

# “ FULL WAVEFORM INVERSION BASED ON MORPHOLOGICAL COMPONENT ANALYSIS SEISMIC DATA RECONSTRUCTION ”

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## ABSTRACT

Seismic exploration is the most important and the most effective method of solving the petroleum exploration problem which uses seismic data to cognize the underground geological structure and locate oil and gas traps. Full waveform inversion (FWI) is able to build high precision velocity model of earth medium through extracting full information from the seismic data. The quality of seismic data has significant impact on the full waveform inversion. However, field seismic data acquisition is restricted by various conditions, for example, the observation system is often irregular and seismic data may be missing. Irregularity or missing of seismic data will seriously affect the seismic processing and interpretation results. Seismic data reconstruction methods can be used to restore the missing data, then improve the quality of seismic data. In this paper, we utilize the morphological component analysis (MCA) method to reconstruct the seismic data. Then we combine this method with full waveform inversion to provide high quality seismic data as input for FWI. Experimental results show that the MCA method not only can rebuild seismic data accurately but also play a vital role in denoising. The FWI result using regular seismic data is better than the FWI result using irregular seismic data. In other words, we cannot obtain accurate velocity model using the irregular seismic for FWI. Full waveform inversion based on MCA seismic data reconstruction can obtain high precision subsurface velocity by taking advantages of the fact that this method can rebuild regular seismic data before the inversion process. Application of this method to the field seismic data shows that the quality of seismic data is improved, and can provide better inversion results.

## 1. INTRODUCTION

Full waveform inversion (FWI) is an effective method that using observed seismic data to invert for physical property of the earth media. FWI has been proved to be the most effective method of establishing high resolution velocity model of the subsurface [Sirgue, 2009; Warner, 2010]. The velocity model obtained from FWI can be used to improve the quality of migration and provide valuable imaging results. FWI is classically done by reducing the residual between simulated wave field and observed wave field. High precision velocity model is obtained by minimizing the objective function in the iteration process [Tarantola, 1984; Pratt, 1996]. The least squares

objective function in the data space is minimized through seeking the gradient and local descent direction in the model space. Then we can update the velocity model in each iteration and take the updated model as the input for the next iteration. In this process, the quality of seismic data is a key issue for full waveform inversion. The application of full waveform inversion first appeared in the 1980s. The realization of the full waveform inversion of the two-dimensional seismic data proves that the full waveform inversion is a high-precision earth model building method [Gauthier et al., 1986; Mora, 1987]. Three-dimensional full waveform inversion arose from the mid-1990s gradually [Pratt and Sams, 1996]. Then FWI was successfully applied to three-dimensional offshore

seismic data [Plessix et al., 2010; Fichtner et al., 2011; Hu et al., 2012]. However, the application to land data still has great challenges, mainly because it is unable to provide observation seismic data that meet the requirements of full waveform inversion.

In the practical seismic data acquisition process, due to some limitation factors such as topography, surface obstacles and acquisition cost, the survey system is usually not in accordance with the regular grid which will result in on the deviation of receivers from the regular grid points. Some receivers may be even far from the designed position. Consequently, some traces of the observed seismic data will be missing, and bad traces may appear. Irregular or sparse sampling seismic data is lack of some useful information, thereby seriously damaging the results of seismic data processing and interpretation. If we still process the irregular seismic data simply on a regular grid, large error will come out. Meanwhile, we cannot forcibly arrange the data to be on the regular survey grid to get the information of the real reflection point. Otherwise, FWI will provide incorrect inversion results.

Hence, it is very necessary to reconstruct the irregular seismic data.

Seismic data reconstruction is a method that interpolates the irregular seismic data through a certain strategy and algorithm to rebuild more complete data with higher sampling rate, hence improving the quality of the data. In recent years, the sparse signal theory has developed rapidly [Starck, 2010], which can be a powerful tool to reconstruct the irregular seismic data. The morphological component analysis (MCA) proposed by Stack is based on sparse representation and morphological diversity of the signal. By utilizing its morphological differences, the signal can be separated into several sparse components [Li et al., 2012; Du et al., 2015].

In this paper, we apply the MCA method to seismic data reconstruction on the basis of the principle of this method. Seismic data are divided into two morphological components according to the morphological differences of the components. The components of the two forms are reconstructed, and then the reconstructed results are combined to obtain the result of complete seismic data. We incorporate the data reconstruction method in the full waveform inversion process, in order to provide high quality seismic data for full waveform inversion. We test the data reconstruction method with a numerical model, and the results show that the MCA method can rebuild seismic data accurately and has the ability of denoising. Full waveform inversion based on MCA data reconstruction

gets higher precision results. At last, we apply this method to the field seismic data. It shows that the quality of seismic data is improved, and inversion results become better.

## 2. SEISMIC DATA RECONSTRUCTION METHOD

Sparse representation is a representation approach that accounts for most or all information of a signal with a linear combination of a small number of elementary signals called atoms. The procedure of expressing the signal in sparse representation by using an over-complete dictionary is called signal sparse decomposition. A signal is usually represented in time domain or a proper transform domain. The purpose of signal sparse decomposition is to process the signal more easily in transform domain.

A sparse signal  $\mathbf{x}$  is usually expressed as the linear sum of a set of basic spread functions or signal atoms:

$$\mathbf{x} = \Phi \alpha = \sum_k^K \alpha_k \varphi_k \quad (1)$$

where  $\alpha_k$  is the sparse representation coefficient of the dictionary  $\Phi$ .  $\Phi$  is a  $N \times K$  matrix which is made up of signal atoms  $\varphi_k$  ( $\|\varphi_k\|^2 = 1$ ).

Starck [2004, 2005] found that a single transformation could not always represent the signal well. Therefore, the morphological component analysis method is proposed. Assuming a signal  $\mathbf{x}$  has  $N$  different morphological components  $\mathbf{x}_n$ , then it can be expressed as sum of all the components

$$\mathbf{x} = \sum_{n=1}^N \mathbf{x}_n = \sum_{n=1}^N \Phi_n \alpha_n \quad (2)$$

The actual seismic data is usually composed of many elements, that is, the seismic data has the feature morphological diversity. For the seismic data  $\mathbf{d}$ , assuming it is composed of  $N$  different components  $\mathbf{d}_n$  and has the noise  $\epsilon$ , it can be represented as

$$\mathbf{d} = \sum_{n=1}^N \mathbf{d}_n + \epsilon \quad (3)$$

If the seismic data  $\mathbf{d}_{obs}$  is irregular or incomplete, we need to reconstruct the missing information. This problem can be expressed as

$$\mathbf{d}_{rec} = \mathbf{R} \mathbf{d}_{obs} = \left( \sum_{n=1}^N \mathbf{d}_n + \epsilon \right) \quad (4)$$

where  $\mathbf{d}_{rec}$  is reconstructed data,  $\mathbf{R}$  is the seismic data regularization operator. We can get every component by solving the constrained optimization problem

$$\min_{\alpha_1, \dots, \alpha_N} \sum_{n=1}^N \|\alpha_n\|_1 \quad s.t. \quad \left\| \mathbf{d}_{obs} - \mathbf{R}^{-1} \sum_{n=1}^N \Phi_n \alpha_n \right\| \leq \sigma \quad (5)$$

where  $\|\alpha_n\|_1$  is L1 norm,  $\sigma$  is the residual between observed data  $\mathbf{d}_{obs}$  and iterative reconstruction item  $\sum_{n=1}^N \Phi_n \alpha_n$ . We use block coordinate relaxation (BCR) algorithm [Sardy et al., 2000] to solve the Equation (5). Firstly, we analyze the irregular seismic data and affirm the missing traces. Secondly, we extract the structure features and geological information of the seismic data and determine the sparse dictionary  $\Phi_n$ . Then we set the initial morphological component  $\mathbf{d}_n^{(0)} = 0$  and the initial residual  $\mathbf{r}^{(0)} = \mathbf{d}_{obs}$ , calculate the edge residual  $\mathbf{r}_n^{iter} = \mathbf{r}^{(iter-1)} + \mathbf{d}_n^{(iter-1)}$  within the scope of the maximum iteration. Finally, update the coefficient  $\alpha_n^{(iter)}$  and calculate the  $n^{th}$  component  $\mathbf{d}_n^{(iter)} = \Phi_n \alpha_n^{(iter)}$ . Cycle the above steps until the threshold value  $\lambda_n^{(iter)} < \lambda_{min}$ , and get the reconstruction result  $\mathbf{d}_{rec} = \sum_{n=1}^N \mathbf{d}_n^{(N_{iter})}$ .

### 3. FULL WAVEFORM INVERSION METHOD

Usually geophysical inversion problems are based on the Bayes framework [Tarantola, 1987]. Seismic wave propagation can be expressed as

$$\mathbf{d} = G(\mathbf{m}) \quad (6)$$

where  $\mathbf{m}$  is seismic geophysical parameters vector, such as subsurface velocity.  $G(\cdot)$  describes the propagation of seismic wave field which depends on subsurface model. The process of estimating the parameters of the stratigraphic model from seismic data is called seismic inversion. The inversion process can be write as

$$\mathbf{m} = G^{-1}(\mathbf{d}) \quad (7)$$

where  $G^{-1}(\cdot)$  describes the inversion process using proper mathematical algorithms.

Seismic inversion is essentially a nonlinear problem, and the solving process is very complicated. In the practical implementation process, the subsurface model and seismic wave propagation are approximation to the real situation. Because the propagation of seismic waves is a nonlinear process, the operator is not simply the inverse matrix of the forward operator. In the study of nonlinear inversion problems, the scholars often linearize it locally in order to simplify the solving process.

The misfit function of the full waveform inversion based on the regularization of seismic data is defined

as a L2 norm of the residual between the simulated data and the reconstructed data,

$$E(\mathbf{m}) = \frac{1}{2} \sum_{s,r} \int_0^T (\mathbf{S}\mathbf{u} - \mathbf{R}\mathbf{d}_{obs})^2 dt \quad (8)$$

where  $\mathbf{S}\mathbf{u} = G(\mathbf{m})$ ,  $\mathbf{u}$  is the full wave field of forward modeling,  $\mathbf{S}$  indicates the limitation of the location of the receivers.  $\mathbf{r}$  is the geophone point corresponding to the shot point.  $\mathbf{R}$  is the seismic data regularization operator. In this paper, we use the scalar wave equation to describe the seismic wave forward modeling process. The seismic geophysical parameters  $\mathbf{m}$  is the subsurface P-wave velocity. In order to obtain accurate subsurface velocity model, we should minimize the objective function (8) using effective optimization methods.

Methods of solving the objective function mainly include Newton methods and gradient methods. In this paper, we use the gradient method, and the velocity model updates can be expressed as an iteration formula,

$$\mathbf{m}^{(k+1)} = \mathbf{m}^{(k)} - \alpha^{(k)} \nabla_{\mathbf{m}} E^{(k)} \quad (9)$$

where  $k$  is the iteration number, and  $\alpha$  is the step length. The iterative direction is the opposite direction of the gradient of objective function.

Based on the time domain scalar wave equation, the gradient of the objective function is obtained by the adjoint-state method [Plessix, 2006],

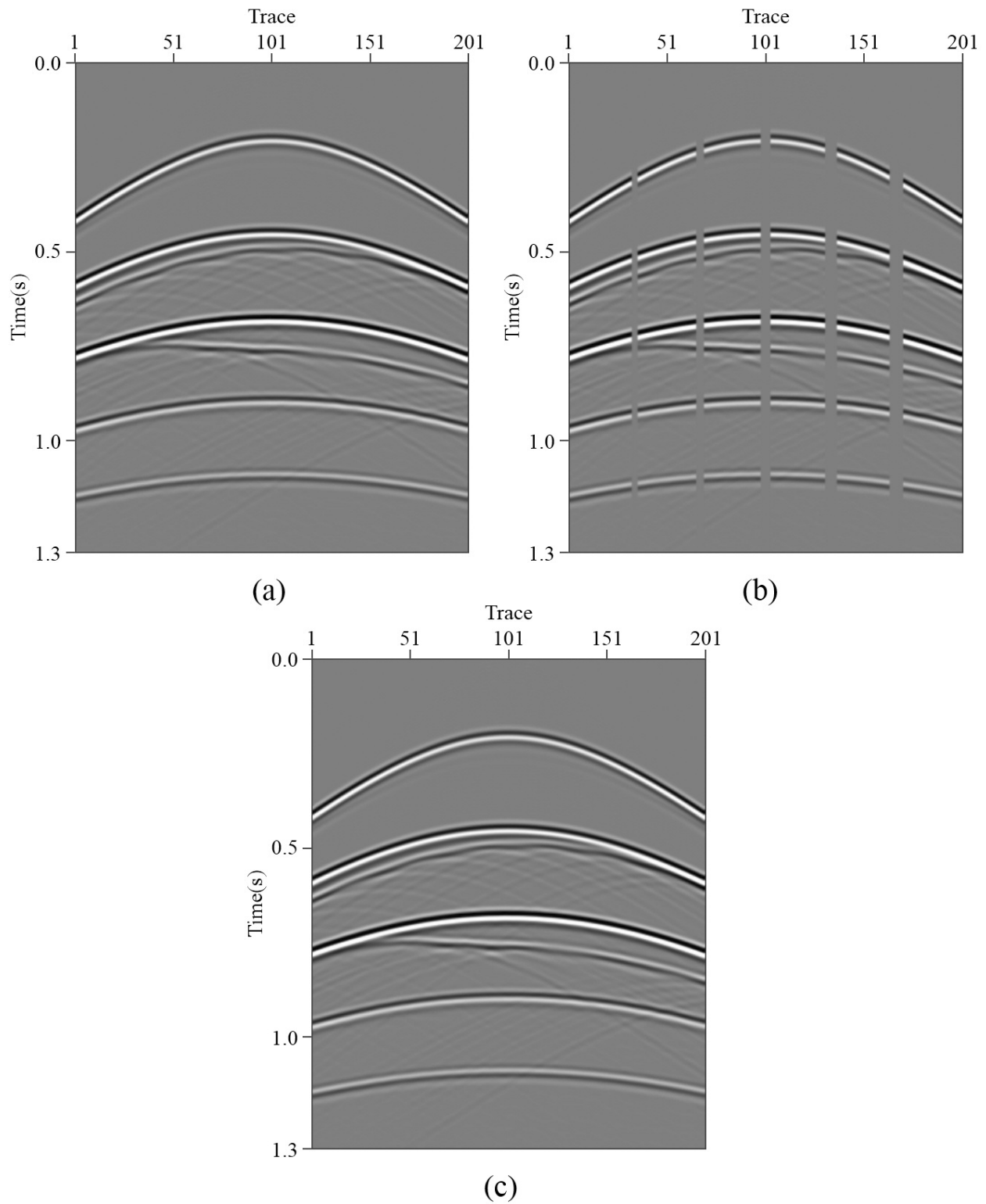
$$\nabla E = \sum_{s,r} \int_{t_0}^t \frac{\partial^2 \mathbf{u}}{\partial t^2} G^*(G(\mathbf{m}) - \mathbf{d}_{reg}) dt \quad (10)$$

where  $G^*(G(\mathbf{m}) - \mathbf{d}_{reg})$  describes the back propagation of wave field residuals in model space. The gradient is the inner product of the second order derivative of the forward wave field in time and the backward wave field of the residual.

## 4. NUMERICAL EXAMPLES

### 4.1 DATA RECONSTRUCTION METHOD TEST

We select a synthetic seismic record (Figure 1a) to verify the effect of MCA data reconstruction method. The data have 201 traces, and the trace interval is 10m. The time length of each trace is 1.3s, and the time interval is 1ms. We set five parts of traces of the original data to zero (Figure 1b). The number of the missing traces of the five parts is 3, 4, 5, 6 and 7 respectively. We apply the MCA data reconstruction method to the incomplete data, and get the reconstructed data (Figure 1c). Figure 1c shows that the MCA reconstruction method can rebuild the incomplete



**FIGURE 1.** a) Synthetic seismic record; b) Non-noise synthetic record with several traces decimated; c) Reconstructed record of b) by MCA method.

data effectively, even at the large gap location.

Meanwhile, the MCA reconstruction method can remove random noise due to the wavelet dictionary, because the random noise can't be sparsely represented in the UWT and Curvelet dictionaries. We add random noise

(SNR=10, SNR=4) to the incomplete shot record (Figure 1b) and get the noisy data (Figure 2a, Figure 2c). Then we reconstruct the noisy data using the MCA method, Figure 2b is the reconstructed data of the noisy record with SNR=10, and Figure 2d is the reconstructed data

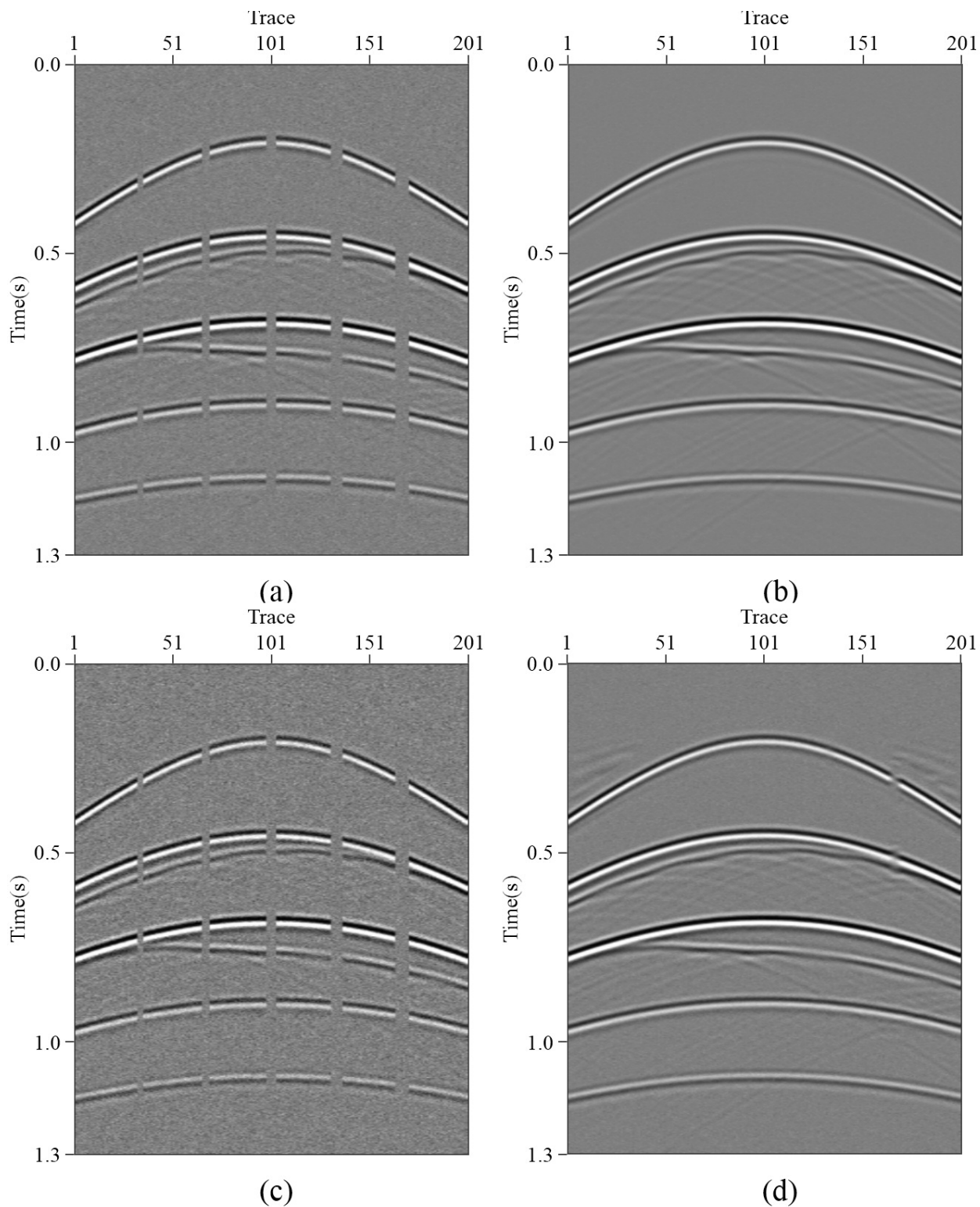
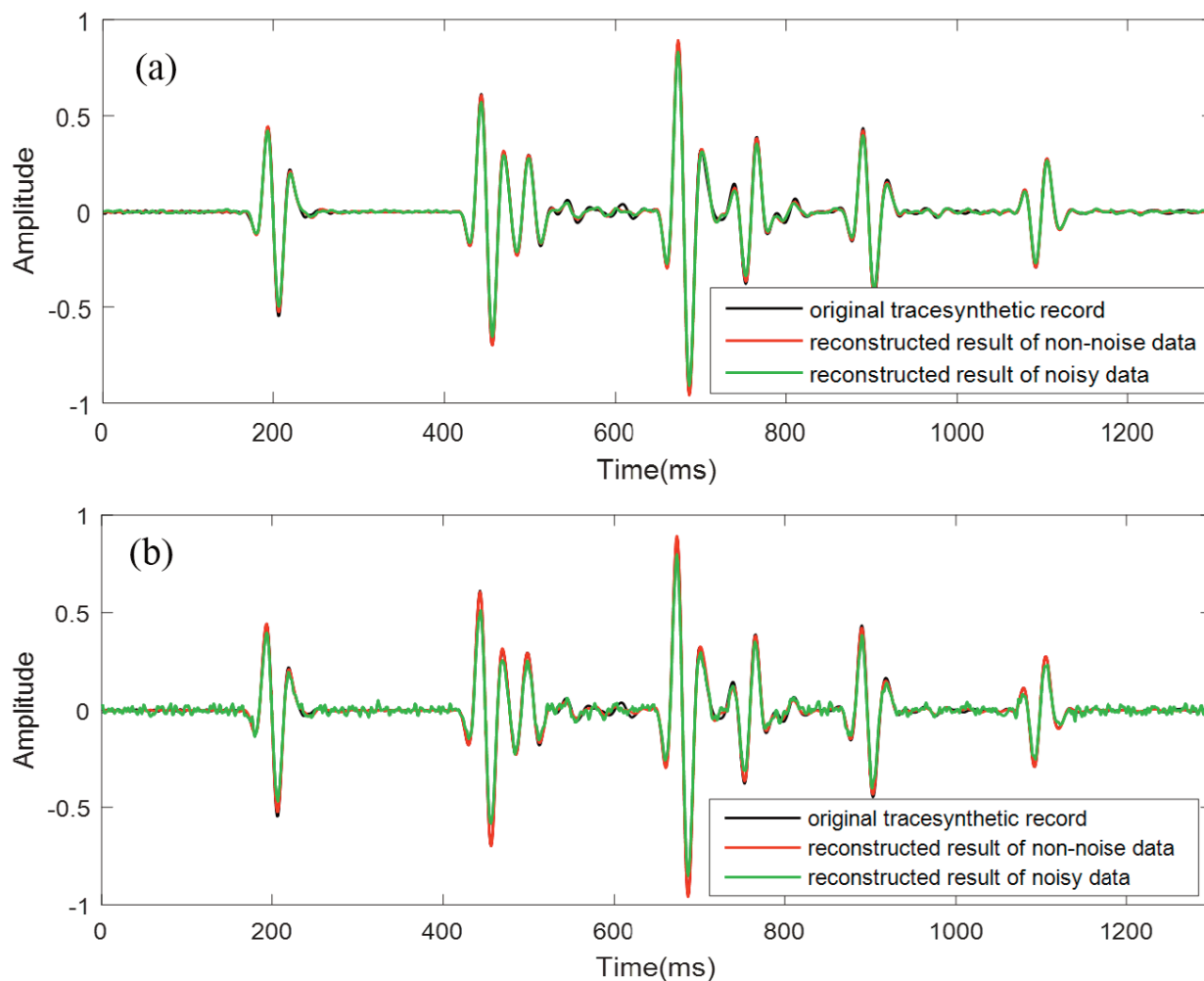


FIGURE 2. a) Noisy synthetic record with several traces decimated; b) Reconstructed record of a) by MCA method.

of the noisy record with SNR=4. We can see that this method can reconstruct the missing traces using the noisy data, and can remove the random noise in the record. But if the record is added too much noise the small details of the record may not be reconstructed well.

Then we select the 101st trace from both the reconstructed data respectively, and compare them with the original trace (Figure 3). The black curve is the original trace, the red curve is the reconstructed result using the non-noise incomplete data, and the green



**FIGURE 3.** Reconstructed results comparison. a) Reconstructed data of the noisy record with SNR=10; b) Reconstructed data of the noisy record with SNR=4; synthetic seismic record (black curve), reconstructed result of non-noise data (red curve), reconstructed result of noisy data (green curve).

curve is the reconstructed result using the noisy incomplete data. The green curve in Figure 3a is the reconstructed data of the noisy record with SNR=10, and the green curve in Figure 3b is the reconstructed data of the noisy record with SNR=4. The Figure shows that the reconstructed results, even the data is noisy, are well matched with the original trace.

#### 4.2 FWI WITH IRREGULAR SHOT GATHER

In practical seismic exploration, the survey system is often not in accordance with the regular grid and the receiver are not on the regular grid points. Some receivers of the same survey line may be get far from the designed location. Moreover, due to the obstacles or instrument factors, there will be missing or bad traces. If we still process the data simply as regular grid, large error will be produced. When the modeling grid of full wave form inversion does not match the

actual acquisition survey, we need to deal with seismic data by means of data regularization method, and obtain high-quality seismic data that suitable for full waveform inversion. We use the SEG/EAGE overthrust velocity model (Figure 4a) to test the result of full waveform inversion where the observed record data is not regular. Figure 4b is the initial velocity model for FWI which is the Gaussian smoothing of the true velocity model. First we generate the forward modeling record as observed data and set some traces to zero. Figure 5a is the forward modeling record of one shot, and Figure 5b is the irregular forward modeling record. Next we use these two kinds of record as the input observed data and carry out full waveform inversion. Figure 6a and Figure 6b show the inversion results using regular record and irregular record respectively. To see the impact of irregular data, we select one trace of the inversion results at the location of

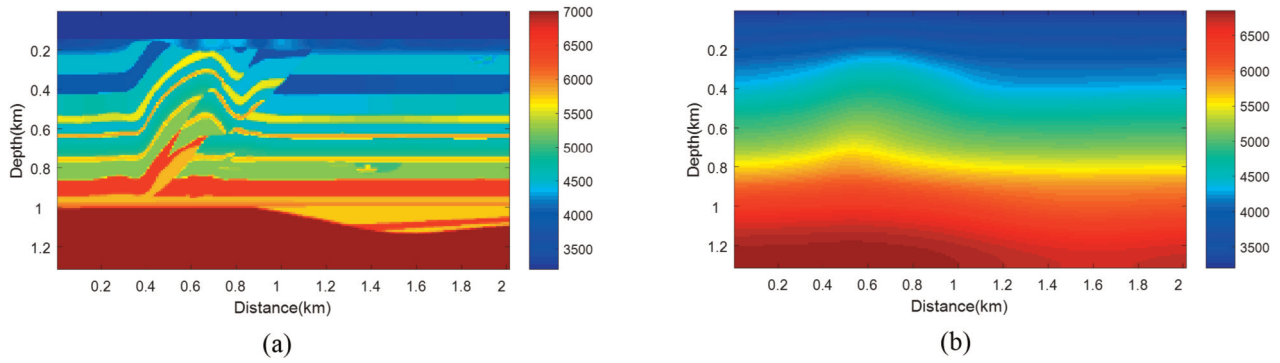


FIGURE 4. SEG/EAGE overthrust model. a) The true velocity model; b) The initial model.

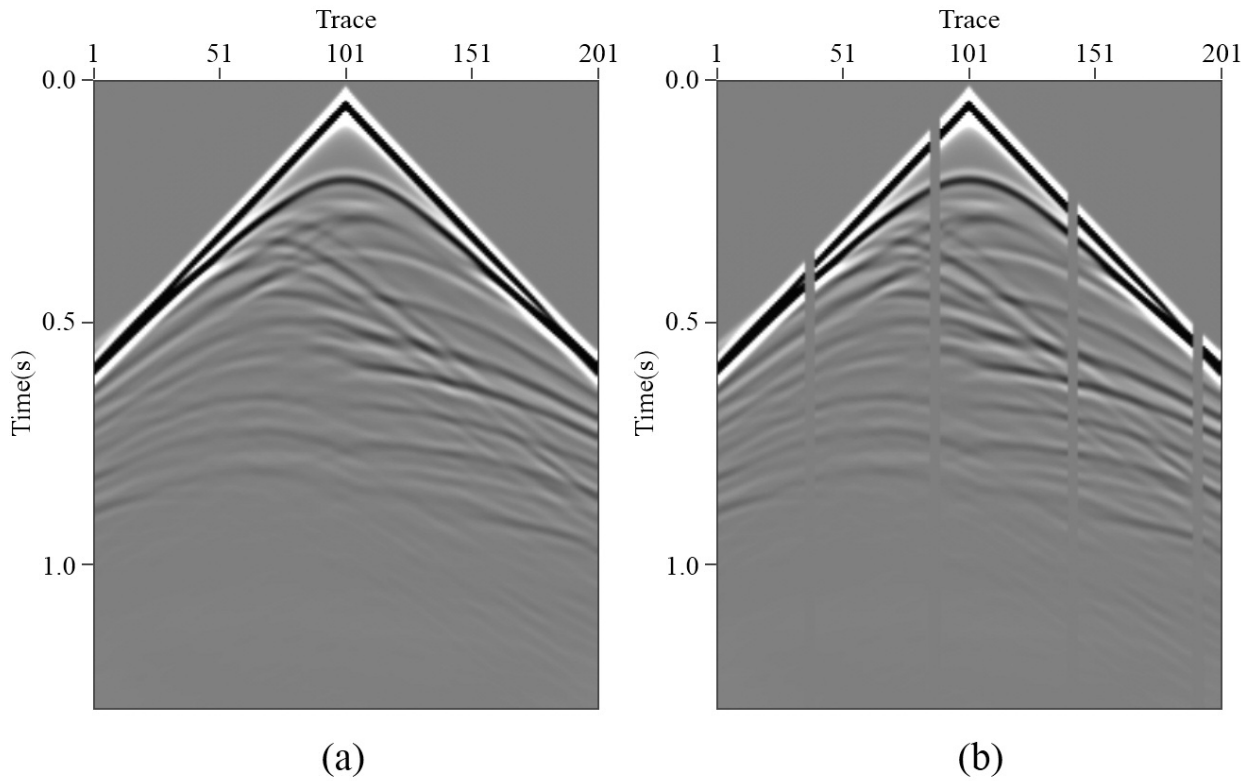


FIGURE 5. a) The regular forward modeling record; b) The irregular forward modeling record.

0.6km, 1.15km and 1.5km respectively as shown in Figure 7. The black curve is the true velocity model, the red curve is the initial model, the blue curve is the inversion result using the regular record and the green curve is the inversion result using the irregular record. We can see that the inversion result using regular record as input is closer to the true model than using irregular record. Figure 8 shows the evolution of the objective function of these two cases. We can see that the objective function value of FWI using regular record decreases faster than the objective function value of FWI using irregular record. The objective function value of FWI using regular record is smaller

than the objective function value of FWI using irregular record in the same iteration. This also shows that the inversion result using regular record is closer to the true model than the inversion result using irregular record.

### 5. FIELD DATA EXAMPLES

We apply this method to a land seismic data set. Figure 9a shows one of the shots record. We can see some traces are missing. First, we use MCA method to get the complete data (Figure 9b). We use these two kinds of data (incomplete data and complete

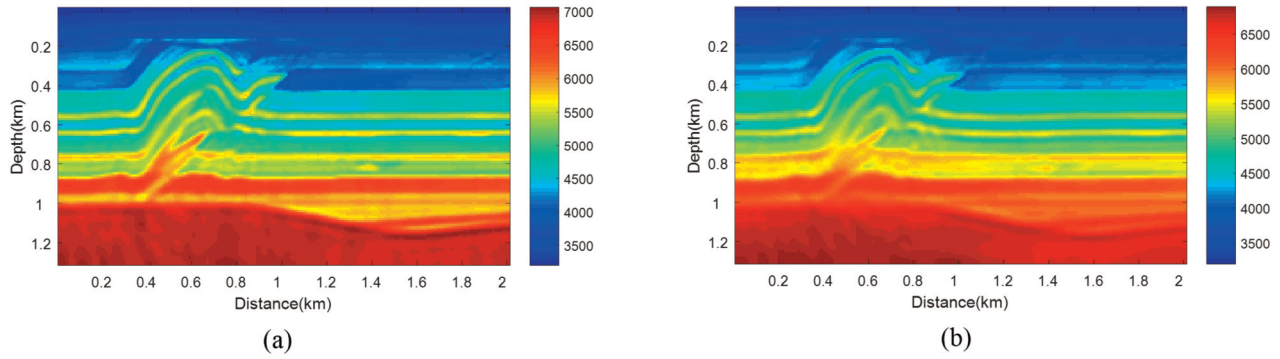


FIGURE 6. a) The inversion result using the regular record; b) The inversion result using the irregular record.

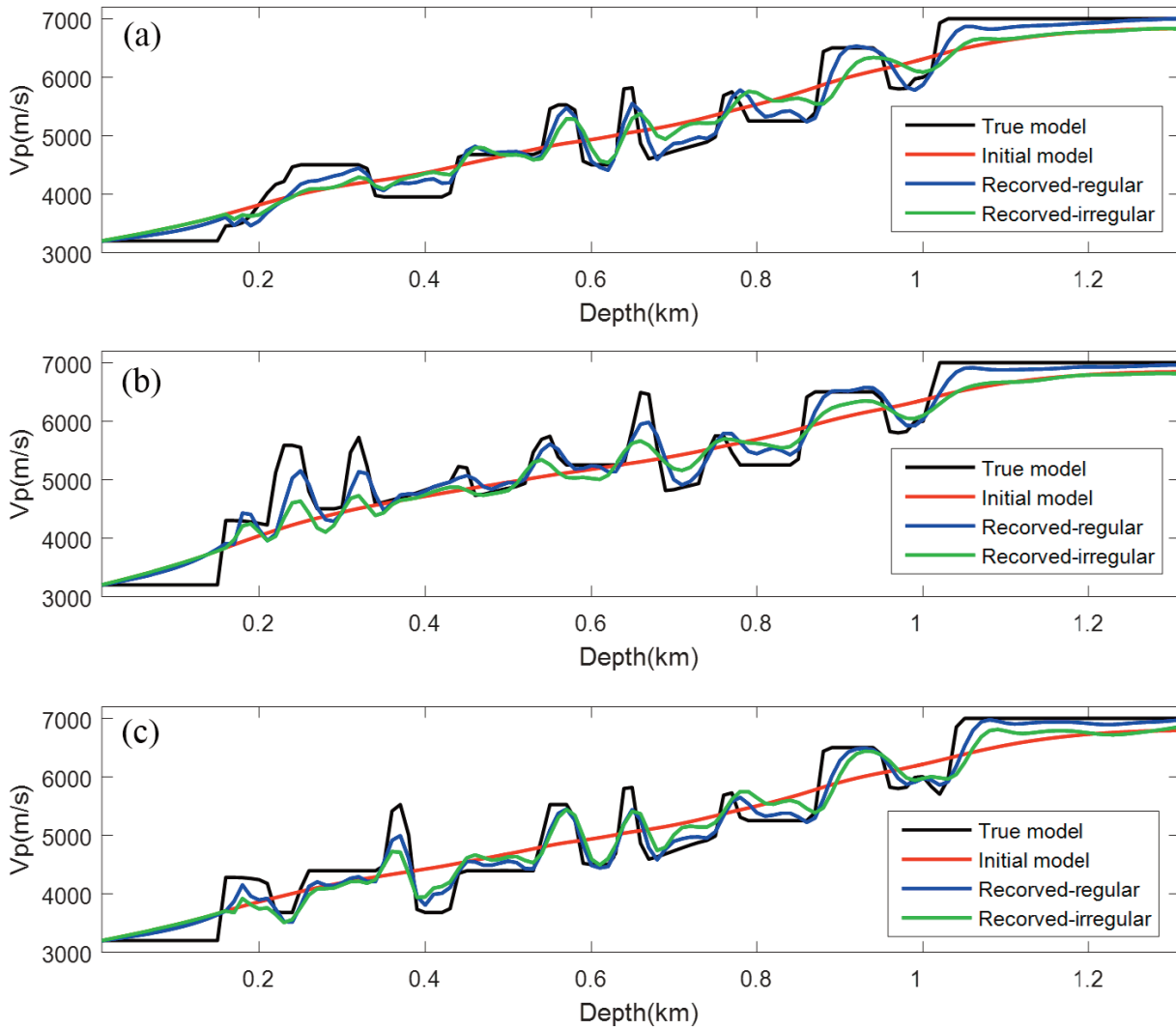


FIGURE 7. Inversion result comparison. (a) Trace at  $x=0.6\text{km}$ ; (b) Trace at  $x=1.15\text{km}$ ; (c) Trace at  $x=1.5\text{km}$ .

data) respectively as the input of FWI, and obtain the inversion results (Figure 10). We can see that the inversion result using reconstructed data is better, as shown in the ellipse location, the resolution of the inverted result using the reconstructed data is higher than the inverted result using the irregular data.

## 6. CONCLUSIONS

In this paper, we studied the influence of seismic irregularity on full wave form inversion. In order to get regular seismic data, we apply the morphological component analysis (MCA) method to the reconstruc-



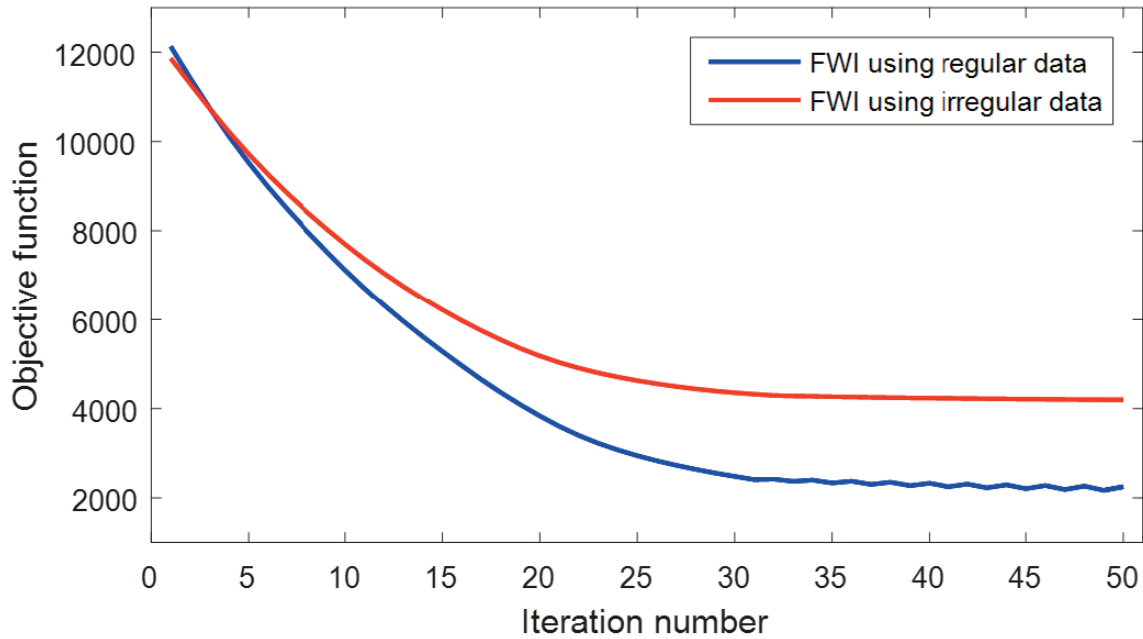


FIGURE 8. Evolution of objective function.

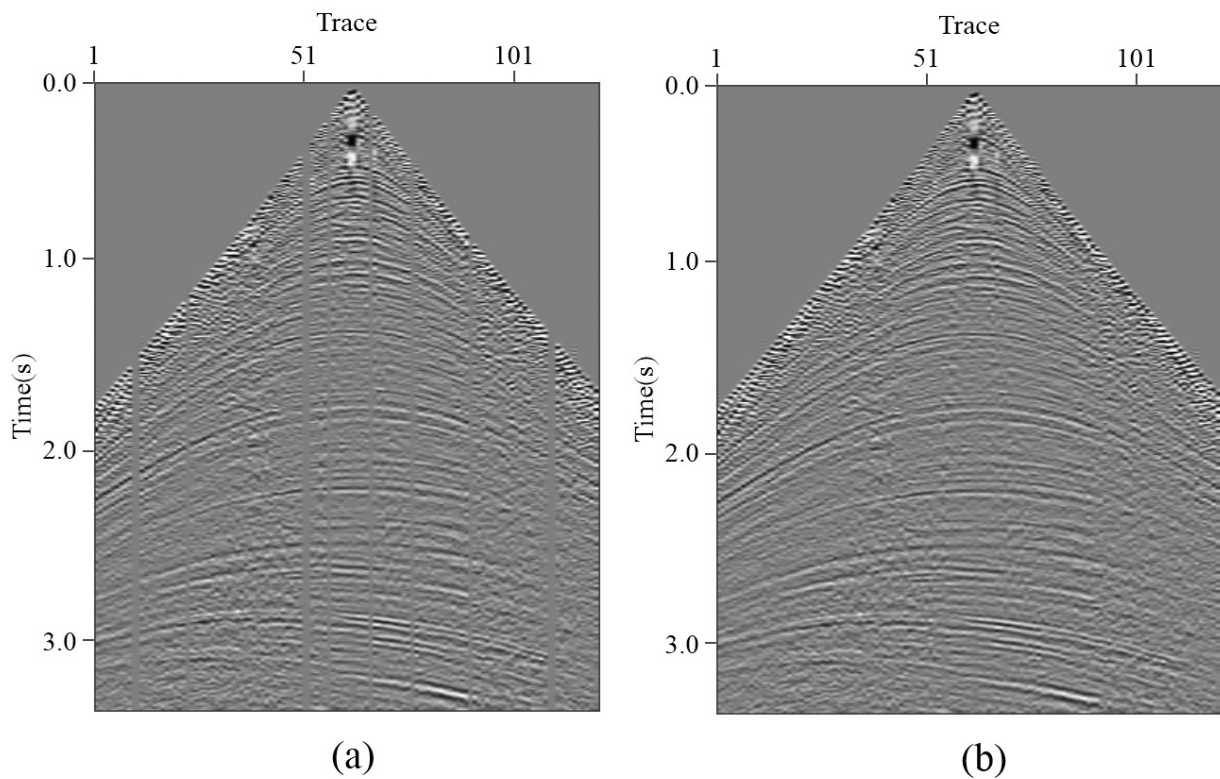


FIGURE 9. a) The original field seismic record; b) The reconstructed seismic record.

tion of irregular seismic data. The amplitude reconstructed traces are consistent with the neighboring traces. We cannot obtain accurate velocity model using the irregular seismic for FWI. The numerical example and field data application proved that the MCA method can be effectively used for seismic data reconstruction, and this method has the effect of

denoising. Full waveform inversion based on MCA seismic data reconstruction can obtain higher precision subsurface velocity than using irregular seismic data for FWI directly. Application of this method to the field seismic data, shows that the quality of seismic data is improved, and can obtain provide better inversion results.

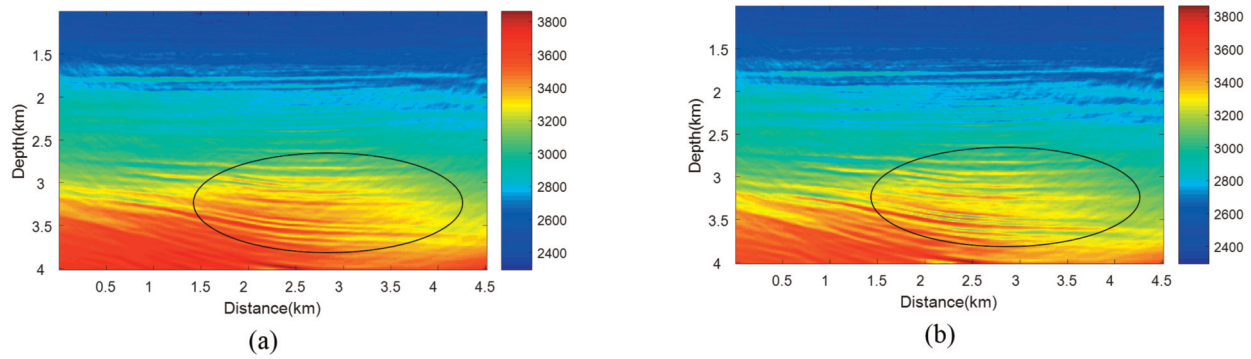


FIGURE 10. FWI results comparison. a) FWI result using irregular field seismic record; b) FWI result using reconstructed seismic record.

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