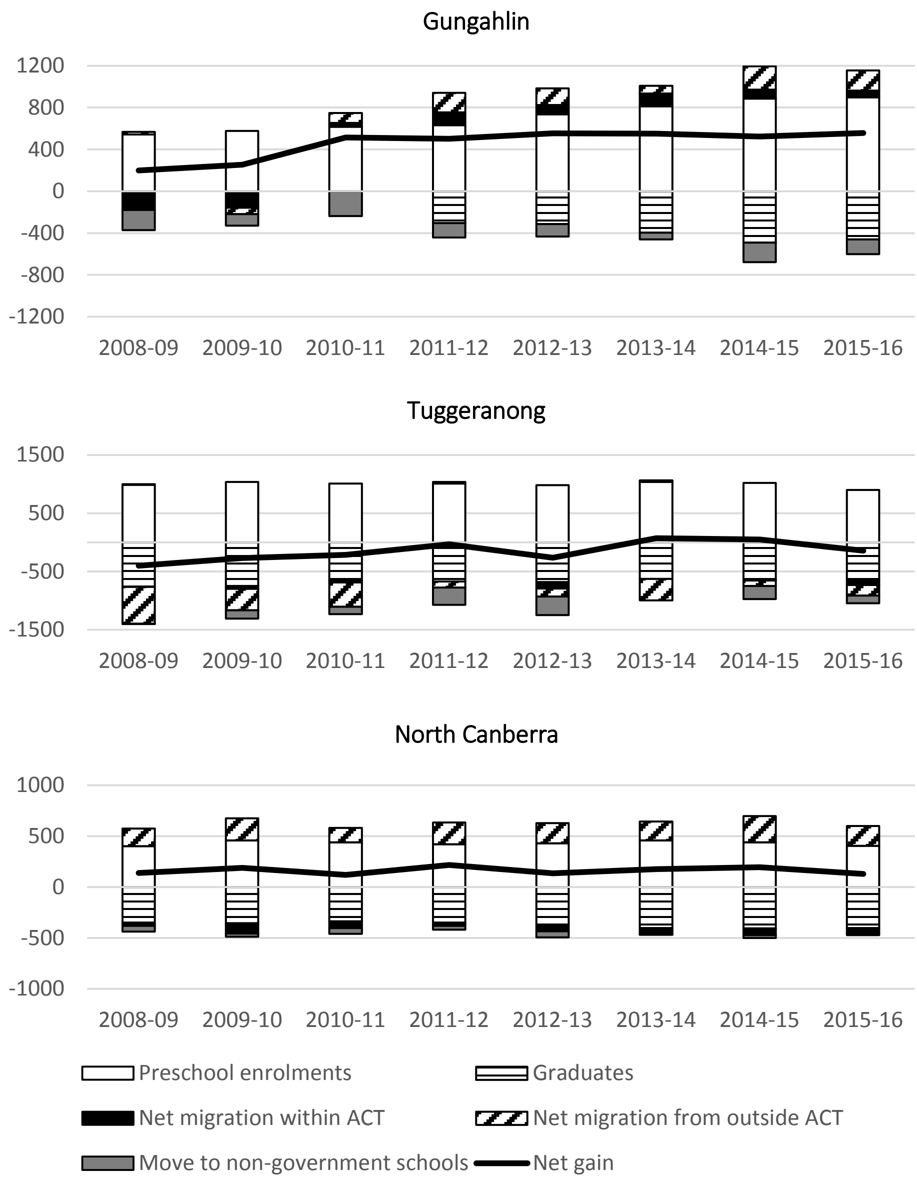

Figure 1: Components of enrolment change for schools in Gungahlin, Tuggeranong and North Canberra, 2008–2016

Between 2008 and 2016 most school regions in the ACT, except Tuggeranong, experienced growth in enrolments each year. The sources and levels of enrolment change, however, varied considerably across the regions with some regions dependent on several sources of growth (e.g., South Canberra, Woden Valley and non-government schools) and others driven primarily by preschool enrolments (e.g., Belconnen, Tuggeranong and Weston Creek).

Consider, for example, the sources of enrolment change during 2008–2016 for schools in the Gungahlin, Tuggeranong and North Canberra regions (Figure 1, previous page). Gungahlin grew the most, with this growth particularly evident from 2011 with 500 to 560 new students added each year. Preschool enrolments were the most significant and increasing component of growth for Gungahlin. However, there was a steady net loss of students to non-government schools each year. Preschool enrolments similarly were the only source of enrolment growth in the Tuggeranong region between 2008 and 2016, declining marginally in 2015–2016.

4. Cohort transition projection models

Three models are introduced for projecting enrolments by academic level in this section. The first is the standard cohort transition model based on annual enrolment numbers by academic level (Johnstone and Philp 1973; Gould 1993; Webster 1970). The second projection model is a multiregional cohort transition model that integrates all of the main sources of enrolment change presented in Table 1. The third projection model is that utilised by the ACT Education Directorate, which, at the individual school level, combines cohort transition information with assumptions on household compositional changes over time in relation to planned developments in the school's priority intake area. Feedback from school principals on the plausibility of the results is also incorporated.

To assess the accuracy of the projections, 2008–2012 data are used to predict the observed 2013–2016 enrolments with the assumption that preschool enrolments are known. The purpose of the analysis is to see how well the two types of data-based projections perform against the ACT Education Directorate's projections.

4.1 Cohort transition model

A simple and effective way to project school enrolments is to calculate the ratios of students progressing from one academic level to the next. While this does not explain changes to enrolment numbers from one year to the next, it does provide a measure of the overall change, which can be compared over longer periods.

Consider, for example, the observed enrolment data for schools in the Gungahlin region from 2008–2016 (Table 4). Here, we see that there were 316 level 1 students in 2008 (shaded grey). In 2009, this cohort of students grew to 325 level 2 students and then to 341 level 3 students in 2010. In 2016, the same cohort represented 385 level 9 students. Note, as mentioned previously, Gungahlin is a relatively new school region and has been growing rapidly due to migration from outside the ACT (level 11 became available in 2011 and led to the first cohort of level 12 students in 2012). Cohort sizes in other school regions remained very similar or declined.

Table 4: Observed enrolment data by academic level for schools in Gungahlin, 2008–2016

Level	2008	2009	2010	2011	2012	2013	2014	2015	2016
P	589	543	579	619	633	736	815	888	898
K	361	402	379	432	516	538	613	699	773
1	316	373	423	397	460	549	580	586	693
2	326	325	346	429	428	471	531	570	598
3	335	331	341	378	433	443	485	553	553
4	296	350	360	316	401	459	468	494	575
5	309	318	357	350	350	423	461	497	521
6	309	303	319	349	390	380	440	485	502
7	242	273	284	264	339	339	332	387	458
8	229	259	266	295	278	354	360	367	392
9	204	231	260	271	284	298	365	380	385
10	203	204	245	261	289	321	316	399	415
11	0	0	0	308	369	413	502	479	570
12	0	0	0	0	303	311	396	489	458

Notes: P = Preschool; K = Kindergarten.

Transition ratios calculated from the enrolment numbers in Table 4 can be used to produce projections of school enrolments. The projection model is specified, in matrix form, as

$$\mathbf{E}_{t+1} = \mathbf{G}\mathbf{E}_t + \mathbf{B}$$

where $\mathbf{E}_t$ are vectors denoting enrolments at time *t* and in academic level *x*, $\mathbf{G}$ is the transition matrix and $\mathbf{B}$ is a vector including the projected preschool enrolments. In detailed form, the matrix equation looks like

$$\begin{bmatrix} e_P^{t+1} \\ e_K^{t+1} \\ e_1^{t+1} \\ e_2^{t+1} \\ e_3^{t+1} \\ e_4^{t+1} \\ e_5^{t+1} \\ e_6^{t+1} \\ e_7^{t+1} \\ e_8^{t+1} \\ e_9^{t+1} \\ e_{10}^{t+1} \\ e_{11}^{t+1} \\ e_{12}^{t+1} \end{bmatrix} = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ g_P&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&g_K&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&g_1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&g_2&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&g_3&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&g_4&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&g_5&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&g_6&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&g_7&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&g_8&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&g_9&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&g_{10}&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&g_{11}&0 \end{bmatrix} \begin{bmatrix} e_P^{t} \\ e_K^{t} \\ e_1^{t} \\ e_2^{t} \\ e_3^{t} \\ e_4^{t} \\ e_5^{t} \\ e_6^{t} \\ e_7^{t} \\ e_8^{t} \\ e_9^{t} \\ e_{10}^{t} \\ e_{11}^{t} \\ e_{12}^{t} \end{bmatrix} + \begin{bmatrix} b^{t-4}g_0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where e_x^t denotes enrolments at time t, g_x denotes transitions between school levels with the subscript marking the origin academic level, b^{t-4} denotes births and g_0 is the ratio of births at time *t*–4 to preschool enrolments. This model could be adapted further to include repetition rates as in Johnstone and Philp (1973).

4.2 Multiregional cohort transition model

The enrolment transition model presented above is designed for projecting enrolments for single regions/schools independent of other regions/schools. To allow multiple regions/schools to be projected simultaneously, the matrix projection model needs to be reworked to include transitions amongst schools as well as academic levels. This matrix projection model is specified as:

$$\mathbf{E}_{t+1,x+1} = \mathbf{G}\mathbf{E}_{t,x} + \mathbf{I}_{x+1}$$

where $\mathbf{E}_{t,x}$ is a vector of enrolments of regions or schools at time *t* and academic level *x*, $\mathbf{G}$ contains the transition probabilities between levels and schools (g_{ij}), and $\mathbf{I}_{x+1}$ is a vector of in-migration counts of enrolments at academic level *x+1*. In detailed form, the matrix equation for an eight school region system looks like

$$\begin{bmatrix} e_{1,x+1}^{t+1} \\ e_{2,x+1}^{t+1} \\ e_{3,x+1}^{t+1} \\ e_{4,x+1}^{t+1} \\ e_{5,x+1}^{t+1} \\ e_{6,x+1}^{t+1} \\ e_{7,x+1}^{t+1} \\ e_{8,x+1}^{t+1} \end{bmatrix} = \begin{bmatrix} r_1 & g_{21} & g_{31} & g_{41} & g_{51} & g_{61} & g_{71} & g_{81} \\ g_{12} & r_2 & g_{32} & g_{42} & g_{52} & g_{62} & g_{72} & g_{82} \\ g_{13} & g_{23} & r_3 & g_{43} & g_{53} & g_{63} & g_{73} & g_{83} \\ g_{14} & g_{24} & g_{34} & r_4 & g_{54} & g_{64} & g_{74} & g_{84} \\ g_{15} & g_{25} & g_{35} & g_{45} & r_5 & g_{65} & g_{75} & g_{85} \\ g_{16} & g_{26} & g_{36} & g_{46} & g_{56} & r_6 & g_{76} & g_{86} \\ g_{17} & g_{27} & g_{37} & g_{47} & g_{57} & g_{67} & r_7 & g_{87} \\ g_{18} & g_{28} & g_{38} & g_{48} & g_{58} & g_{68} & g_{78} & r_8 \end{bmatrix} \begin{bmatrix} e_{1,x}^{t} \\ e_{2,x}^{t} \\ e_{3,x}^{t} \\ e_{4,x}^{t} \\ e_{5,x}^{t} \\ e_{6,x}^{t} \\ e_{7,x}^{t} \\ e_{8,x}^{t} \end{bmatrix} + \begin{bmatrix} i_{1,x+1} \\ i_{2,x+1} \\ i_{3,x+1} \\ i_{4,x+1} \\ i_{5,x+1} \\ i_{6,x+1} \\ i_{7,x+1} \\ i_{8,x+1} \end{bmatrix},$$

where the diagonal elements of the $\mathbf{G}$ matrix is equal to the retention proportions (r_i) and the off-diagonal elements capture the movements between schools or regions (g_{ij}). This model is similar to the one used by Sweeney and Middleton (2005) to study enrolment transfers in the Santa Barbara Elementary School District in California.

5. School enrolment projections

5.1 School regions

To provide an assessment of different projection models, we first focus on school regions. This allows us to make broad comparisons regarding expectations of change across a range of schools, and to work with data that tend to be more stable over time and less prone to specific situations that require expert assumptions or local knowledge. For the assessment, we assume that preschool enrolments are known. Normally these are estimated using observed or projected birth data (*t*–4).

In addition to our comparison of the two projection models against the ACT Education Directorate's projections and the observed data, we show the effects of different assumptions concerning the transitions used for the projections. The enrolment transition projection model uses the average annual transitions from 2008–2012. Three different transitions assumptions are used in the multiregional projection model. The first keeps 2011–2012 transitions constant. The second uses average 2008–2012 transitions and holds them constant. The third incorporates trends based on linear regressions of the observed transitions from 2008–2009 to 2011–2012.

Table 5: Average observed and projected student enrolments by school region, 2013–2016

School Region	Observed	ED	Cohort Transition	Multiregional 2011–12	Multiregional 2008–12	Multiregional Trend
GUN	6,941	7,195	6,229	6,663	5,971	6,911
BEL	10,464	10,683	10,652	10,340	10,636	10,443
SCAN	4,633	4,766	4,517	4,501	4,471	4,789
TUGG	7,942	7,873	7,756	8,181	7,985	8,304
WOD	4,548	4,280	4,325	4,354	4,247	4,476
WEST	2,063	2,178	2,083	2,071	2,041	2,210
NCAN	5,628	5,724	5,637	5,690	5,511	5,860
Total	42,219	42,699	41,198	41,799	40,861	42,994

Notes: GUN = Gungahlin; BEL = Belconnen; SCAN = South Canberra; TUGG = Tuggeranong; WOD = Woden Valley; WEST = Weston Creek; NCAN = North Canberra; ED = ACT Education Directorate.

Table 5 presents the average 2013–2016 results from the school region projections with the best performance shaded grey. Overall, the multiregional projection model with 2011–2012 transitions performed the best, followed closely by the ACT Education Directorate's projections. However, the performance for each model varied by school region:

- The ACT Education Directorate's projection model did well on average for the Tuggeranong school region (-0.9% error), but poorly for Gungahlin (3.7%), Woden Valley (-5.9%) and Weston Creek (5.6%).
- The enrolment transition method performed the best out of all the models for South Canberra (-2.4%) and North Canberra (0.2%), well for Weston Creek (0.9%) but poorly for Gungahlin (9.9%) and Woden Valley (-5.2%).
- The multiregional model with 2011–2012 transitions performed well for Weston Creek (0.4%) but poorly for Gungahlin (-4.5%) and Woden Valley (-4.5%).
- The multiregional model with 2008–2012 transitions performed well for Tuggeranong (0.5%) and Weston Creek (-1.1%), but very poorly for Gungahlin (-14.6%) and poor for Woden Valley (-6.9%).
- The multiregional model with trend transitions did very well for Gungahlin (-0.5%) and Belconnen (-0.2%), but not well for South Canberra (3.5%), Tuggeranong (4.5%), Weston Creek (7.2%) and North Canberra (4.2%).

The year-to-year projection results and observed enrolment numbers for Gungahlin, Tuggeranong and North Canberra are presented in Figure 2 (next page).

In summary, the multiregional models applied to school regions appear to work best overall (especially considering the objective nature of the information required). Average transition models work well when the school regions are relatively stable but poorly when regions are growing, such as Gungahlin and Belconnen. Here, incorporating trends or auxiliary information on planning developments in the transitions makes sense. The big advantage of the multiregional projection models is that the patterns of change (and error) can be explained and linked to observed trends. There is also consistency in the projection framework for school region moves within the ACT. That is, a departure from one school region must be an entry into another. This is particularly important when considering the transition between primary and high school levels and between high school and secondary college levels. The other factor to consider is efficiency.

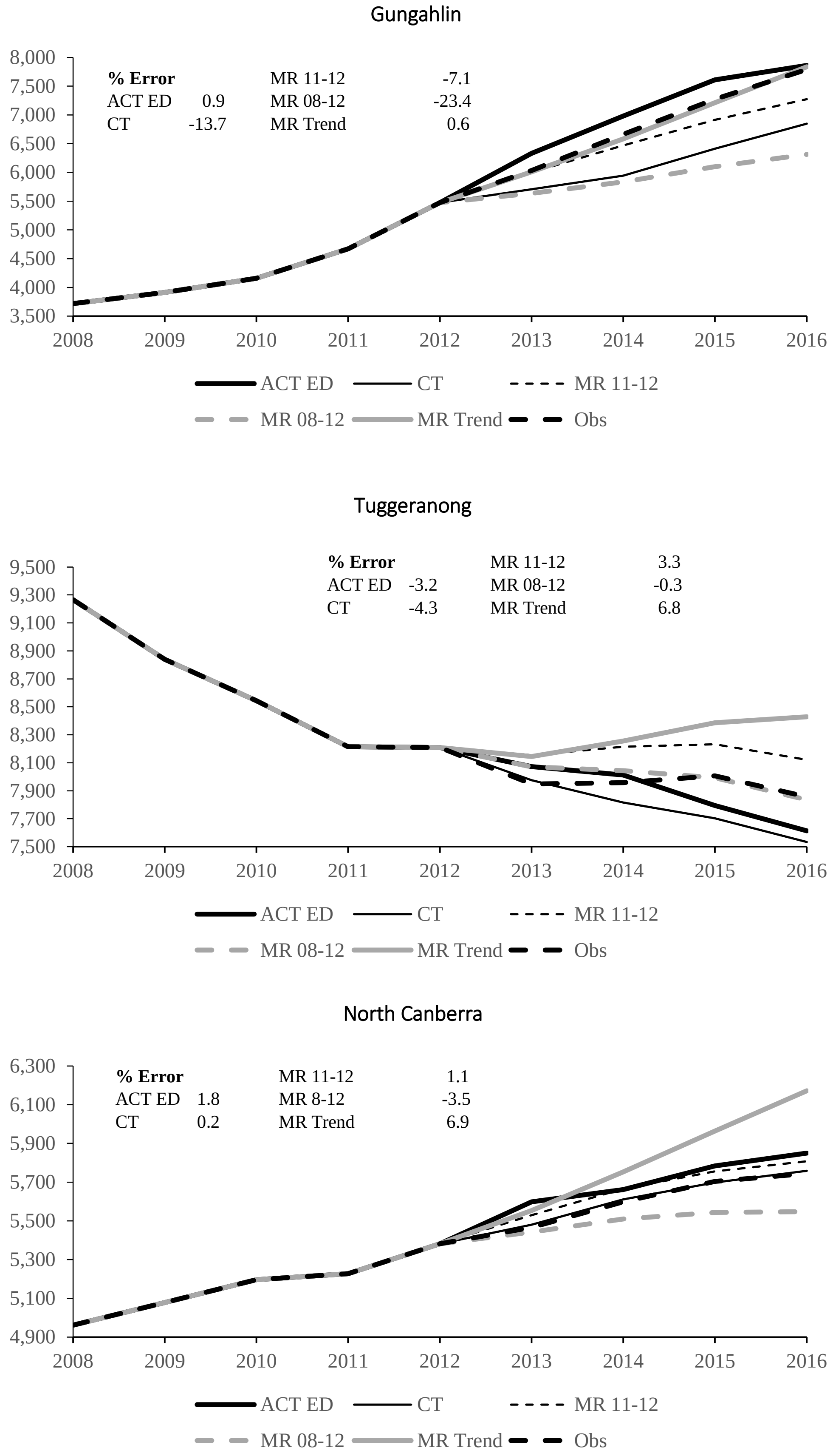

Figure 2: Observed and estimated enrolments for schools in Gungahlin, Tuggeranong and North Canberra, 2008–2016

Notes: ACT ED = ACT Education Directorate's projection; CT = cohort transition; MR = multiregional projection; Obs = observed values.

5.2 School projections for North Canberra

The multiregional projection method is particularly useful for capturing students transitioning between primary and high schools and between high schools and secondary colleges. This is further illustrated with schools in North Canberra, an area with a relatively stable total population in terms of age composition and growth.

Table 6: Observed and projected enrolments for schools in North Canberra, 2013–2016

	2013			2014			2015			2016		
School	Obs	ED	MR	Obs	ED	MR	Obs	ED	MR	Obs	ED	MR
					A. Observed and projected enrolments							
A	424	436	442	410	432	434	405	443	420	396	480	412
B	315	344	331	327	335	343	350	345	345	350	336	338
C	421	429	426	432	423	432	471	456	446	480	471	439
D	567	632	591	585	662	604	598	675	614	600	656	621
E	483	425	443	554	436	447	556	456	466	611	462	466
F	77	84	79	76	84	78	70	85	74	77	84	72
G	545	587	541	553	588	544	562	600	532	573	622	515
H	718	744	711	741	797	721	723	823	719	721	842	710
I	1,068	1,064	1,059	1,054	1,064	1,082	1,050	1,064	1,073	1,065	1,064	1,084
J	848	854	809	866	841	805	919	837	824	871	833	835
Total	5,466	5,599	5,433	5,598	5,662	5,490	5,704	5,784	5,510	5,744	5,850	5,491
					B. Difference between projected and observed							
A		12	18		22	24		38	15		84	16
B		29	16		8	16		-5	-5		-14	-12
C		8	5		-9	0		-15	-25		-9	-41
D		65	24		77	19		77	16		56	21
E		-58	-40		-118	-107		-100	-90		-149	-145
F		7	2		8	2		15	4		7	-5
G		42	-4		35	-9		38	-30		49	-58
H		26	-7		56	-20		100	-4		121	-11
I		-4	-9		10	28		14	23		-1	19
J		6	-39		-25	-61		-82	-95		-38	-36
Total		133	-33		64	-108		80	-194		106	-253

Notes: Obs = observed values; ED = ACT Education Directorate projection; MR = multiregional projection.

The school level projections for North Canberra focus on the level and between-school movements across seven primary schools (schools A–G), two high schools (schools H–I) and one secondary college (school J). In this illustration, only schools in North Canberra are included, but the model framework could be extended to include any set of schools. Also, for illustration purposes, we apply the average 2008–2012 transitions as the basis for estimating transitions for 2013–2016.

The projection results for schools in North Canberra are presented in Table 6 (panel A). Also included are the corresponding observed values and the projections made by the ACT Education Directorate (panel B) with the closest value to the observed value shown in grey. The multiregional projection model out-performed the ACT Education Directorate's projections for seven out of the 10 schools in each of the four projection years. These differences, however, are fairly modest. The real benefits of the multiregional model are that: (i) less assumptions are required; and (ii) there is a capacity to disaggregate the projection error.

6. Discussion and conclusion

Understanding the demographic drivers of enrolment change is essential for making accurate and informed projections. Variables influencing change include the birth of children in school intake areas, movements between government schools in a city or district, migration into or out of a city or region, and movements to or from non-government schools. The multiregional cohort projection model presented in this paper provides a flexible platform for including the main transitions affecting school enrolment change. By grouping schools in a multiregional model, understanding about student movements is increased and projection bias reduced.

In order to make and assess projections, some understanding of the components of enrolment change and their trends is required. There is no single projection model that performs best in all situations. School districts should consider a variety of assumptions when developing projection models and adapt them to meet local differences. They need also to incorporate proposed or new housing developments and land releases which may result in increased migration by young families into the area and rapid population growth impacting education provision for the areas affected.

As projections are often heavily scrutinised, the ability to explain trends in terms of the sources of growth provides a stronger evidence base and argument for school infrastructure, personnel and resource requirements. It also allows the projections producer to understand where the sources of error occurred in relation to changing enrolments over time. This is the main motivation underlying the multiregional cohort projections.

We have demonstrated the variability in the sources of student enrolment change across the ACT and the relative stability in the different sources of enrolment change and cohort transitions. Movements within and outside the ACT government school system can have large effects and are more difficult to predict. The multiregional school enrolment projection model captures these movements and may be applied to a wide array of situations. However, the transition data must be available.

Key messages

- Accurate projections of school enrolments and understanding of sources of error are essential for good school planning.
- The multiregional projection model used in our study performed as well or better than a cohort transition model and current projection model used by the ACT Education Directorate.
- A multiregional model that interrogates enrolment change can be used for short- to medium-term enrolment projections in the ACT.

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