A Short Note on Non-isothermal Diffusion Models

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Asymptotic behaviour of the DIAL and DRAL non-isothermal models, derived previously for the diffusion of water vapour through a porous building structure, is studied under the assumption that the initially non-isothermal structure becomes purely isothermal.

Keywords: Isothermal and non-isothermal diffusions, diffusion models, condensation in building structures.

1 Introduction

In a previous research communication [1], the DIAL and DRAL non-isothermal diffusion models were derived and compared with the standard isothermal models commonly used in building thermal technology. In a further related communication [2] we applied these models to the Glaser condensation scheme. The Glaser scheme enables, among others, an assessment of condensate for a one-year period. Naturally, throughout the year the structure is mostly exposed to non-isothermal states but, especially in the summer season, it may be subjected to a purely isothermal state. The DIAL and DRAL models provide the basic relations solely for non-isothermal conditions, i.e., they contain different temperatures T_1, T_2 belonging to the opposite sides of the structure. When approaching the isothermal state ($T_2 \to T_1$), the relations give an uncertain expression 0/0 and, at first sight, it is not clear how these relations should be applied to the isothermal state ($T_2 = T_1 = T$).

This short communication is aimed at deriving asymptotic DIAL and DRAL relations holding for the isothermal state of a building structure possessing the diffusion resistance factor μ.

2 Asymptotic DIAL relations

For the DIAL model the generalised diffusion resistance R_d^* and diffusion 'conductivity' D_{eff}^* read [1]

$$R_{\text{eff}}^* = \frac{d}{D_{\text{eff}}^*}, \quad D_{\text{eff}}^* = \frac{(2-n)\,k\,p_a}{\mu R_a} \frac{T_1 - T_2}{T_1^{2-n} - T_2^{2-n}}, \tag{1}$$

$$p_a = 98066.5\ \text{Pa},\ k = 8.9718 \cdot 10^{-10}\ \text{m}^2\text{s}^{-1}\text{K}^{-1.81},\ n = 1.81.$$

To determine the relations describing the isothermal state ($T_2 = T_1 = T$) it is necessary to use the l'Hospital rule

$$\lim_{T_2 \to T_1} \frac{T_1 - T_2}{T_1^{2-n} - T_2^{2-n}} = \lim_{T_2 \to T_1} \frac{\frac{d}{dT_2}(T_1 - T_2)}{\frac{d}{dT_2}\left(T_1^{2-n} - T_2^{2-n}\right)} = \frac{T_2^{n-1}}{2-n} = \frac{T^{n-1}}{2-n}, \tag{2}$$

which leads to the following result

$$R_{\text{eff}}^* = \frac{d}{D_{\text{eff}}^*}, \quad D_{\text{eff}}^* = \frac{k\,p_a}{\mu R_a} T^{n-1} = \frac{k\,p_a}{\mu R_a} T^{0.81}. \tag{3}$$

3 Asymptotic DRAL relations

Repeating the same procedure as in the foregoing section, we can rewrite the original DRAL non-isothermal relations [1]

$$R_{\text{eff}} = \frac{d}{D_{\text{eff}}}, \quad D_{\text{eff}} = \frac{(n-1)k}{\mu} \frac{T_1 - T_2}{T_2^{1-n} - T_1^{1-n}} \tag{4}$$

using the l'Hospital rule

$$\lim_{T_2 \to T_1} \frac{T_1 - T_2}{T_2^{1-n} - T_1^{1-n}} = \lim_{T_2 \to T_1} \frac{\frac{d}{dT_2}(T_1 - T_2)}{\frac{d}{dT_2}\left(T_2^{1-n} - T_1^{1-n}\right)} = \frac{T_2^n}{n-1} = \frac{T^n}{n-1} \tag{5}$$

into the purely isothermal relations ($T_2 = T_1 = T$)

$$R_{\text{eff}} = \frac{d}{D_{\text{eff}}}, \quad D_{\text{eff}} = \frac{k}{\mu} T^n = \frac{k}{\mu} T^{1.81}. \tag{6}$$

Relation (6) exactly corresponds to the isothermal IM-TDR result [1], which confirms the consistency of the developed models.

4 Conclusion

Derived asymptotic relations (3) and (6) represent a necessary complement for the DIAL and DRAL *non-isothermal* models when they are faced with the task of estimating the water condensate inside building structures exposed to purely *isothermal* conditions. Such problems may appear in building thermal technology when calculating the one-year balance of condensate inside the building envelopes.

References

[1] Ficker, T., Podešvová, Z.: *Models for Non-Isothermal Steady-State Diffusion in Porous Building Materials*. Acta Polytechnica (accepted for publication).

[2] Ficker, T., Podešvová, Z.: *Modified Glaser's Condensation Model*. Acta Polytechnica (accepted for publication).

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