On the Extensometer Whose Magnifier is a Zöllner Suspension Type Tiltmeter, and the Observation of the Earth's Strains by Means of the Instruments

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SUMMARY. — In order to observe the ground-strain due to the tide generating force of the earth, it is necessary to record measurable the linear strains, which are as large as 10^{-10}. For that purpose, it should not only make longer the base line on the ground, but also magnify the relative displacement of the both edge-points of the base line extremely. A new type of highly sensitive extensometer of which magnifier is a Zöllner suspension type tiltmeter is devised. In the mechanism of the extensometer, a super-invar rod whose the length is constant, is put parallel with the base line, and an edge of the rod is fixed on one edge of the line so that the relative displacement of the unfixed edge of the rod and the other edge of the line may be transformed into the inclination of the support of the Zöllner pendulum. Consequently, the pendulum of the tiltmeter rotates magniloquently. And also the rotational angle of the pendulum is enlarged and recorded with an optical lever on a photographic printing paper. The extensometer has a sensitivity in the order that is higher than the formerly used one, and the sensitivity is able to be perfectly controlled at will. As the sensitivity to the linear strain is more than 100 times as large as that of the extensometer as to the ground-tilting, the influence of the tilting may be neglected.

The linear strains of the earth's surface in earth tide are observed with this extensometer at Osakayama Observatory, and are obtained the analyzed values that are subtracted the influence of the ground-tilting, in four directions N-S, E-W, S 38° W and S 52° E. Then, the earth-tidal constants $h - 3l$, h and l are obtained as 0.440, 0.599 and 0.053 respectively. And also the records (of the ground-strain) which are observed with new type extensometer are compared with them which are observed with other types'. In these records, the interesting phenomena which are generated within a short hour, are found.

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RIASSUNTO. — Allo scopo di osservare la deformazione della crosta della terra dovuta alla forza generatrice delle maree terrestri è necessario registrare le sollecitazioni lineari misurabili, dell'ordine di 10^{-10}. A tale scopo, non soltanto si deve allungare di più la linea di base sul terreno, ma anche aumentare grandemente il relativo spostamento di ambedue i punti estremi del piano di base. È stato ideato un nuovo tipo di estensimetro ad alta sensibilità il cui amplificatore è un clinometro Zöllner di tipo a sospensione. Nel meccanismo dell'estensimetro un'asta di superinvar, la cui lunghezza è costante, è posta parallelamente al piano di base; un estremo dell'asta è fissato in modo che lo spostamento relativo dell'estremo non fisso dell'asta rispetto all'estremo del piano di base, possa essere trasformato in una inclinazione del supporto del pendolo Zöllner. Di conseguenza, il pendolo del clinometro ruota in modo accentuato; ne risulta ingrandito anche l'angolo di rotazione del pendolo che viene registrato con sistema ottico su carta fotografica. L'estensimetro ha una sensibilità maggiore di quello usato precedentemente, e la sensibilità può essere controllata a volontà. Siccome la sensibilità alla sollecitazione lineare è 100 volte maggiore di quella dell'estensimetro rispetto alla deformazione del terreno, l'influenza di quest'ultima può essere trascurata.

Le sollecitazioni lineari della superficie della terra nella marea terrestre vengono osservate con questo estensimetro presso l'Osservatorio Osakayama: dai valori analizzati ottenuti si sottrae l'influenza della deformazione del terreno, nelle quattro direzioni N-S, E-W, S 39° W, S 52° E. Le costanti della marea terrestre $h - 3l$, h ed l sono 0.440, 0.599 e 0.053 rispettivamente. Inoltre le registrazioni della sollecitazione del terreno che sono osservate col nuovo tipo di estensimetro, sono confrontate con quelle osservate con altri tipi e mettono in evidenza interessanti fenomeni che avvengono perfino nel termine di un'ora.

1) INTRODUCTIONS.

In order to observe the strainnings with extensometers for the studies of foreknowledge of the outbreaks of the earthquakes (1) and one of earth's free oscillations (2), it is necessary to make use of the extensometers whose orders of their sensitivities are 10^{-10} per millimeter on their records. One of the method to make them highly sensitive is to make their spans longer. It is, however difficult to make the span of the extensometer longer unrestictedly, because it is restricted by the conditions of the observatory and the material of the instrument. Therefore we also should devise the method to amplify the relative displacement on both side of the base line of the extensometer. And the magnifier is necessary to be fit for continuous and long date's observations. The present author has devised the new type of the extensometer whose amplifier consists of Zöllner suspension type's horizontal pendulum. In this

method, the relative displacement of the both edge points is transformed into the inclination of the rotational axis of the horizontal pendulum and then the rotational angle is magnified by the optical lever on the records. This method is so excellent that we have nice observations of the crustal strains as the earth-tide, earth's free oscillation and crustal movements.

2) Instruments.

This new type's extensometer consists of three parts, the part of the standard scale, the one of the magnifier (enlarger) of the relative displacement and the one of the recording.

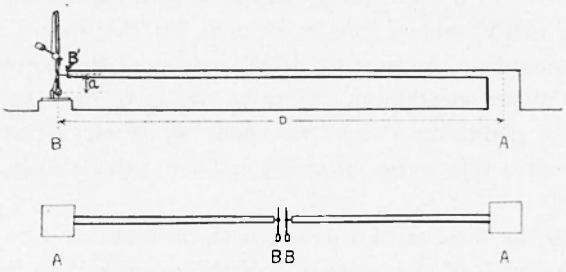

Fig. 1

The standard scale (P) consists of rods, about 1 cm. diameter or pipes of about 3 cm in diameter, which are made of super-invar. The one edge of the scale is fixed on the base point (A) to observe the relative displacements in the direction of the scale between the other edge of the scale and the other close base point on the ground. In some case, the fixed edge micro-adjuster, but this adjuster is included in the magnifier part in the improved type. The standard scale is set horizontally in the horizontal component and is set vertically in the vertical one. The magnifier part consists of the tiltmeter of Zöllner suspension type and the micro-adjustor of the interval between the unfixed edge of the scale and the joint of its tiltmeter on the base B (Fig. 1).

The support S of the tilmeter is about 20 cm in length and its below edge is supported with the pivot C or with two sheets of the plate springs on the board N. The horizontal pendulum H is suspended with stainless steel strings of 50μ in diameter from the upper edge of the support S and the other point J on the board N. The support S and the

unfixed edge B' of the scale is connected with the plate spring E. The distance between the fulcrum C and the connecting point of E is about 3 cm (2-5 cm). The relative displacement between the unfixed edge B' and B or C is transformed into the inclination of the support S then the horizontal pendulum is rotated magniloquently. The rotation of the pendulum is enlarged with the optical beam and the reflecting mirror fixed on the pendulum. The weight of the pendulum is about 15 grams and the length of its equivalent simple pendulum is about 5.0 cm. The period of the pendulum is adjusted with the horizontal movement of the suspension point J as in Fig. 2′ or the tilting of the board N as Fig. 2 and is able to prolong till about twenty seconds easily and to make the desired period. The pendulum has a weak damper of oil or magnet: (the greater part of them has an oil damper). The fundamental measurement of its sensitivity is performed by means of the measuring the relation between the magnitude of the movement on the record and the one of the adjustment of the interval (BB') by use of the adjuster G' at the fixed period of the pendulum. Usually, we have only to measure the periods of the pendulums, then we check up their sensitivities. The sensitivity is also able to be obtained by the calculations as following Fig. 2, Fig. 2′.

Let D be the interval AB of the both base point, a be the distance from the fulcrum C of the support S of the pendulum to the connecting point E of the support and the scale and e be the change of the meau of the linear strain along AB. Then the tilting angle ϕ of the support is given as

$$\phi = \frac{D}{a} e \qquad [1]$$

and the rotational angle ϑ of the horizontal pendulum is given as

$$\vartheta = \frac{T^2 g}{4 \pi^2 l'} \phi = \frac{T^2 g D e}{4 \pi^2 l' a}, \qquad [2]$$

where T is the period of the pendulum, l' is the length of the equivalent simple pendulum and g is the acceleration of the gravity.

Using the optical lever of L in length, the magnitude y of the change on the recording paper is given as

$$y = 2 L \vartheta = \frac{Lg T^2 De}{2 \pi^2 l' a}. \qquad [3]$$

Then the sensitivity S of the extensometer is expressed as

$$S = \frac{e}{y} = \frac{2\pi^2 l' a}{Lg D T^2} . \qquad [4]$$

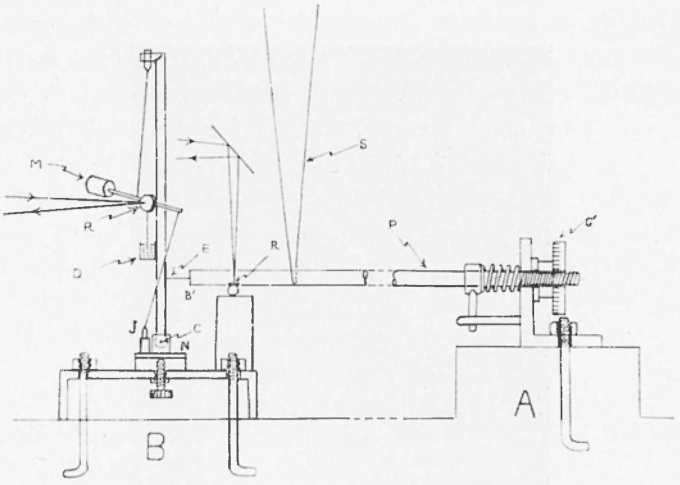

Fig. 2

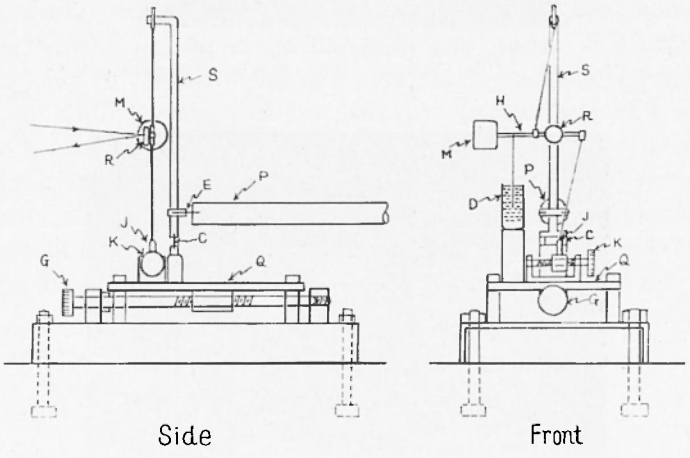

Fig. 2′ – The sketch of Amplifier Part of the Extensometer. C is the Support of the Pendulum, D is the Damper, E is the Connector, G is the Span-Adjuster, H is the Horizontal Pendulum, K is the Period-Adjuster, P is the Rod Scale, Q is the Sliding Plate, R is the Reflectiong Mirror and S is the Support of the Pendulum.

Calculating the sensitivity by use of actual dimensions of the instruments are expressed as $l' = 5.0$ cm, $a = 3.5$ cm, $L = 200$ cm, $D = 10$ cm, $P = 10$ sec, and formula [4], the change of 1 mm on the recording paper is equivalent to 1.76×10^{-9} in the linear strain of the crust.

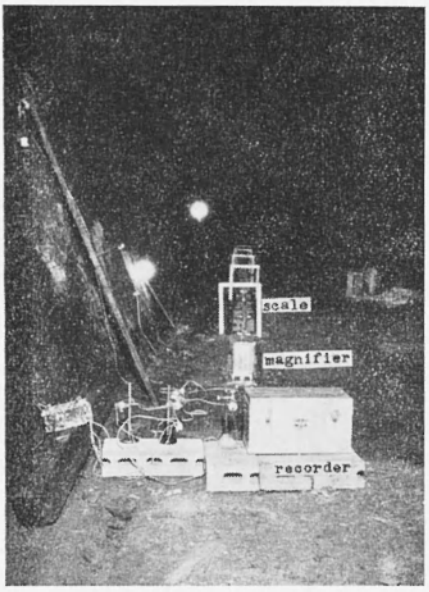

Photo 1 – The Whole View of the Extensometer

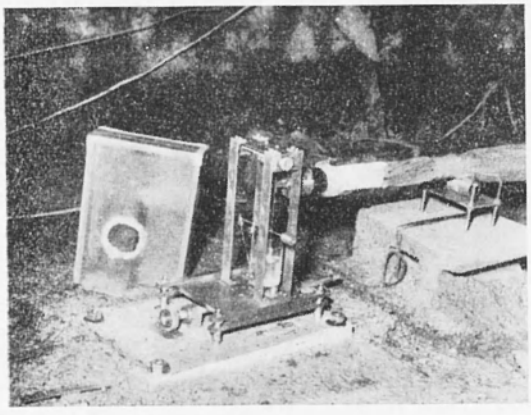

Photo 1*a* – The Magnifier Part of the Extensometer. This type is equipment with the vertical pendulum instead of the inverted pendulum.

Let the period T be 20 sec, then S is 4.4×10^{-10} per one millimeter on the records. Measuring the changes till the dimension of 0.1 millimeter on the records, we are able to distinguish them till the order of 10^{-10} in the changings of the linear strains. We have obtained good agreements between the sensitivities obtained experimentally and the ones calculated by use of the formula [4]. We have to consider the effects of the inclinations of the ground because the amplifier consists of tiltmeter. Let φ be the tilting of the ground. Then, the tilting ($\dot{\phi}$) of the support of the pendulum of the magnifier is given as

$$\dot{\phi} = \frac{D\,e}{a} + \varphi\,. \qquad [5]$$

Let e_o/φ_o be the specific ratio between e and φ when the first term is equal to the second in the formula [5]. Then we have

$$\frac{e_o}{\varphi_o} = \frac{a}{D}\,. \qquad [6]$$

In actual case, $a < 4$ cm. and $D > 400$ cm. Then the magnitude of e_o/φ_o is larger than 100.

Therefore, if the length of the extensometer larger than is longer than four meters, the magnification of the earth's strain is 100 times of the one of the tilt; the longer the length of the extensometer is, the less the noise caused by earth's tilt we are able to make. We need not consider the effect caused by, the earth tiltings in usual cases. In a stricter case, we have only to eliminate the effect due to the tiltings by use of the results of the observation of low sensitive tiltmeter at the near place. An another method is to combinate two extensometers whose amplifier parts are connected in the opposit sides respectively; these arrangement are shown in Fig. 1. In the vertical component of the extensometer, the construction of the amplifier is similar to the horizontal one. It seems that the methods of subtraction of the noise caused by earth-tilt is possible, but the cancelling method by the reversed instruments is complcated in usually.

3) Observation.

This type's extensometers are being used for the observations of the crustal deformations at Osakayama Observatory (Shiga Prefecture), Kishu Mine (Mie Prefecture) and Suhara Observatory (Wakayama

Prefecture). In the present paper, the observations with the newly devised extensometers at Osakayama Observatory are reported. Osakayama Observatory is situated at 135° 51′.5. E longitude and 34° 59′.6. S latitude, and is in 100 meters above the sea level. And the Observatory is about 65 km. to the nearest sea that is the Japan sea. The room of the observatory is about 100 meters under the ground. The amplitude of annual variation is 0.4° C and the one of the daily variation is less than 0.01° C. The newly devised extensometers have been set in the directions of East to West and North to South in 1958, and afterward nine components of the extensometer have been set in Osakayama Observatory. The study of the comparison between the observations by the extensometers situated newly and the ones by the extensometers set formerly.

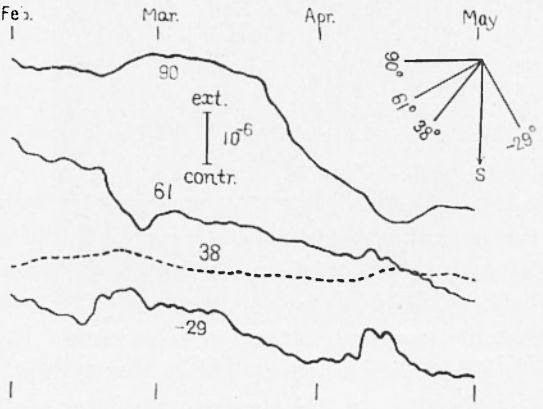

Fig. 3

The sensitivities of the instruments are adjusted to the magnitude of the seasonal change in the routine observations, and usually keeps up ample scope to ability of more precise observations for the special time.

Photo 1 shows the *E-W* and *N-S* components of these instruments.

Fig. 3 shows the comparison of the seasonal changes which are observed with newly devised and old types ([3]) ([1]) extensometers in the period from February to April in 1959, when there are many perturbations. The numerical signs on the curves show azimuths (measured from north in degrees) of the observing linear strains; curves 90 and 0 are the strainning with the newly devised types of extensometers along the directions of *E-W* and *N-S*, curve 61 is the one with the extensometer which

consists of the combination of the horizontally slackening wire and galvanometer of the torsional wire, curve 29 is one of Sassa type's extensometer which consists of the vertically slackening wire and tri-fiillar suspension type pendulum and curve 38 is one of the rod and roller type's. As the observing directions are various, the forms of their changes are also various, but their absolute magnitude are almost equal and happened at the same times.

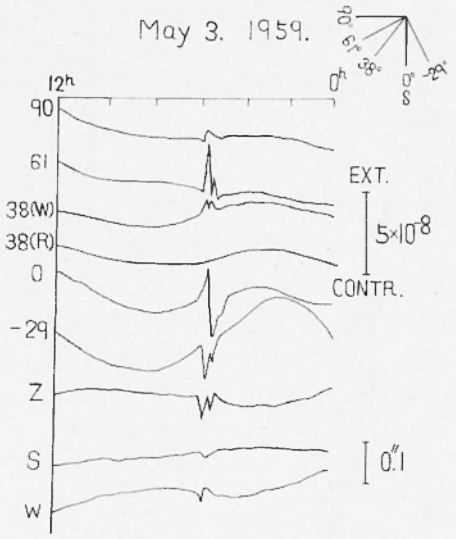

Fig. 4

Fig. 4 shows the sudden changes of the linear strains observed with various type's extensometers in the various directions, and the ground tiltings observed with the horizontal pendulum type's tiltmeters: 38 (W) is the record by Sassa type's extensometer and 38 (R) is one of the roller type's one, Z is one of the vertical component of the extensometer, S and W is the one of south and west components of the ground-tilt, and others are the same as them in Fig. 3. From these changes, it seems that voller type extensometer does not record such changes because the roller of the amplifier has some frictions or the limit of linearity of the sensitivity is worse. The form of the short period's change in curve 90 looks like the one of curve W, but the long periods one does not. And also the one of 0 does not look like the one of S. From this fact and the result of formula [5], we understand that the perturbations in curve 0 and 90, are not the effects of the ground tiltings.

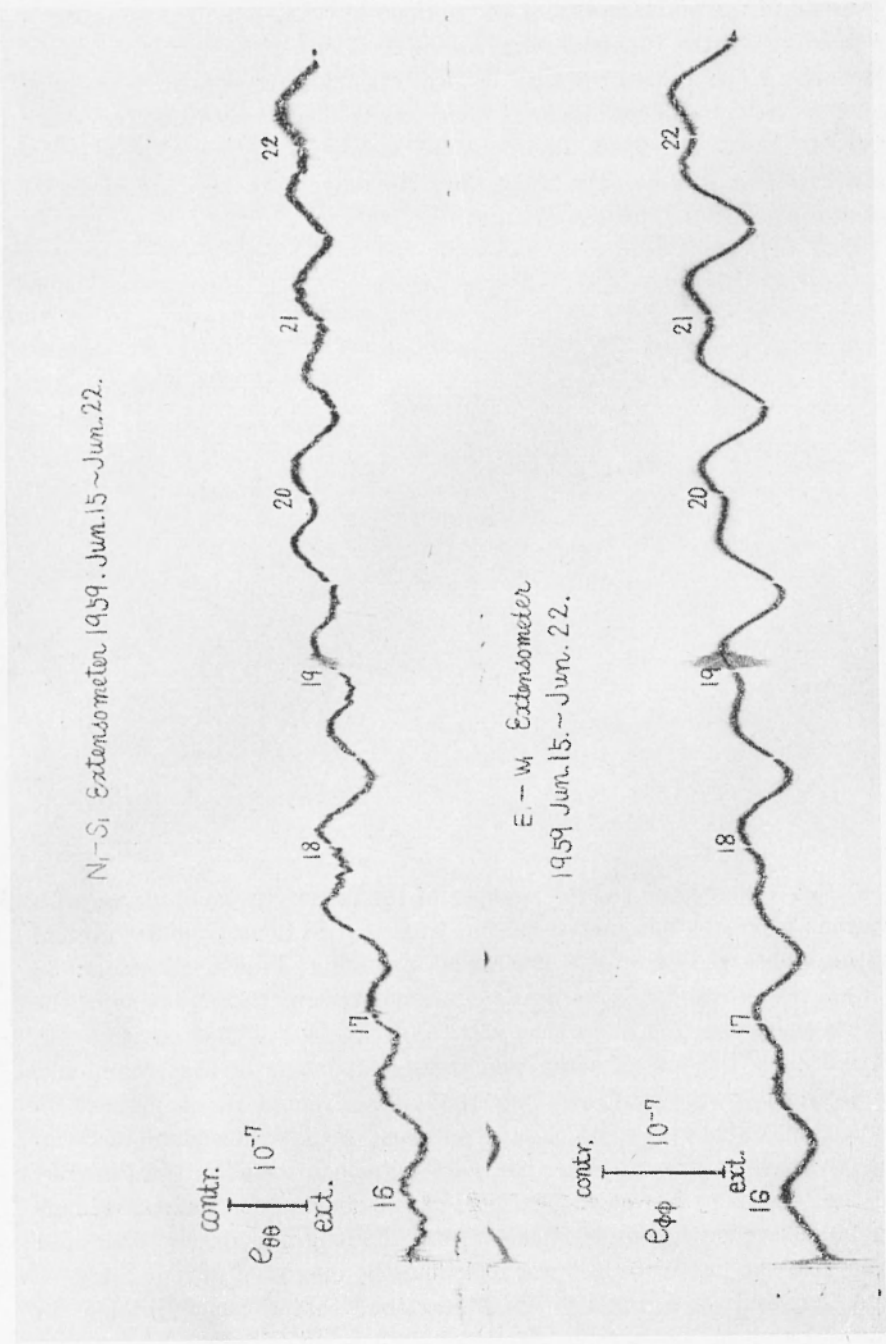
N-S Extensometer 1959. Jun.15~Jun.22.
contr.
$e_{\theta\theta}$ 10^{-7}
ext.
16
17
18
19
20
21
22
E-W Extensometer
1959 Jun.15.~Jun.22.
contr.
$e_{\phi\phi}$ 10^{-7}
ext.
16
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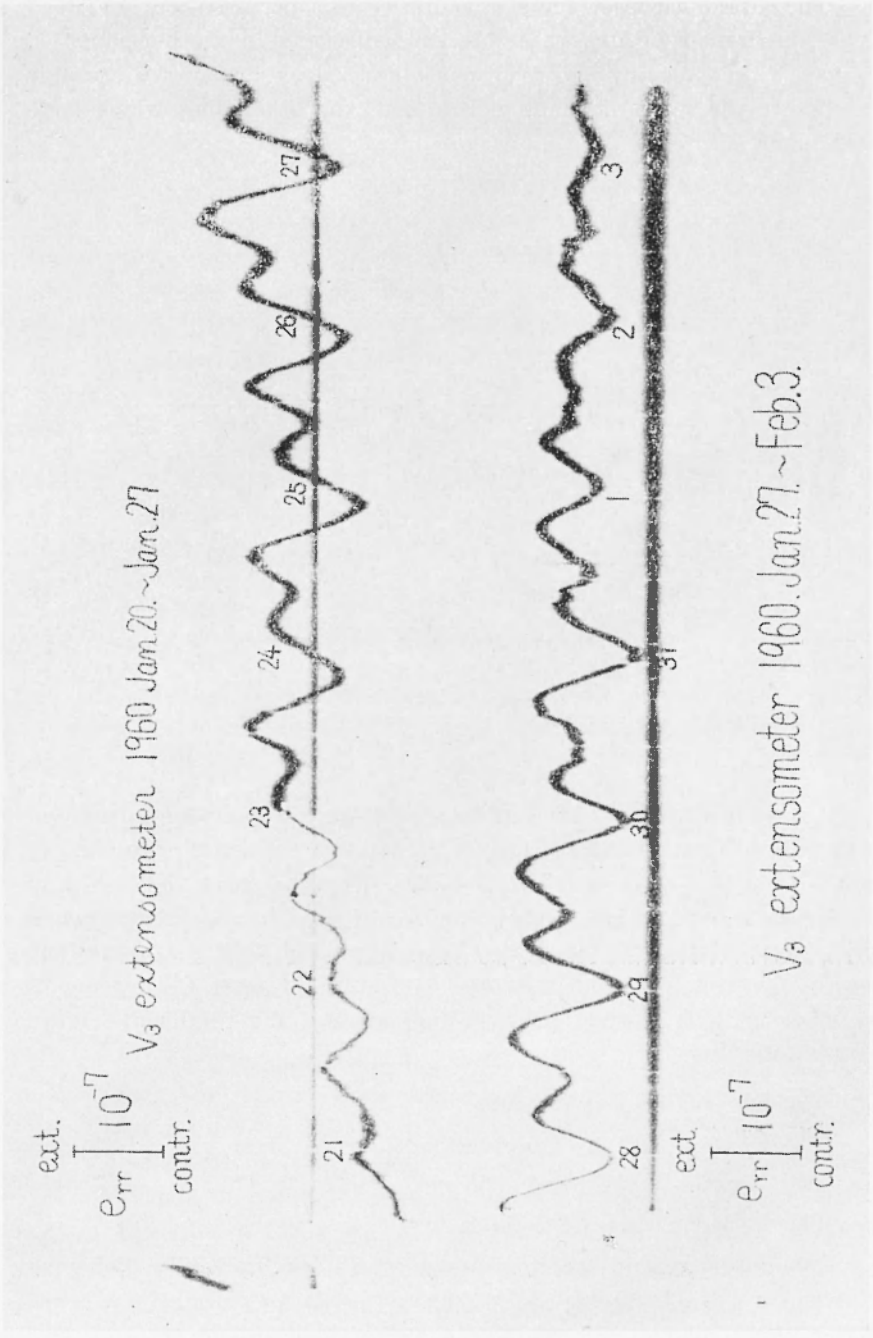
Photo 2


ext.
e_{rr} 10^{-7}
contr.
V_3 extensometer 1960 Jan.20.~Jan.27
21
22
23
24
25
26
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28
29
30
31
1
2
3
ext.
e_{rr} 10^{-7}
contr.
V_3 extensometer 1960. Jan.27.~Feb.3.

Photo 2*a*

The distributions of the strain due to this perturbation, is solved as the quadrant-type and these question is discussed in another paper (5). It goes without saying that the newly devised extensometers are able to observe the micro strainning, less than the other ones which have been made till now.

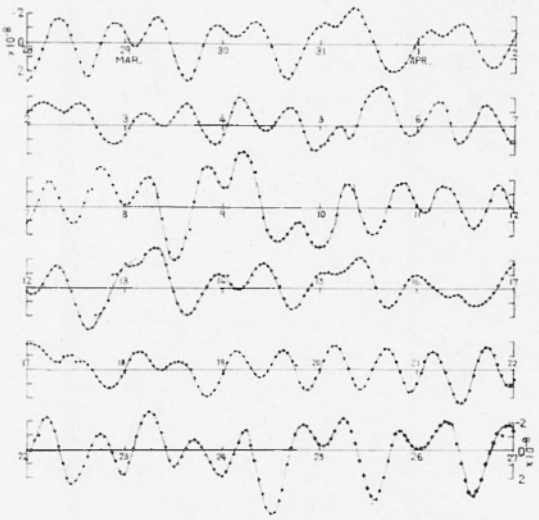

Fig. 5

Let us measure first the hourly values on the observed record and then subtract the running means of 25 hours of the series of the hourly means from the every measuring value. Then we have the series of the hourly means which consists mainly of the tidal variations and show the variations of the period from March 28 1959 to April 26 1959 in Fig. 5. Next we calculated tidal constants M_2 and O_1, by means of improved *T.I.* method (Darwin-method) and the results are shown in the following.

The Observatory: Osakayama

The Direction of the Observation: East to West

The Length of the Span: 530 cm.

The Period of the Analysis I:

The Tensitivity of the Instruments: 1.81×10^{-8}/mm ($T = 5{,}08$ sec.).

The Analyzed Values: $M_2 = 0.632 \times 10^{-8} \cos (2\,t - 4.6^{\circ})$.

$O_1 = 0.672 \times 10^{-8} \cos (\ t - 22.0^{\circ})$.

The Period of the Analysis II:

The Sensitivity of the Instruments: 0.53×10^{-8}/mm ($T = 8.6$ sec.).

The Analyzed Values: $M_2 = 0.758\times10^{-8}\cos(2t-359.7^\circ)$,

$O_1 = 0.735\times10^{-8}\cos(t-16.3^\circ)$.

The values obtained above contain a little noise caused by the tidal tiltings, which had been analyzed by use of the tilting's observations with horizontal pendulum type tiltmeters in the period from January, 1948 to February, 1949 at Osakayama. The M_2 and O_1-components of this tidal tilt in the prime vertical direction are expressed as

M_2: $0.0042''\times\cos(2t-70^\circ)$, O_1: $0.0007''\times\cos(t-218^\circ)$ in west-tilt,

M_2: $0.0028''\times\cos(2t-230^\circ)$, O_1: $0.0067''\times\cos(t-220^\circ)$ in north-tilt.

As the dimension of a/D of the extensometer above mentioned is 0.0070, the noises caused by the both components of the west tidal tilt are given as

$$M_2\text{: } 0.014\times18^{-8}\cos(2t-70^\circ),$$
$$O_1\text{: } 0.003\times10^{-8}\cos(t-218^\circ).$$

Subtracting these noise from the analyzed values of the extensometer, the mean values of the tidal component of the linear strains in the period I and II are given as

$$M_2\text{: } 0.69\times10^{-8}\cos(2t-0.9^\circ),$$
$$O_1\text{: } 0.70\times10^{-8}\cos(t-19.1^\circ).$$

By the same method, the more precise observations are performed afterward with the higher sensitivity and the longer periods. Now, these results are shown in the Table 1. The values of 142° and 38° have been obtained by use of every 30 minutes' levels of the strain curves (6).

In Table 1, the numerals in parenthesis are the uncorrected values from which the tilting effects are not subtracted.

If the ratio h/l is about 0.1, which is obtained afterward as $h/l = 11.1$ (7), where h and l are Love's (8) and Shida's (9) numbers respectively; the amplitude of the diurnal component of the tidal strain is largest in the direction of the East to West and the one of the semi-diurnal component is largest in the one of the South to North. And also the amplitude of O_1-component is larger than the one of M_2-component. And so these relations are satisfied nearly in the Photo 1 and Table 1,

Table 1

Azimuth of Observation	Period of Analysis	Span of Observation	Sensitivity of Instrument	M_2-component		O_1-component	
				Amplitude	Phase Lag	Amplitude	Phase Lag
		m	$\times 10^{-8}$/mm	$\times 10^{-8}$		$\times 10^{-8}$	
90° (East)	1959.1.27	5.3	1.81	(0.632)	(4°.6)	(0.672)	(22°.0)
	— 2.25			0.634)	358.9	0.675	22.1
»	1959.3.28	»	0.53	(0.758)	(359°.7)	(0.735	(16°.3)
	— 4.26			0.753	0.0	0.737	16.4
»	1959.5.24	»	0.37	0.880	357°.8	0.760	3°.5
	— 8.23						
0° (North)	1959.4.23	6.6	0.57	1.362	10°.9	0.640	20°.8
	— 7.22						
142° (S52°E)	1960.2.19	10.0	0.064	1.63	0°.9		
	— 3.19						
38° (S38°W)	1960.8.25	26.0	0.209	0.464	16°.9		
	— 9.23						

which show the diurnal components are remarkable in the E-W component but are little in N-S component.

Let W_2 be the tide-generating potential which is a spherical solid harmonic of 2nd order, $F(r)$ and $G(r)$ be the functions of the distance r from the earth's center only. Then the radial, colatitudinal and longitudinal displacements, U_r, U_θ and U_ϕ respectively, are able to expressed as ([10]),

$$\left\{\begin{aligned} U_r &= \frac{2F(r) + G(r)\,r^2}{r} W_2 \\ U_\theta &= \frac{F(r)}{r} \cdot \frac{\partial W_2}{\partial \theta} \\ U_\phi &= \frac{F(r)}{r \sin\theta} \cdot \frac{\partial W_2}{\partial \phi} \end{aligned}\right. \qquad [7]$$

From [7] and the formulas of strain elements, we have the prime vertical component $e_{\phi\phi}$ and the areal dilatation of the tidal strain Σ as follows ([11]) ([7])

$$\left\{\begin{aligned} &e_{\phi\phi} = G(r)\, W_2 \text{ in diurnal component,} \\ &\Sigma = e_{\phi\phi} + e_{\theta\theta} = 2\,(h - 3\,l)\,\frac{W_2}{a\,g}\,. \end{aligned}\right. \qquad [8]$$

From the mean values weighted with observing spans and the period of analyses of M_2 components shown in Table 1 and formulas [8], we have following as

$$\begin{aligned} G &= 0.74 \times 10^{-12} \\ h - 3\,l &= 0.440. \end{aligned} \qquad [9]$$

The meanings of $G(r)$ are the $G = (h - 2\,l)/ag$ and one of the functions concerned with the radial displacement only, and so we are able to obtain from this relation as $h - 2\,l = 0.493$, then $h = 0.599$ and $l = 0.053$.

Summarizing this paper, we have devised the experiment with the new type's extensometers whose sensitivities are able to be maintained in 10^{-10} per one millimeter on the record. Their sensitivities are able to be adjusted easily to any dimensions and at any time. We have been performing the much better observations than the ones of formerly used extensometers at Osakayama Observatory. From this observation we have obtained the values of $G(a)$ and $h - 3\,l$ directly.

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