Gravitomagnetism

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In the Introduction to our talk, we explained a simple thought experiment to indicate that gravitomagnetism has to arise if some basic relations, such as the dependence of inertial mass on velocity given by the standard special-relativistic formula, the dependence of active gravity on the inertial mass, and the Lorentz invariance, take place: the effect on a test mass between two uniformly moving parallel streams of linear distributions of masses was considered from the frame in which they both move with the same velocity but in opposite directions, and from the frame in which one of the streams is at rest (see, e.g., [1] for details).

We then summarized the basic ideas and results of the recent experiment *Gravity Probe B*, in which four gyroscopes placed in a satellite orbiting the Earth measure the gravitomagnetic field caused by Earth's rotation. After a long period of preparation, observation and data analysis it has now been concluded that "the combined four-gyro result gives a statistical uncertainty of 14 % (∼ 5 marcsec/yr) for frame-dragging" (in other words, for the presence of gravitomagnetic effects) – see http://einstein.stanford.edu/highlights/status1.html.

In the main part of the talk, we summarized our recent works with Joseph Katz from the Hebrew University in Jerusalem and Donald Lynden-Bell from the Institute of Astronomy of the University of Cambridge on Mach's principle and gravitomagnetic effects in general relativity and cosmology.

Einstein was strongly influenced by Mach's idea that the inertia of a particle here and now arises as a consequence of its interaction with other particles in the universe. What do we understand by "Mach's principle" today? In our comprehensive work [2] summarizing a number of our preceding contributions to gravitomagnetic effects and Mach's principle, we start out from the general formulation of Mach's principle by Hermann Bondi in his *Cosmology* [3]: Local inertial frames are determined by the distribution of energy and momentum in the Universe by suitable averages. In mathematical terms, we investigate the validity of such a formulation for the case of general linear perturbations of standard "background" models, i.e., of isotropic and homogeneous cosmological models described by the Friedmann-Lemaître-Robertson-Walker solutions of Einstein's field equations. In particular, we focus on those of Einstein's equations for linear perturbations which represent constraints on initial data. In suitable coordinates ("gauges"), these constraints are represented by elliptic equations which connect the distribution of matter and energy described by an energy-momentum tensor of physical matter and fields with the geometry described by the metric tensor and its derivatives. In these gauges, the local inertial frames are determined *instantaneously* by the distribution of matter and energy. One has to realize that the physical effects associated with Mach's ideas, e.g. the "dragging of inertial frames" ("gravitomagnetic effects") have a global character and require special coordinate systems, special gauges. As was noted by Dieter Brill in the discussion during the conference on various aspects of Mach's principle (on the basis of which a very valuable book [4] arose), "Mach's principle can show the way to give physical meaning to quantities which are usually considered as coordinate dependent."

In more recent papers [5, 6], we show that within General Relativity, any general statement of Mach's principle that attributes all dragging of inertial frames solely to the distribution of energy and momentum of matter as the origin of inertia is false: *Gravitomagnetic effects are also caused by gravitational waves.* To show this, we investigate waves which do not depend on one spatial coordinate (say the "$z-$coordinate"). We find that there is an almost flat cylindrical region near the z-axis of a revolving gravitational wave pulse (which inevitably has no "physical" energy-momentum) and demonstrate that the inertial frame in the cylindrical interior rotates relative to the inertial frame at great distances. Our aim was to produce a nice clean example of the rotation of the inertial frame in an almost flat region surrounded by rotating gravitational waves. An extreme example of inertia due to gravitational waves alone is provided by Gowdy's universe [7], a closed world that contains nothing except gravitational waves. One of our ultimate aims is to discuss the meaning of Mach's principle in fully nonlinear general relativity, and particularly its application to such systems as Gowdy's universe. Another question of great interest is what bearing Mach's principle has on the existence of dark energy.

References

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This lecture, here briefly summarized, was dedicated to Jiří Niederle, whose broad interests and contributions in mathematical physics also include gravity theories.

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