The characterization of diffusion tensor for mid-latitude ionospheric plasma

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ABSTRACT

In this study, the relationship between diffusion and the electrical conductivity tensor is investigated by using the real geometry of the Earth's magnetic field in the Northern Hemisphere for mid-latitudes. This relationship is derived from equation. Only in the elements of the diagonal of the tensor is an equal relationship found between the electrical conductivity called the longitudinal conductivity and the diffusion coefficient. The diagonal elements are equal to each other and do not depend on the Earth's magnetic field; however, the other elements of the tensor strongly depend on the Earth's magnetic field and they have characteristic of Bohm diffusion became a semi-empirical model ($D_B = (1/16)kT/eB$) but they do not depend on the electrical conductivity or the classical diffusion coefficient.

1. Introduction

The ionosphere is the layer of the Earth's atmosphere that extends from 50 to 1000 km. The ionosphere significantly affects the propagation of radio waves so the variability of the ionosphere is important for ionospheric physics and radio communications. The ionosphere can be characterized by its electron density distribution, which is a complex function of spatial and temporal variations and geomagnetic and solar activity [Rishbeth 1965, Rishbeth and Garriott 1969, Whitten and Poppoff 1971, Banks and Kockarts 1973]. Usually, the ionosphere with respect to its electron density distribution is separated into five independent regions, sometimes called layers. The bottom one, from 50 to 80 km, is called the D layer, from 80 to 130 km is the E layer, and above 150 km is the F layer. The F region is usually separated at the F1 layer, from 130 to 200-250 km, and the F2 layer is above 250 km [Rishbeth and Garriott 1969, Whitten and Poppoff 1971, Banks and Kockarts 1973, Rishbeth 1977]. The ionosphere is a natural plasma. Many plasma phenomena can be analyzed using macroscopic transport equations, either by considering the plasma as a multi-constituent fluid or by treating the plasma as a single conducting fluid [Rishbeth 1975, St.-Maurice and Schunk 1977]. In some cases, however, a satisfactory description can be obtained only through the use of kinetic theory. The presence of a pressure gradient term in the momentum transport equation provides a force that tends to smooth out any inhomogeneities in the plasma density [Zhilinskiĭ and Tsendin 1980]. Ion electrons diffuse under the influence of partial pressure gradients and gravity. The diffusion of the particles in a plasma results from this pressure gradient force. To deduce the expression for the electron diffusion coefficient for a warm weakly ionized element, the momentum transport equations for the electrons are used with a constant electron-neutral collision frequency in the ionospheric plasma [Banks and Kockarts 1973, St.-Maurice and Schunk 1977, Abbas et al. 2011]. The electrons have far lower masses than ions, and they have far higher typical speeds at fixed temperature and are much more easily accelerated (i.e., they are much more mobile). As a result, it is the motion of the electrons, not the ions, that is responsible for the transport of heat and charge through plasma. Transport properties can now be computed, including the electric conductivity and thermal conductivity, on the presumption that it is Coulomb collisions that determine the electron mean free paths and that magnetic fields are unimportant in providing a more serious impediment to charge and heat flow than Coulomb collisions and thus dominate the conductivities [Rishbeth and Garriott 1969, Whitten and Poppoff 1971, Banks and Kockarts 1973].

This motion is hindered by collisions with the neutral air. The equation for the diffusion of plasma is obtained by adding the ion and electron force equations. The gravity term for electrons is omitted as it is quite negligible [Banks and Kockarts 1973]. The diffusion coefficient, called diffusivity, is an important parameter indicative of the diffusion mobility. The diffusion coefficient is not only encountered in Fick's law, but is found in numerous other equations of physics and chemistry. The diffusion coefficient is generally prescribed for a given pair of species. For a multi-component system, it is prescribed for each species in the ionospheric plasma. The higher the diffusivity, the faster the diffusion. The diffusion coefficient is proportionally constant between the diffusion flux and gradient in the concentration of the diffusing species, and it depends on both temperature and pressure [Rishbeth and Garriott 1969, Whitten and Poppoff 1971, Banks and Kockarts 1973].

2. The relationship between diffusion and electrical conductivity tensor

The behavior of the ionospheric plasma and the neutral thermosphere is subject to the equation of state (perfect gas law) and the general conservation equations for mass, momentum and energy [Rishbeth 1965, Bittencourt 1995]:

a) Continuity equation:
Density change = Production–Loss–Transport
b) Equation of motion:
Acceleration = Force–Drag–Transport
c)Temperature change = Heating–Cooling–Conduction.

If it does not have an impact outside, the transport of particles in the ionosphere plasma from place to place results from the pressure-gradient (∇P). This force occurs in any part of the plasma density to eliminate inhomogeneity. If $\mathbf{B}\neq 0$, the medium is called the anisotropic [Banks and Kockarts 1973]. Due to this, the ionospheric plasma is anisotropic. The real geometry of the Earth's magnetic field for the Northern Hemisphere is shown in Figure 1, where $B_x = B\mathrm{Cos}I\mathrm{Sin}d$,

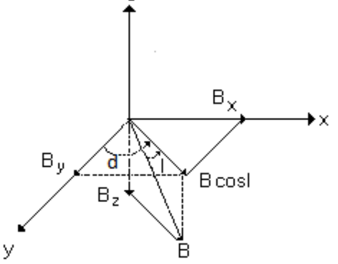

Figure 1. The geometry of the Earth's Magnetic Field (for the Northern Hemisphere).

$B_y = B\mathrm{Cos}I\mathrm{Cos}d$, and $B_z = -B\mathrm{Sin}I$. I is the magnetic dip angle and d is the magnetic declination angle. If **U** and m are the velocity and mass of the electron, respectively, then the force acting on the electron is as follows:

$$m\frac{d\mathbf{U}}{dt} = -e(\mathbf{U}\times\mathbf{B}+\mathbf{E}) - m\nu\mathbf{U} \quad (1)$$

where $\nu = \nu_{ei} + \nu_{en}$

and,

$$\nu_{ei} = N\left[59 + 4.18\log\left[\frac{T_e^3}{N}\right]\right]\times 10^{-6}T_e^{-3/2}[\mathrm{m.k.s}] \text{ and}$$
$$\nu_{en} = 5.4\times 10^{-16}N_n T_e^{1/2}[\mathrm{m.k.s}]$$

are the electron-ion and electron-neutral collision frequencies, respectively. It is assumed that the velocity and fields vary as $e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$, where ω is angular wave frequency, and ω_c is electron angular gyro frequency.

Its vector components are:

$$\omega_{cx} = \frac{-eB_x}{m},\ \omega_{cy} = \frac{-eB_y}{m} \text{ and } \omega_{cz} = \frac{-eB_z}{m}.$$

The current density can be obtained as follows:

$$\mathbf{J} = \sigma_0\mathbf{E} - \frac{e}{m(\nu - i\omega)}\mathbf{J}\times\mathbf{B} \quad (2)$$

in which

$$\sigma_0 = \frac{Ne^2}{m(\nu - i\omega)}$$

is the longitudinal conductivity. We assumed that the z-axis of the coordinate system with its origin located on the ground is vertical upwards. The x-and y-axes are geographic eastward and northward in the Northern Hemisphere [Aydoğdu et al. 2004]. When the real geometry of the Earth's magnetic field is used for the steady-state ($\partial/\partial t=0$, that is, $\omega=0$), the conductivity tensor of the solution of Equation (2) is obtained as follows:

$$\mathbf{J} = \sigma\mathbf{E} \quad (3)$$

with

$$\sigma = \frac{\sigma_0}{\nu^2+\omega_c^2}\begin{bmatrix} \nu^2+\omega_{cx}^2 & \omega_{cx}\omega_{cy}+\omega_{cz}\nu & \omega_{cx}\omega_{cz}-\omega_{cy}\nu \\ \omega_{cx}\omega_{cy}-\omega_{cz}\nu & \nu^2+\omega_{cy}^2 & \omega_{cy}\omega_{cz}+\omega_{cx}\nu \\ \omega_{cx}\omega_{cz}+\omega_{cy}\nu & \omega_{cy}\omega_{cz}-\omega_{cx}\nu & \nu^2+\omega_{cz}^2 \end{bmatrix} \quad (4)$$

All of the tensor elements of Equation (4) are real if Equation (1) is used for the diffusion coefficient, depending on the real geometry of the Earth:

$$\mathbf{\Gamma} = \mu(\mathbf{\Gamma}\times\mathbf{B} + n\mathbf{E}) - D\nabla n \quad (5)$$

where,

$$\mu = \frac{-e}{m\nu},\ D = \frac{k_b T}{m\nu} \text{ and } \mathbf{\Gamma} = (n\mathbf{U})$$

are respectively: the electron mobility, the electron diffusion coefficient and the flux of density. The flux of density in terms of the current density is as follows:

$$\mathbf{\Gamma} = \mathrm{n}\mathbf{U} \quad \text{and} \quad \mathbf{J} = -\mathrm{en}\mathbf{U} = \sigma\mathbf{E} \tag{6}$$

From here, the electric field vector that depends on the flux of density could be written as follows:

$$\mathbf{E} = \frac{-\mathrm{e}}{\sigma}\mathbf{\Gamma} \tag{7}$$

If this Equation (7) is used Equation (5), Equation (8) is obtained as follows:

$$\mathbf{D}\nabla\mathbf{n} = \left(\frac{-\mathrm{e}\mu\mathbf{n}}{\sigma} - \mathrm{I}\right)\mathbf{\Gamma} + \mu(\mathbf{\Gamma}\times\mathbf{B}) \tag{8}$$

where I is the unit matrix and σ is the conductivity tensor. From here, the diffusion tensor depends on both the classical diffusion coefficient [Abbas et al. 2011] and the electrical conductivity:

$$\begin{bmatrix}(\nabla \mathrm{n})_x \\ (\nabla \mathrm{n})_y \\ (\nabla \mathrm{n})_z\end{bmatrix} = \tag{9}$$

$$\begin{bmatrix} \frac{1-\sigma_0}{\mathrm{D}} & \frac{1}{\mathrm{D_{Bz}}} + \frac{\omega_{cz}}{\nu} & -\left(\frac{1}{\mathrm{D_{By}}} + \frac{\omega_{cy}}{\nu}\right) \\ \frac{1}{\mathrm{D_{Bz}}} + \frac{\omega_{cz}}{\nu} & \frac{1-\sigma_0}{\mathrm{D}} & \frac{1}{\mathrm{D_{Bx}}} + \frac{\omega_{cx}}{\nu} \\ \frac{1}{\mathrm{D_{By}}} + \frac{\omega_{cy}}{\nu} & -\left(\frac{1}{\mathrm{D_{Bx}}} + \frac{\omega_{cx}}{\nu}\right) & \frac{1-\sigma_0}{\mathrm{D}} \end{bmatrix} \cdot \begin{bmatrix}\mathbf{\Gamma}_x \\ \mathbf{\Gamma}_y \\ \mathbf{\Gamma}_z\end{bmatrix}$$

where,

$$\mathrm{D_{Bz}} = \frac{\mathrm{k_b T}}{\mathrm{eB_z}}, \; \mathrm{D_{By}} = \frac{\mathrm{k_b T}}{\mathrm{eB_y}} \text{ and } \mathrm{D_{Bx}} = \frac{\mathrm{k_b T}}{\mathrm{eB_x}}.$$

With respect to Equation (9), the diffusion tensor is given by:

$$\mathrm{D_B} = \begin{bmatrix} \frac{1-\sigma_0}{\mathrm{D}} & \frac{1}{\mathrm{D_{Bz}}} + \frac{\omega_{cz}}{\nu} & -\left(\frac{1}{\mathrm{D_{By}}} + \frac{\omega_{cy}}{\nu}\right) \\ \frac{1}{\mathrm{D_{Bz}}} + \frac{\omega_{cz}}{\nu} & \frac{1-\sigma_0}{\mathrm{D}} & \frac{1}{\mathrm{D_{Bx}}} + \frac{\omega_{cx}}{\nu} \\ \frac{1}{\mathrm{D_{By}}} + \frac{\omega_{cy}}{\nu} & -\left(\frac{1}{\mathrm{D_{Bx}}} + \frac{\omega_{cx}}{\nu}\right) & \frac{1-\sigma_0}{\mathrm{D}} \end{bmatrix} \tag{10}$$

The diagonal elements of tensor (Equation 10) are equal to each other and they only depend on the classical diffusion coefficient (D) and the electrical conductivity (σ_0). They do not depend on the Earth's magnetic field. However, the tensor is symmetric and they have characteristic of Bohm diffusion became a semi-empirical model (D_B= (1/16)kT/eB). This diffusion coefficient does not depend on the density but it depends on magnetic field (that is, $D_{Bxy} = -D_{Byx}$, $D_{Bxz} = -D_{Bzx}$, and $D_{Byz} = -D_{Bzy}$). These results are interesting because the tensor (Equation 10) is a mixture of the electrical conductivity, the classical diffusion coefficient, and the diffusion depending on the magnetic field.

3. Numerical analysis and results

The general system of transport equations is applied to the mid-latitude topside ionosphere. The restriction to this region of the ionosphere enables us to make several simplifying assumptions that significantly reduce the general system of transport equations. First is fully ionized plasma composed of two major ions, electrons, and a number of minors. Next, the steady state and the species temperature and flow velocity differences are small in the mid-latitude topside ionosphere. This enables us to neglect stress and nonlinear acceleration terms [Rishbeth 1975]. We wondered what the relationship between the electron diffusion and conductivity became in the ionospheric plasma by using the real geometry of the Earth's magnetic field. This relationship is given by Equation (10). According to Equation (10), only in the elements of the diagonal of the tensor is the relationship found between the electrical conductivity called the longitudinal conductivity and the diffusion coefficient; the diagonal elements are equal and thus do not depend on the Earth's magnetic field. However, the other elements of the tensor strongly depend on the Earth's magnetic field, but they do not depend on the electrical conductivity or the classical diffusion.

In this study, the relationship between the diffusion tensor and the electrical conductivity is investigated by using the real geometry of the Earth's magnetic field in the Northern Hemisphere for mid-latitudes ionosphere plasma. The results obtained: I (dip angle)= 55.6°, d (Declination) = 3°, R=10, by using Equations (4)-(10). The ionospheric parameters used for calculations are obtained using the IRI (International Reference Ionosphere) model. For a given location, time, and date, the IRI model describes the electron density, electron temperature, ion temperature, and ion composition in the altitude range from about 50 to about 2000 km; it also describes the electron content. The modelprovides monthly averages in the non-auroral ionosphere for magnetically quiet conditions. The diagonal elements of Equation (10) are equal and only depend on the classic diffusion coefficient and longitudinal conductivity (σ_0). The seasonal diagram of the diagonal element for mid-latitude in ionospheric plasma is given in Figure 1 for local time (12.00-24.00). According to Figure 1, the diagonal element that depends on both the classical diffusion coefficient and electric conductivity decreases with altitude in mid-latitudes ionospheric plasma at 12.00 LT (Local Time) and for every season. It takes the minimum value and is negative at around

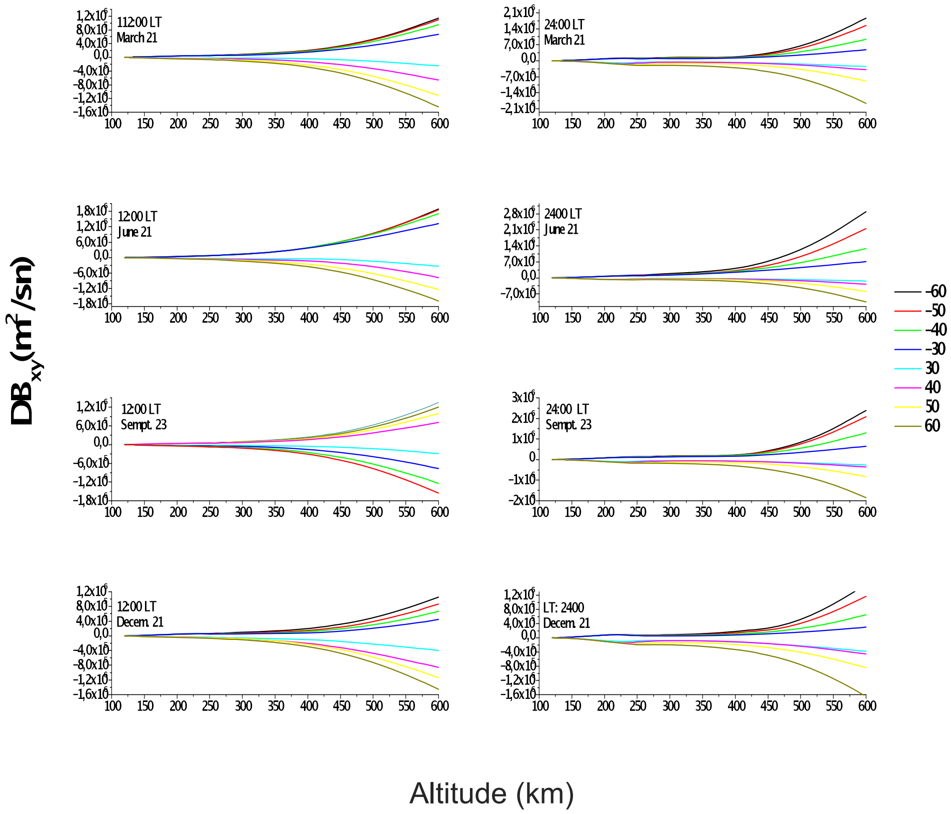

Figure 2. The seasonal diagram of diagonal elements in Equation (10) for mid-latitude in ionospheric plasma.

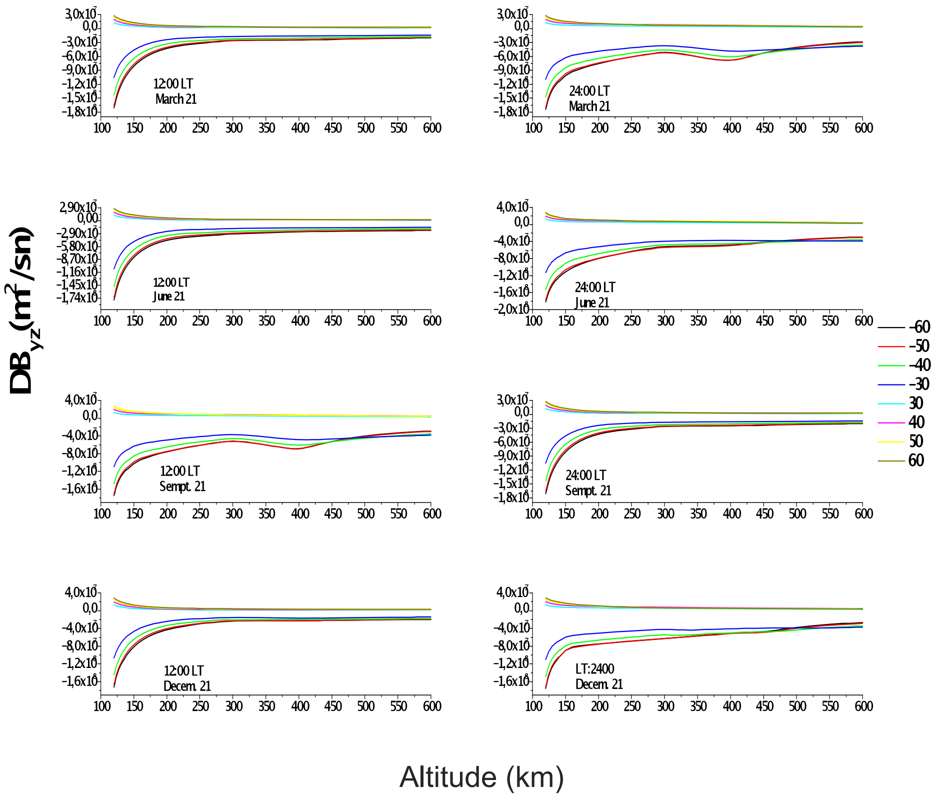

Figure 3. The seasonal diagram of DB_{xy} (m^2/sn) in Equation (10) for mid-latitude in ionospheric plasma.

225-400 km for March 21 and September 23 in general. However, for the same seasons, it suddenly decreases to north latitude (30°N) and is approximately -1.2×10^{-6} for March 21 and -7.0×10^{-6} at 300 km. For June 21 and December 21, it takes two times minimum values for south latitudes with an altitude (200-250 km; 275-400 km) except for the north latitudes. In addition, the diagonal elements of Equation (10) increased about 300 km after

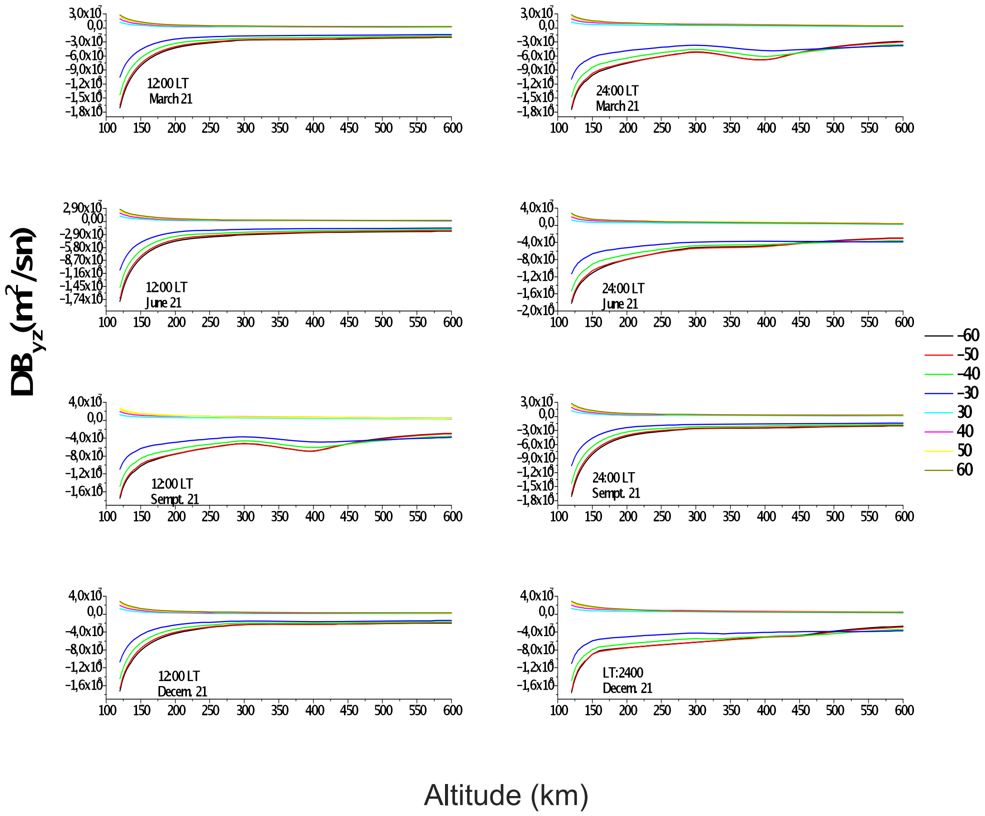

Figure 4. The seasonal diagram of $DB_{yz}(m^2/sn)$ in Equation (10) for mid-latitude in ionospheric plasma.

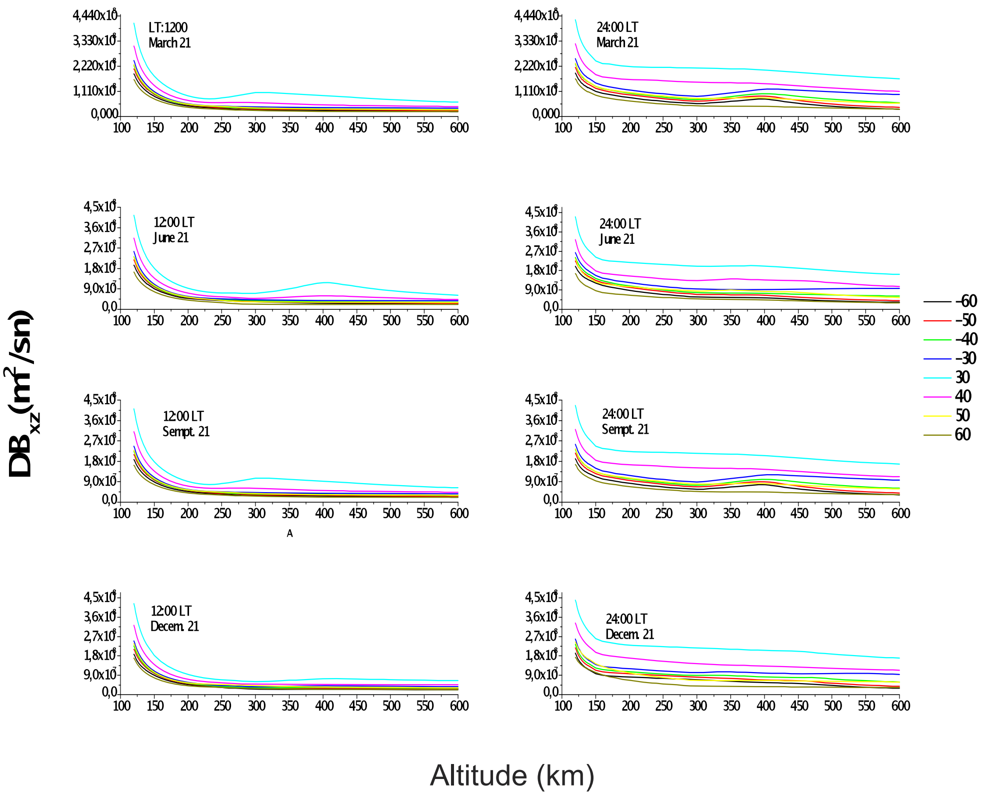

Figure 5. The seasonal diagram of $DB_{xz}(m^2/sn)$ in Equation (10) for mid-latitude in ionospheric plasma.

every season and latitude, but are negative. For 24.00 LT, the trend in change of altitude of the diagonal elements of Equation (10) is similar to the change of in 12.00 LT for every season. It is also the minimum altitude (about 300 km) but its magnitude has decreased with respect to 12.00 LT. The seasonal diagram of $DB_{xy}(m^2/sn)$ is given for 12.00-14.00 LT in Figure 2. $DB_{xy}(m^2/sn)$ is positive both for 12.00 LT and 24.00 LT in mid-latitude ionospheric plasma for the Northern Hemisphere and all seasons, but is negative for the Southern Hemisphere. The magnitude of $DB_{xy}(m^2/sn)$ is bigger at 12.00 LT than 24.00 LT and seasonal. The change in altitude of $DB_{yz}(m^2/sn)$ is the same as both the local time and the magnitude in Figure 2; however, the decrease in south latitudes and increase in north latitudes is exponential. The change in altitude of $DB_{xz}(m^2/sn)$ is shown in Figure 4. $DB_{xz}(m^2/sn)$ decreases with altitude for both seasonal and mid-latitudes (12.00-24.00LT) but the most noticeable results occur at 30° south latitude. $DB_{xz}(m^2/sn)$ quickly decreases with altitude for both season and latitude. $DB_{xz}(m^2/sn)$ is approximately maximum between 300 and 450 km depending on the season 30° south latitude. The relationship between the magnitudes is $DB_{xz}(m^2/sn) > DB_{yz}(m^2/sn) > DB_{xy}(m^2/sn)$.

4. Conclusion

This article has reviewed the diffusion tensor for electrons in the mid-latitude ionospheric plasma and investigated whether any relationship between the diffusion and electrical conductivity exists. The findings indicate that the diffusion coefficient depends only on 0 (longitudinal conductivity) in Equation (10). However, the other conductivities, such as Pedersen and Hall conductivity, do not affect the diffusion in ionosopheric plasma. The diagonal elements of tensor (Equation 10) are equal to each other and they only depend on the classical diffusion coefficient (D) and the electrical conductivity (σ_0). They do not depend on the Earth's magnetic field. However, the tensor is symmetric and they have the characteristic of Bohm diffusion became a semi-empirical model ($D_B=(1/16)kT/eB$). This diffusion coefficient does not depend on the density but it depends on magnetic field. (That is, $D_{Bxy}=-D_{Byx}$,$D_{Bxz}=-D_{Bzx}$, and $D_{Byz}=-D_{Bzy}$). These results are interesting because the tensor (Equation 10) is a mixture of the electrical conductivity, the classical diffusion coefficient, and the diffusion depending on the magnetic field.

Based on our results, the diagonal elements in Equation (10) take the minimum value around 200-400 km as both seasonal and latitudes are negative in general. This altitude (200-400 km) is the peak height of the region and has generally the maximum electron density in this region for every season. The tensorial elements of the diffusion expected for the diagonal element in Equation (10) do not depend on electrical conductivity, but the tensorial elements relate to the electron temperature and collision frequency due to a magnetic field constant in mid-latitudes ionospheric plasma. The diffusion coefficients with respect to all diagrams change to linear with electron temperatures for mid-latitudes ionospheric plasma.

Our procedure may be useful in assessing ionospheric variables such as conductivity and it can be used for ionospheric plasma density measurements. This study could become important in mid-latitude stations for ionosonde measurements because the electron concentration of the medium determines the behavior of the medium against any external influence.

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