Crustal thickness in Fennoscandia from phase velocities of Rayleigh waves

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INTRODUCTION.

It has been shown previously, that local variations in crustal thickness can be determined from phase velocities of Rayleigh waves over relatively dense nets of seismograph stations (Press 1956, Press 1957, Ewing and Press 1959). The method needs a net of stations with homogeneous instrumentation of vertical seismographs. This net must be sufficiently dense to allow identification of each individual wave crest at every station inside the area under investigation. Furthermore, the method needs a dispersed train of Rayleigh waves of sufficiently large amplitudes to be clearly recorded by the seismographs used. The earthquake producing this wave train must be at a great distance from the area under investigation, and the wave path should be over homogeneous crustal structure to avoid disturbing interference.

In this paper an attempt is made to use the phase-velocity method to determine the thickness of the earth's crust in Fennoscandia. Two earthquakes are selected for this study, one with surface-wave propagation nearly perpendicular to the west coast of Norway, the other with wave propagation nearly parallel to this.

METHODS AND MATERIALS USED.

Data about the earthquakes, selected for this study, are given in Table I. Table II gives the coordinates of the seismograph stations

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and the distances and directions to the epicenters, and Fig. 1 shows the station net.

Table I. – Earthquakes selected for the study (Gutenberg 1959 b and 1960) (M = Magnitude, m = Unified magnitude).

Date	Time	Location	Depth	M	m	
1957 July 28	08 40 05	16 ¾ N 99 W	Normal	7.9	7.0	Mexico
1958 Nov. 6	22 58 06	44 ½ N 148 ¾ E	75 km	8.7	8.0	Kurile Islands

At all stations the records of short-period vertical seismographs were used. The time of each wave crest was measured to the nearest second. If the amplitude was very small and the period large, the accuracy of the measurement was of the order of 5 sec, but for larger amplitudes, the accuracy was about 1 sec.

Table II. – Seismograph stations, distance and direction to the epicenters (h = height of station above sea level, distance in degrees, azimuth in degrees clockwise from north).

Station		Latitude North	Longitude East	h m	Mexico earthquake		Kurile Islands earthquake	
					Distance	Azimuth	Distance	Azimuth
Goteborg	(*Gb*)	57° 42′	11° 59′	66	—	—	72.0	30.9
Helsinki	(*Hel*)	60° 10′	24° 58′	20	91.1	307.3	66.0	39.7
Kiruna	(*Ki*)	67° 50′	20° 25′	390	85.0	303.0	61.5	39.6
Köbenhavn	(*Kob*)	55° 41′	12° 26′	13	87.8	296.6	73.6	30.8
Lund	(*Lu*)	55° 42′	13° 11′	32	88.2	297.3	—	—
Skalstugan	(*Sk*)	63° 35′	12° 17′	580	84.2	296.1	66.9	32.3
Sodankylä	(*Sod*)	67° 22′	26° 39′	—	87.3	308.7	60.2	44.1
Uppsala	(*Up*)	59° 52′	17° 38′	14	88.2	301.0	68.6	35.3

To correct these observed arrival times (t) for reading errors, they are compared with the number of the waves (N), as Fig. 2 shows. If an observed point lies definitely off the smooth curve, it is corrected to lie on the smooth curve. The wave period (T) is found from the slope of this curve.

Fig. 1. – Locations of seismograph stations used in this study.

Another method for determining the wave period, used on the waves from the Mexico earthquake, is illustrated in Fig. 3. Here the observed period is plotted against log ($N + 2$). The points thus obtained should lie on a smooth curve, which is almost a straight line for low values of N.

The identification of the wave crests at different stations involves some problems. The seismograms look so different (Fig. 4) that direct comparison between these gives no definite result. The procedure used

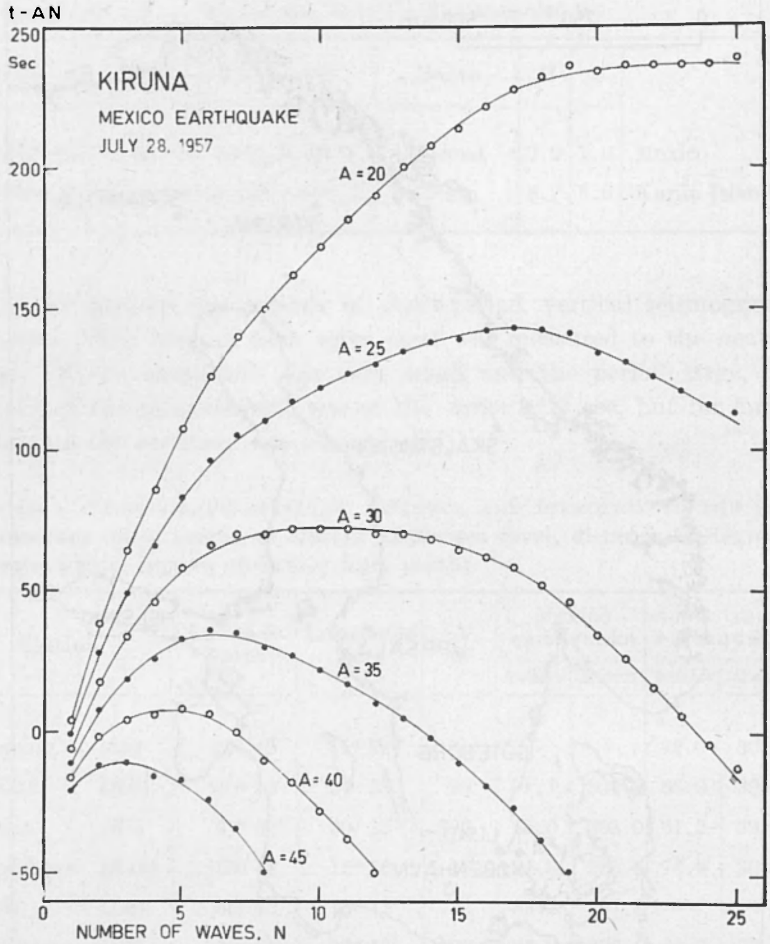

Fig. 2. – Illustration of method used in correcting observed arrival times and wave periods (see text).

is based on the assumption, that the direction of wave propagation is nearly along great circles through the epicenters, and that the phase velocities of the first observed Rayleigh waves in each earthquake are about 4 km/sec. This method gave only one possible solution for the first arrived waves, and it could also be used to determine the unknown

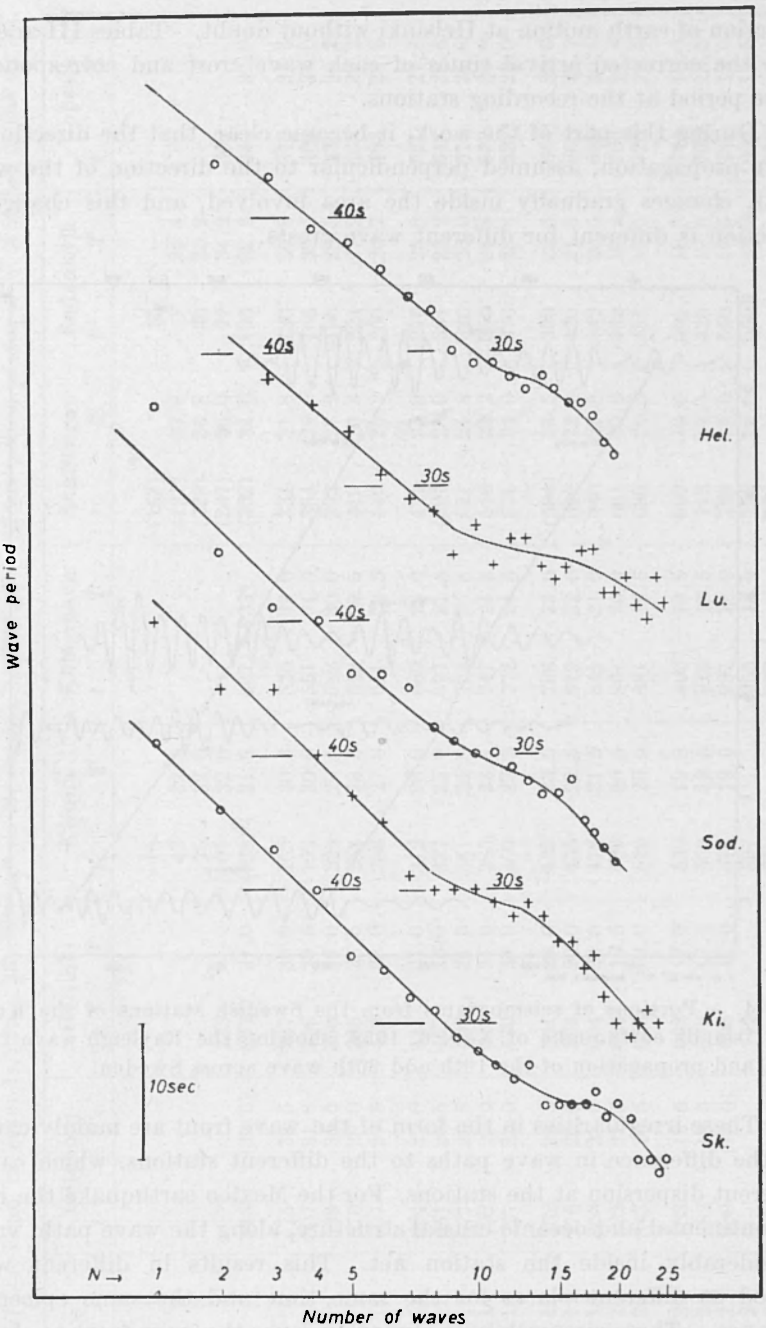

Fig. 3. – Illustration of method of correcting observed wave periods (see text).

direction of earth motion at Helsinki without doubt. Tables III and IV give the corrected arrival times of each wave crest and corresponding wave period at the recording stations.

During this part of the work, it became clear, that the direction of wave propagation, assumed perpendicular to the direction of the wave front, changes gradually inside the area involved, and this change in direction is different for different wave crests.

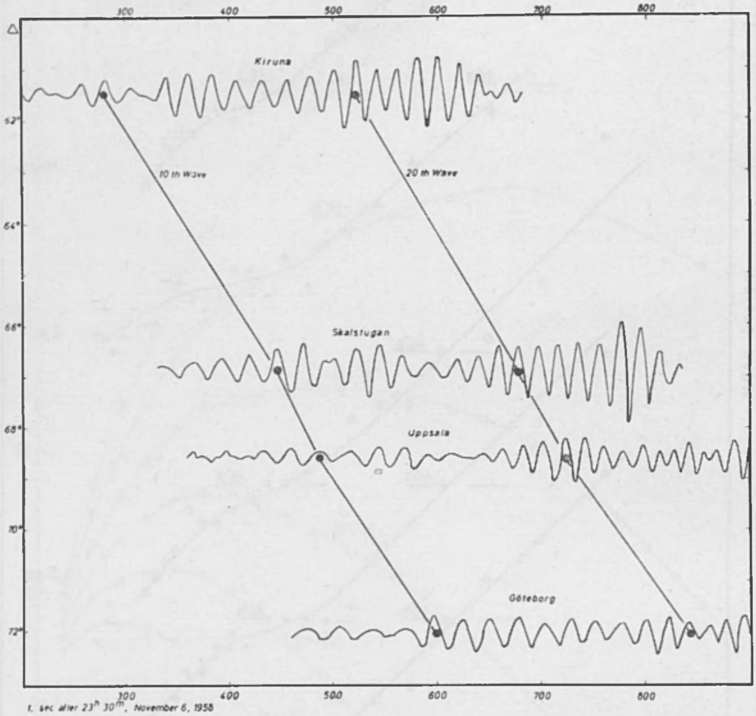

Fig. 4. – Portions of seismograms from the Swedish stations of the Kurile Islands earthquake of Nov. 6, 1958, showing the Rayleigh-wave train and propagation of the 10th and 20th wave across Sweden.

These irregularities in the form of the wave front are mainly caused by the difference in wave paths to the different stations, which causes different dispersion at the stations. For the Mexico earthquake the ratio of continental and oceanic crustal structure, along the wave path, varies considerably inside the station net. This results in different wave period at different places for the same time and the same epicentral distance. These circumstances obviously cause the wave front to change form and direction.

Table III. – Arrival times of Rayleigh wave crests from Kurile Islands earthquake of Nov. 6, 1958 (t = Time in seconds after $23^h\ 30^m\ 00^s$ on November 6, 1958; T = Wave period in sec; N — Number of the wave).

N	Goteborg		Helsinki		Kiruna		Kobenhavn		Skalstugan		Sodankylä		Uppsala	
	t	*T*	*t*	*T*	*t*	*T*	*t*	*T*	*t*	*T*	*t*	*T*	*t*	*T*
1					10				(153)		— 35			
2					43	32.5			(191)	37.8	1	35.8		
3					75	32.0			(229)	36.8	36	35.2		
4			235		107	32.0			(267)	36.0	72	34.2	309	
5	443	(33.5)	267	31.0	(138)	31.5	487	(33)	(302)	34.0	105	32.5	342	32.0
6	475	32.5	297	30.0	169	30.5	520	32.0	334	31.8	136	30.8	373	31.5
7	507	31.8	326	29.5	199	30.0	551	31.0	364	29.0	166	29.5	404	31.0
8	539	30.6	356	29.0	229	29.5	582	30.0	392	27.2	196	28.5	435	30.2
9	570	29.0	385	28.5	258	28.5	612	29.5	419	26.8	224	27.2	465	29.2
10	597	27.8	413	28.0	286	27.0	641	28.5	446	26.2	250	26.2	493	28.2
11	624	27.0	441	27.0	312	26.0	669	28.0	471	25.5	276	25.8	521	27.8
12	651	26.5	467	26.0	337	25.0	697	27.5	(497)	25.0	301	25.2	548	26.6
13	677	25.5	492	24.5	361	24.0	722	25.5	521	24.8	326	25.0	573	24.8
14	702	25.0	516	24.0	385	23.8	747	24.5	546	24.2	350	24.8	597	24.0
15	727	25.0	539	23.0	409	23.5	772	24.5	571	24.0	376	23.8	(621)	23.0
16	752	24.5	561	23.0	432	23.0	796	23.0	(594)	23.5	399	22.5	(644)	22.0
17	776	24.0	584	22.0	455	22.5	819	22.0	617	22.8	420	21.5	665	21.2
18	800	23.0	604	20.0	478	21.5	841	21.0	640	21.8	442	20.5	686	20.5
19	822	21.5	624	19.5	499	20.5	861	20.0	661	20.0	462	19.8	706	19.8
20	843	20.5	643	19.0	519	20.0	881	19.5	680	20.0	481	19.5	726	19.5
21	863	19.5	662	18.0	539	19.8	900	19.0	699	19.5	501	19.5	745	19.0
22	882	18.5	679	17.5	559	19.5	919	18.5	718	19.5	520	19.5	763	18.5
23	900	(18.0)	696	17.0	578	19.0	937	18.0	738	19.0	540	19.5	781	18.0
24			712		597		955	18.0	758	19.0	560	19.5	800	17.5
25					616		973		778				816	

The direction of the wave front is found for each station triangle, where the wave velocity is computed. The arrival times of a wave front at the three stations I, II and III are t_1, t_2 and t_3 (Fig. 5). The

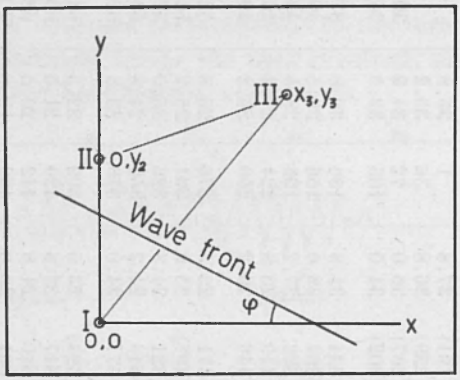

Fig. 5. – Geometry of plane wave front traversing station triangle.

equations giving apparent phase velocity c and direction of wave propagation φ inside the triangle, assuming a plane wave front are:

$$\tan \varphi = \frac{y_3}{x_3} - \frac{y_2 (t_3 - t_1)}{x_3 (t_2 - t_1)} \qquad [1]$$

$$c = \frac{y_2 \cos \varphi}{t_2 - t_1} \qquad [2]$$

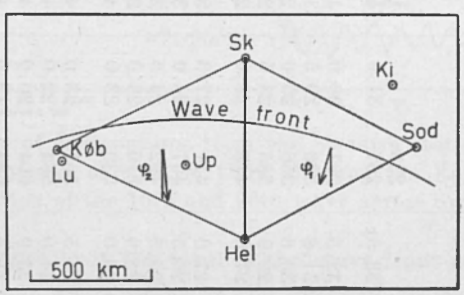

Fig. 6. – Geometry of curved wave front traversing the station net from the west (Mexico earthquake).

The value of φ was found to vary rather regularly inside the area. This indicated a possibility of including the curvature of the wave front in the velocity computation, by assuming a simple smooth form of the

Table IV. – Arrival times of Rayleigh wave crests from Mexico earthquake of July 28, 1957 (t = Time in seconds after $09^h\ 20^m\ 00^s$ on July 28, 1957; T = Wave period in sec.; N = Number of the wave).

N	Helsinki		Kiruna		København		Lund		Skalstugan		Sodankyla		Uppsala	
	t	*T*	*t*	*T*	*t*	*T*	*t*	*T*	*t*	*T*	*t*	*T*	*t*	*T*
1			28		130				14		86			
2	257	(48)	78	48.0	171	43.6			65	48.4	142	49.2	178	(48)
3	301	44.0	123	44.8	210	39.9	220	40.0	111	44.1	187	44.9	225	43.8
4	344	41.0	168	41.1	246	36.7	258	37.0	154	40.6	228	41.3	267	40.5
5	383	38.6	208	38.3	282	34.2	294	34.4	194	37.8	268	38.4	306	37.8
6	421	36.4	245	35.8	317	32.1	328	32.3	229	35.4	304	36.2	340	35.4
7	457	34.8	280	33.6	348	30.2	359	30.4	263	33.2	340	34.3	374	33.4
8	491	33.2	311	31.7	375	28.4	388	28.7	295	31.6	375	32.7	406	31.6
9	524	31.8	341	30.6	402	27.5	416	27.3	326	30.0	407	31.4	437	29.8
10	554	30.4	371	29.9	429	26.3	441	26.4	355	28.6	438	30.5	465	(27.7)
11	583	29.4	401	29.3	454	25.7	468	25.8	383	27.6	468	29.7		
12	612	28.5	430	28.8	480	25.2	492	25.3	410	26.6	498	29.0		
13	640	28.0	458	28.4	505	24.8	518	25.1	436	25.8	527	28.4		
14	667	27.7	487	27.9	530	24.6	544	24.9	463	25.3	555	28.0		
15	695	27.2	515	27.0	554	24.5	568	24.6	(487)	24.6	582	27.6		
16	722	26.6	541	26.0	578	24.3	591	24.4	(511)	24.3	609	27.0		
17	748	25.8	567	25.0	602	24.1	615	24.1	(535)	24.1	637	26.0		
18	774	24.8	591	24.2	626	23.9	640	23.8	(559)	24.0	662	25.0	673	(22.5)
19	799	23.8	614	23.4	651	23.6	665	23.5	584	23.9	686	23.9	696	(22.0)
20	821	22.8	636	22.4	675	23.3	(687)	23.1	607	23.5	709	22.7	717	(22.0)
21	843	(22.0)	656	21.4	697	22.8	709	22.6	631	22.6	731		739	(21.5)
22			675	20.6	719	22.4	732	22.1	654	21.6			761	(21.5)
23			695	19.8	742	21.9	753	21.7	674	20.8			783	(21.5)
24			716	19.0	763	21.5	773	21.4	694	20.0				
25			736		785	21.0	796	21.0	713	19.4				
26					804		817		733					

wave front. The form chosen is constant curvature for each wave front, where the center of curvature moves with the wave front in direction of the great circle through the epicenter and central Scandinavia.

The curvature was computed by different methods for the two earthquakes.

In case of the Mexico earthquake, the direction of the wave front was found by using equation [1] on the two large station triangles

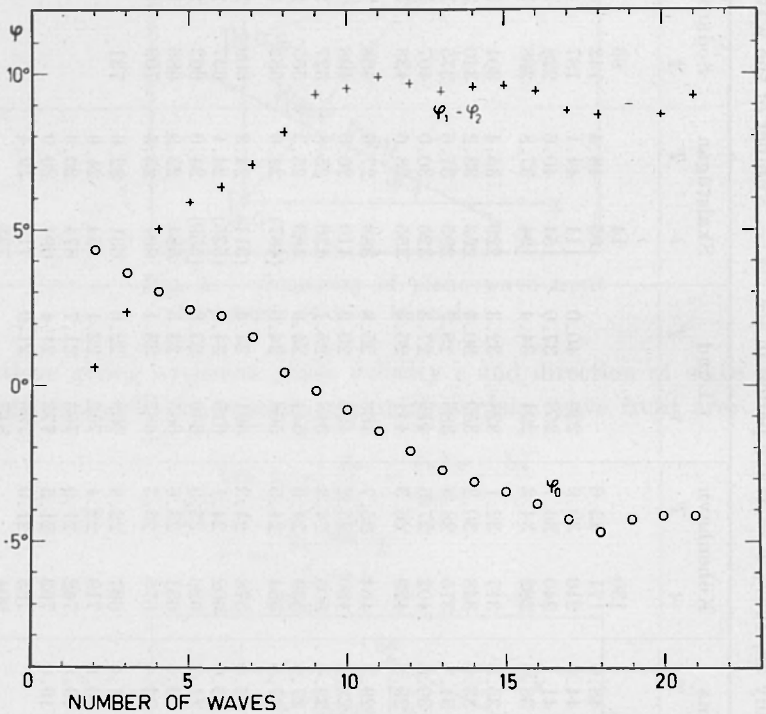

Fig. 7. – Observed direction of wave propagation of the Mexico earthquake in degrees from the line Helsinki-Skalstugan (see text).

Hel–Sk–Sod and *Hel–Sk–Köb* (Fig. 6). The directions φ_1, and φ_2 thus found were assumed to represent correct directions of wave propagation on lines parallel to the line *Hel–Sk* and halfways between this and *Sod* and *Kob* respectively.

Our assumptions make the direction of each wave front to depend only on the distance x from the line *Hel–Sk* and the direction φ_0 on this line. The equation used to find approximate values of φ is:

$$\varphi = \varphi_0 + \frac{2\,x}{L}(\varphi_1 - \varphi_2) \quad , \tag{3}$$

where φ_0 is the value of φ on the line *Hel-Sk*, and L is the distance between Köbenhavn and Sodankylä.

The values of φ_0 and $\varphi_1 - \varphi_2$ are shown in Fig. 7.

The method used to compute the curvature of wave fronts of the Kurile Islands earthquake is illustrated in Fig. 8. The apparent phase velocity c_0 along the line *Sod–Gb* is found for each wave. The lines *Sk–A* and *Hel–B* are parallel to the line *Sod-Gb*. A circle through A,

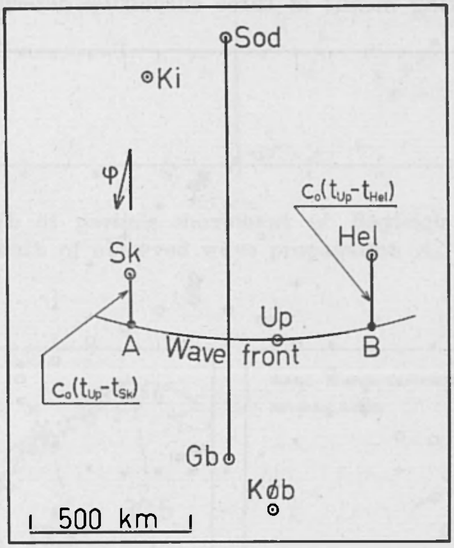

Fig. 8. – Geometry of curved wave front traversing the station net from the north (Kurile Islands earthquake).

Up and B is assumed to represent the true form of the wave front and the centers of these circles are assumed to move with the velocity c_0 along lines parallel to *Sod–Gb*. The values of φ_0, φ_{Sk} and φ_{Hel} are shown in Fig. 9. φ_0 gives the direction of the wave propagation on the line *Sod–Gb*.

Direction of approach of Rayleigh waves from the Mexico earthquake, as indicated by records of long-period seismographs at Uppsala and Kiruna.

The records of long-period seismographs at Uppsala and Kiruna were used to determine the apparent direction of particle movement of the surface waves recorded by the short-period vertical seismometers at these stations. At Uppsala, records of the long-period Benioff seismometers were used, and at Kiruna those of Galitzin seismometers.

The azimuth of particle movement is computed for every readable wave maximum or minimum. The values found (A_p) are given in Fig. 10 together with the azimuths of wave propagation (A_t) found by the method of correlation between stations. The values of A_p are smoothed by taking means of three successive determinations.

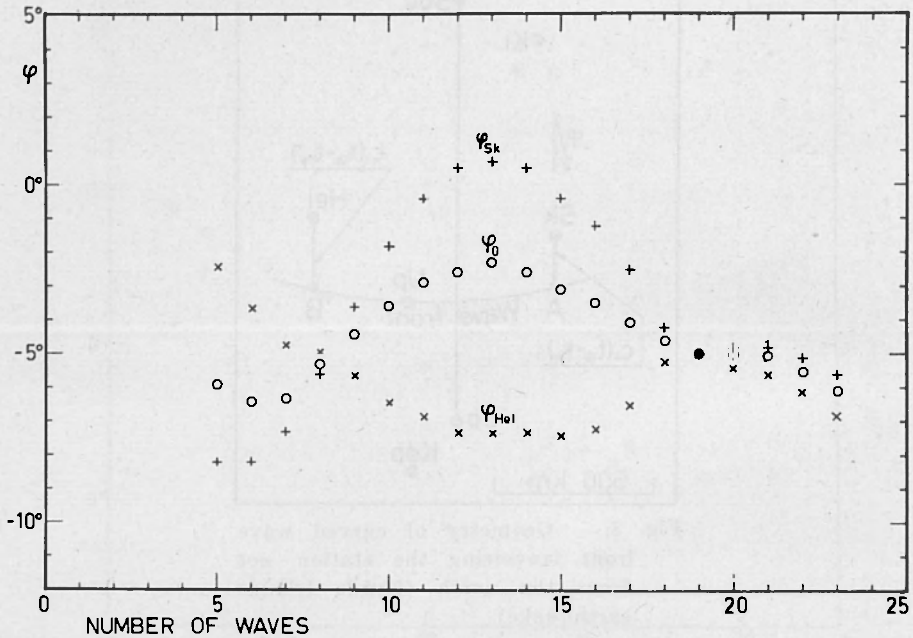

Fig. 9. – Observed direction of wave propagation of the Kurile Islands earthquake in degrees from the line Göteborg-Sodankyla (see text).

Fig. 10 shows a significant difference between A_p and A_t at both stations. This indicates a very irregular form of the wave front, or else the observed particle movements are not perpendicular to the wave front. Such effects can possibly be caused by irregularities in the geological structure near the stations. Moreover, the local geological structure may affect the seismometers to respond differently in different directions. Another possible cause of error in computed azimuths of particle movement is the use of incorrect instrumental constants. In our case, however, these constants are known relatively exact. A mixture of Love waves in the Rayleigh-wave train may influence the result of A_p, but

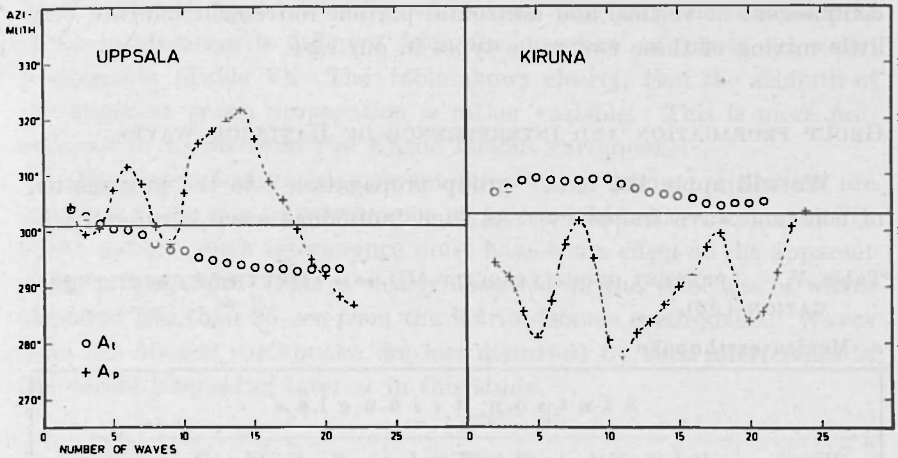

Fig. 10. – Azimuth of particle movement of Rayleigh waves (smoothed) A_p and azimuth of observed wave propagation A_t.

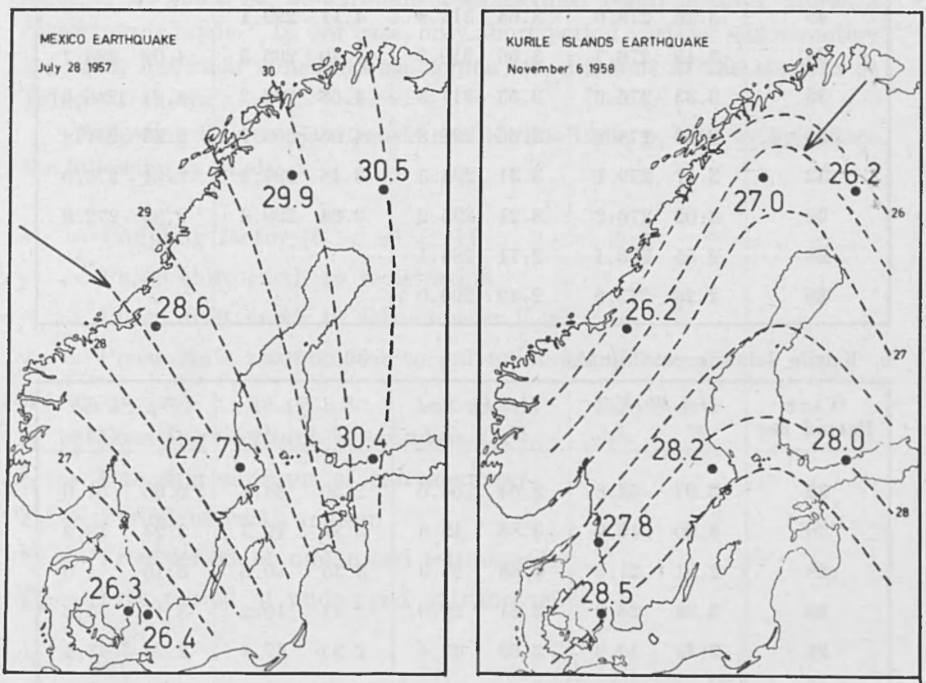

Fig. 11. – Period of tenth wave of the Mexico earthquake and Kurile Islands earthquake. The arrows show directions of great circles through the epicenters.

comparisons of vertical and horizontal particle movement indicate very little mixing of these two wave types in our case.

Group propagation and interference of Rayleigh waves.

We will apply the term " group propagation " to the propagation of constant wave frequency. As each individual wave is observed to

Table V. – Apparent group velocity (U) and azimuth of group propagation (Ag).

a. Mexico earthquake

Wave Period sec	Hel-Sk-Kob U	Hel-Sk-Kob Ag	Hel-Sk-Sod U	Hel-Sk-Sod Ag	Up-Ki-Sk U	Up-Ki-Sk Ag	Up-Sk-Lu U	Up-Sk-Lu Ag
44			3.41	303.6	3.02	297.6		
42	3.39	281.7	3.50	308.1	3.46	297.3		
40	3.49	278.6	3.65	310.8	4.11	299.1		
38	3.45	276.7	3.66	310.2	4.40	295.3	4.08	286.7
36	3.33	276.6	3.53	311.3	4.58	299.2	4.04	286.0
34	3.19	278.6	3.36	298.8	4.05	298.7	3.36	281.8
32	3.17	279.1	3.31	294.5	3.48	297.8	2.61	275.9
30	3.02	276.2	3.21	295.2	3.64	299.3	2.53	272.9
28	2.55	273.1	2.71	288.1				
26	2.26	273.6	2.42	290.0				

b. Kurile Islands earthquake

Wave Period sec	Hel-Sk-Kōb U	Hel-Sk-Kōb Ag	Hel-Sk-Sod U	Hel-Sk-Sod Ag	Up-Ki-Sk U	Up-Ki-Sk Ag	Up-Sk-Gb U	Up-Sk-Gb Ag
32	2.97	52.8	3.64	64.0	2.50	54.1	2.56	51.6
30	3.00	40.0	3.85	48.6	3.39	19.7	3.27	18.9
28	2.91	21.5	3.68	24.9	3.35	−0.6	3.16	1.9
26	2.98	23.8	3.61	28.0	3.41	10.2	3.11	13.3
24	3.33	40.9	3.69	47.4	2.95	47.4	2.26	47.4
22	3.59	45.0	3.46	49.4	3.42	52.2	2.38	52.1
20	3.65	45.6	3.39	49.3	3.65	47.9	2.71	48.0

be of different period at different stations (Fig. 11), the direction of group propagation is different from the direction of phase (or wave) propagation (Table V). The table shows clearly, that the azimuth of the apparent group propagation is rather variable. This is more pronounced in waves from the Kurile Islands earthquake.

The great changes in apparent direction of group propagation are supposed to be due to interference of waves, which have traversed different paths. Such interference must have some effect on the apparent phase propagation. This is clearly observed on the velocities of waves of period less than 25 sec from the Kurile Islands earthquake. Waves from the Mexico earthquake are less disturbed by such interference in the period interval of interest in this study.

PHASE SHIFT EARTH-SEISMOGRAM.

The method used here depends on the type of seismometers used, as different phase lag at different stations must result in error unless a correction is made. In our case, only short-period vertical seismometers are used, but their constants are somewhat different at the stations as Table VI shows.

Following Kirnos (Sawarenski and Kirnos 1960, p. 413) we introduce the following symbols:

σ^2 = Coupling factor ($0 < \sigma^2 < 1$)
$\bar{\gamma}$ = Phase shift earth to registration
γ_1 = Phase shift earth to seismometer if $\sigma^2 = 0$
γ_2 = Phase shift seismometer to galvanometer if $\sigma^2 = 0$
$\gamma_0 = \gamma_1 + \gamma_2$
h_1 = Damping constant of seismometer
h_2 = Damping constant of galvanometer
T_e = Period of earth motion
T_1 = Free period of undamped seismometer
T_2 = Free period of undamped galvanometer
$u_1 = \frac{T_e}{T_1}$
$u_2 = \frac{T_e}{T_2}$

Then we have:

$$\tan \bar{\gamma} = (1 + \delta) \tan \gamma_0, \qquad [4]$$

where

$$\delta = \sigma^2 \frac{4\, h_1\, h_2\, u_1\, u_2}{1 + u_1^2\, u_2^2 - (u_1^2 + u_2^2 + 4\, h_1\, h_2\, u_1\, u_2)} \qquad [5]$$

$$\tan \gamma_1 = \frac{2\, h_1\, u_1}{1 - u_1^2} \qquad [6]$$

$$\tan \gamma_2 = \frac{u_2^2 - 1}{2\, h_2\, u_2} \qquad [7]$$

In Table VI the valnes of γ_0 and $\Delta\gamma = \bar{\gamma} - \gamma_0$ if $\sigma^2 = 1$, are given. Furthermore, $\Delta\, t = \frac{{}^3/_2\, \pi - \gamma_0}{2\, \pi}\, .\, T_e$ which gives the time difference between the phase shift $\gamma = {}^3/_2\, \pi$, and the actual γ_0. The table shows, that the extreme time-lag difference between the seismograms involved in the present investigation is about 0.5 sec, and the error introduced by neglecting the value of σ cannot exeed 0.01 sec in the material under consideration.

No correction is made for the different phase shifts, as these are small ($< 1\%$) compared with the time differences used in computing, the phase velocity.

Phase velocity of Rayleigh waves.

The local phase velocity of Rayleigh waves depends on the thickness of the crustal layers and the wave velocities in the crust and upper mantle.

As our intention is to evaluate the crustal thickness from observed phase velocities, we have to make some assumptions regarding the wave velocities in the crust and upper mantle. We assume no horizontal gradient of wave velocities inside each crustal layer or in the upper mantle. This is not quite true (e. g. Gutenberg 1959 a, p. 28) and may introduce some errors. The observed dispersion curves of continental Rayleigh waves differ somewhat from each other (Båth 1959) and this difference cannot be explained wholly by difference in crustal thickness. Other authors have stated that the observed continental dispersion curves show remarkably little difference (Gutenberg 1959 a, p. 44; Ewing et al 1957, p. 198).

Table VI. – CONSTANTS OF SEISMOGRAPHS AND PHASE SHIFT EARTH TO SEISMOGRAM. h_1 AND h_2 SUPPOSED TO BE EQUAL 1. (For explanation see text).

Station	Instrument	T_1	T_2	γ_0 (degrees)			Δt (sec)			$\Delta\gamma$ (degrees)		
		sec	sec	$T_e=20$	$T_e=30$	$T_e=40$	$T_e=20$	$T_e=30$	$T_e=40$	$T_e=20$	$T_e=30$	$T_e=40$
Göteborg	Grenet Z′	1.4	0.5	259.1	262.8	264.5	0.61	0.60	0.61	—0.077	—0.022	—0.010
Helsinki	Nurmia Z′	0.5	0.2	265.9	267.3	268.0	0.23	0.22	0.22	—0.004	—0.001	—0.0005
Kiruna	Grenet Z′	1.4	0.8	257.4	261.6	263.6	0.70	0.70	0.71	—0.144	—0.042	—0.018
Köbenhavn	Benioff Z′	1.0	0.25	262.9	265.3	266.4	0.39	0.39	0.40	—0.018	—0.005	—0.002
Lund	Grenet Z′	1.4	0.5	259.1	262.8	264.5	0.61	0.60	0.61	—0.077	—0.022	—0.010
Skalstugan	Grenet Z′	1.4	0.8	257.4	261.6	263.6	0.70	0.70	0.71	—0.144	—0.042	—0.018
Sodankylä	Benioff Z′	1.0	0.25	262.9	265.3	266.4	0.39	0.39	0.40	—0.018	—0.005	—0.002
Uppsala	Benioff Z′	1.0	0.7	260.3	263.5	265.1	0.54	0.54	0 54	—0.069	—0.020	—0.009

The phase-velocity method gives rather a relative, than an absolute measure of the crustal thickness, as the thickness computations are based on dispersion curves, either computed from simplified crustal models, or observed over long paths of inexactly known crustal structure.

We will take the observed dispersion curve of Press, Ewing and Oliver (1956) for the wave path Algeria to Natal, as one representing Rayleigh wave dispersion of normal continental crustal structure. This dispersion curve is almost identical to that found across North America (Oliver and Ewing 1958), but differs considerably from that across Euro-Asia (Båth 1959).

In computing phase velocity, the observed group velocities across Africa are used in the period range 10-70 sec, and constant group velocity is assumed in the period range 70-100 sec. The phase velocity of 100 sec period Rayleigh waves is set to 4.2 km/sec (Nafe and Brune 1960). Press (1956) used the same group velocity data in constructing his phase velocity curves, but different assumptions for the lacking data.

Our values of phase velocity are somewhat lower than those computed by Press, especially for large periods. This results in somewhat smaller values of the crustal thickness than Press found by using the same observational data. The differences in thickness depend on the total thickness and the periods and amount to less than five km.

In computing the phase velocity the following procedure was applied:

The equation giving the group velocity U as function of the phase velocity c and the period T is:

$$U = c + k \frac{dc}{dk} , \tag{8}$$

where

$$k = \frac{2\pi}{cT} . \tag{9}$$

These equations lead to:

$$\frac{dc}{dT} = \frac{c^2 - Uc}{TU} \tag{10}$$

which is used to find $\frac{\Delta c}{\Delta T}$. For two periods T_1 and T_2 near each other, the corresponding phase velocities c_1 and c_2 are found by using the equation

$$\frac{c_1 - c_2}{T_1 - T_2} = \frac{\Delta c}{\Delta T} = \frac{\left(\frac{dc}{dT}\right)_1 + \left(\frac{dc}{dT}\right)_2}{2} . \tag{11}$$

Table VII. – Group velocity U (H assumed 35 km) and computed phase velocity c of Rayleigh waves across Africa. Corresponding periods for different crustal thickness H.

T sec	U km/sec	c km/sec	Periods in sec for H other than 35 km									
			H=10km	15 km	20 km	25 km	30 km	40 km	45 km	50 km	60 km	70 km
10	3.030	3.245	2.8	4.3	5.7	7.1	8.6	11.4	12.9	14.3	17.1	20
12	2.988	3.297	3.4	5.1	6.9	8.6	10.3	13.7	15.4	17.1	20.6	24
14	2.968	3.357	4.0	6.0	8.0	10.0	12.0	16.0	18.0	20.0	24.0	28
16	2.955	3.422	4.6	6.9	9.1	11.4	13.7	18.3	20.6	22.9	27.4	32
18	2.950	3.492	5.1	7.7	10.3	12.9	15.4	20.6	23.1	25.7	30.9	36
20	2.965	3.563	5.7	8.6	11.4	14.3	17.1	22.9	25.7	28.6	34.3	40
22	3.038	3.631	6.3	9.4	12.6	15.7	18.9	25.1	28.3	31.4	37.7	44
24	3.125	3.692	6.9	10.3	13.7	17.1	20.6	27.4	30.9	34.3	41.1	48
26	3.208	3.743	7.4	11.1	14.9	18.6	22.3	29.7	33.4	37.1	44.6	52
28	3.273	3.789	8.0	12.0	16.0	20.0	24.0	32.0	36.0	40.0	48.0	56
30	3.320	3.830	8.6	12.9	17.1	21.4	25.7	34.3	38.6	42.9	51.4	60
32	3.402	3.866	9.1	13.7	18.3	22.9	27.4	36.6	41.1	45.7	54.9	64
34	3.510	3.895	9.7	14.6	19.4	24.3	29.1	38.9	43.7	48.6	58.3	68
36	3.580	3.918	10.3	15.4	20.6	25.7	30.9	41.1	46.3	51.4	61.7	72
38	3.617	3.937	10.9	16.3	21.7	27.1	32.6	43.7	48.9	54.3	65.1	76
40	3.645	3.954	11.4	17.1	22.9	28.6	34.3	45.7	51.4	57.1	68.6	80
45	3.718	3.992										
50	3.775	4.022										
55	3.815	4.046										
60	3.846	4.067										
65	3.875	4.085										
70	3.887	4.102										
80	3.890	4.134										
90	3.890	4.167										
100	3.890	4.200										

The phase-velocity curve thus found is assumed to represent a crustal thickness H of 35 km, which agrees with other observations of crustal thickness in Africa (Båth 1961). Phase-velocity curves for other values of H are constructed in the velocity interval $3.25 < c < 4.0$ km/sec by setting T proportional to H (Ewing, Jardetzky and Press 1957, chapter 4). The phase-velocity curves thus found are given in Table VII and Fig. 12.

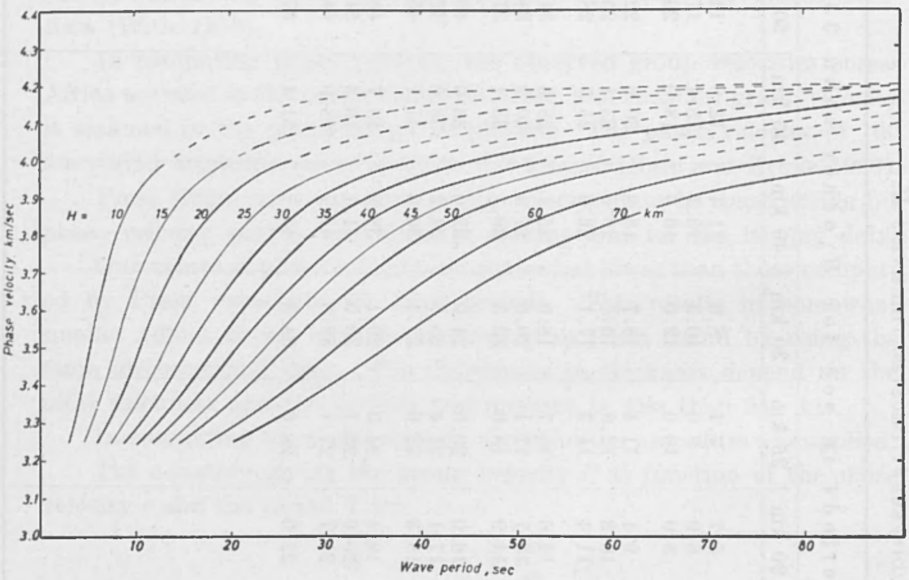

Fig. 12. – Computed phase-velocity curves for different crustal thickness.

Result.

The main result of this investigation is presented in Tables VIII and IX and Fig. 14. Fig. 13 shows one phase-velocity diagram for computing crustal thickness.

There is rather good agreement between the thickness values obtained from the two earthquakes, but the scatter of the individual observations is larger for the Mexico earthquake. This may be caused by lower reading accuracy of the records of the Mexico earthquake. The values given for the Kurile Islands earthquake are computed from waves $N = 5\text{-}14$ (see Table III), as serious interference causes great scatter in results obtained from later waves.

Table VIII. – CRUSTAL THICKNESS AS COMPUTED FROM PHASE VELOCITY OF RAYLEIGH WAVES BETWEEN TWO STATIONS, TAKING ACCOUNT OF DIRECTION AND CURVATURE OF THE WAVE FRONTS. (H = crustal thickness [km]; Sd = standard deviation of single observations; n = number of observations [waves]; δH = error in H for 1 sec error in time difference).

Stations	Mexico earthquake 27 July 1957				Kurile Isl. earthquake 6 Nov 1958			
	H	Sd	n	δH	H	Sd	n	δH
Gb – Hel					36.3	1.9	10	3.5
Gb – Sk					36.5	2.0	10	3.7
Gb – Sod					34.6	0.6	10	0.9
Gb – Up					40.8	2.1	10	3.4
Hel – Ki	39.0	5.8	20	2.8	38.5	5.9	10	8.2
Hel – Kob	34.4	6.9	20	5.2	35.9	1.1	10	2.2
Hel – Sk	34.7	4.8	20	1.1				
Hel – Sod	35.8	5.0	20	5.0	33.2	1.3	10	4.5
Hel – Up	34.4	14.7	13	4.0	28.4	3.0	10	6.1
Ki – Sk				(25.9)	38.6	6.5	10	1.4
Ki – Sod	(53.2)	(9.7)	20	4.4				(13.5)
Ki – Up	39.0	7.0	13	6.8	38.4	4.5	10	3.1
Kob – Sk	34.0	6.5	20	8.4	33.9	1.8	10	3.7
Kob – Up					36.0	1.7	10	1.3
Sk – Sod	34.9	4.9	20	6.9	33.1	2.1	10	2.5
Sk – Up	31.5	3.5	13	3.2				
Sod – Up					34.2	0.8	10	1.8

The column headed δH in Table VIII gives approximately the error in H produced by ½ sec error in opposite sense at the two stations in the first column, in addition to ½ sec error at the stations, used to compute the direction of wave propagation.

Table IX. – CRUSTAL THICKNESS AS COMPUTED FROM PHASE VELOCITY ACROSS STATION TRIANGLES, ASSUMING PLANE WAVE FRONT (For explanation see Table VIII).

Stations	July 28, 1957 - Mexico			Nov. 6, 1958 - Kurile Isl.		
	H	*Sd*	*n*	*H*	*Sd*	*n*
Gb – Kob – Up				36.7	2.0	10
Gb – Sk – Up				39.1	1.4	10
Hel – Ki – Sk				38.7	5.9	11
Hel – Ki – Sod				26.2	4.3	11
Hel – Ki – Up	48.2	10.4	14	37.2	2.0	11
Hel – Kob – Sk	34.6	6.6	20	35.6	1.5	10
Hel – Kob – Up	(17.2)	(13.2)	9	35.7	1.3	10
Hel – Sk – Sod	35.0	4.7	20	33.5	3.4	11
Hel – Sk – Up	36.6	4.5	14			
Hel – Sod – Up	(52.6)	(14.0)	12	33.9	1.6	11
Ki – Sk – Up	(6.6)	(?)	9	39.7	6.1	11
Ki – Sod – Up	31.8	17.2	13	30.7	1.6	11
Lu – Sk – Up	34.6	5.5	8			

The values of Table VIII, where account is taken of the curvature of the wave front, are clearly much more reliable than those of Table IX, assuming plane wave fronts, especially for the Mexico earthquake.

DISCUSSION.

The phase-velocity method here applied to Fennoscandia in order to determine the crustal thickness, has previously been used with good result in the United States (Press 1956, Press 1957, Ewing and Press

1959, Oliver et al 1961). In Fennoscandia the circumstances in respect to wave paths are rather different from North America, where earthquakes in the South Pacific area produced the surface waves and a homogeneous oceanic path produced the dispersion. This has led to some deviations from the method as used in North America. We had to account for the curvature of the wave front, produced by the heterogeneous wave path. This heterogeneity of the wave paths made it impossible to use waves of period smaller than about 25 sec due to great

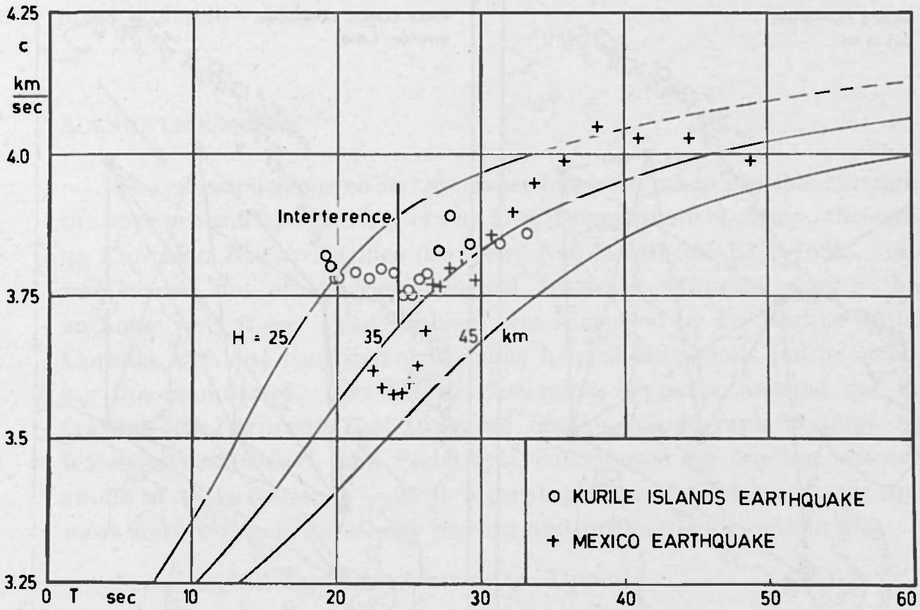

Fig. 13. – Phase velocity (*c*) along the line Skalstugan-Sodankylä, taking account of direction and curvature of the wave fronts. *T* is the wave period. Dispersion curves of Fig. 12 are shown.

irregularities in the propagation of these waves. In addition, the large distances between the stations in our case contributed to the difficulties in identifying the waves.

The result given in Table VIII shows nearly constant thickness, about 35 km, for the whole area. There are indications of a slight increase in the thickness to the west and north to 35-38 km as a mean for northern Sweden against 33-35 km in Finland and southern Sweden.

As the crustal thickness is obtained from phase-velocity curves computed from group velocities measured across Africa, the result must depend on the correctness of the assumptions involved. These are:

1. – Crustal thickness of Africa is 35 km.

2. – The wave velocity and relative thickness of each layer is the same for Fennoscandia as for Africa.

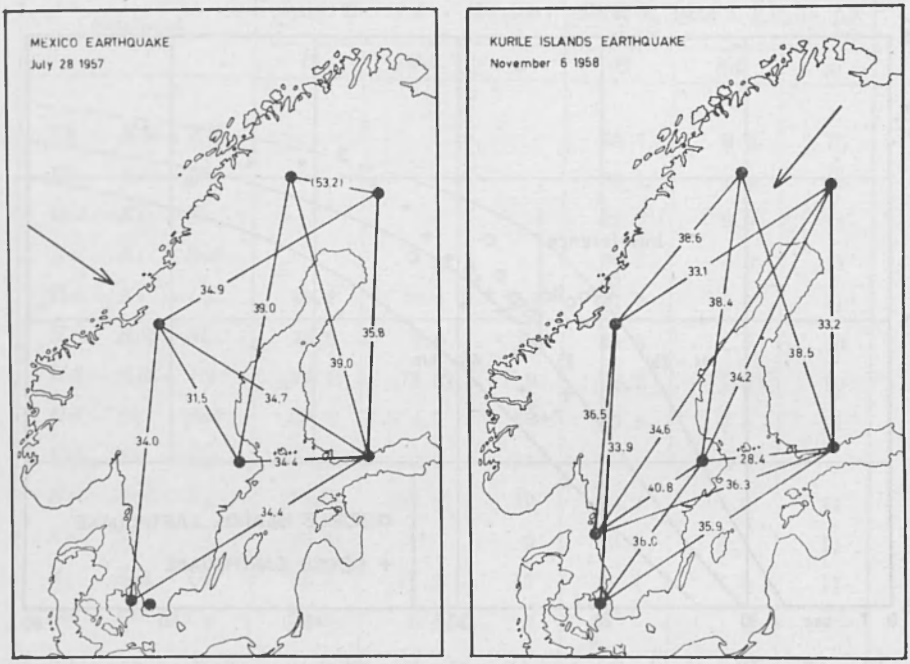

Fig. 14. – Crustal thickness in km as computed from phase velocity of Rayleigh waves between two stations, taking account of direction and curvature of the wave fronts.

3. – Group velocity of Rayleigh waves of period 80-100 sec is 3.890 km/sec across Africa.

4. – Phase velocity of Rayleigh waves of 100 sec period is 4.2 km/sec.

If the result here obtained is compared with results given by other authors, the agreement is usually good.

Porkka (1960) gives $H = 34$ km in northern Fennoscandia by using P and S waves from near earthquakes.

Penttilä et al (1960) found from explosion seismic methods, $H = 26.4 - 29.3$ km decreasing westwards in south Finland. They give the thickness of the granitic layer as $18.4 - 21.3$ km.

Wideland (1954) has estimated the crustal thickness in Sweden from gravity data. His values as read from a diagram are: $H = 23$ km is southern Sweden, 36 km in central Sweden and 52 km in northern Sweden. As a mean for the whole country he gives $H = 30$ km as the most probable value.

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SUMMARY

The phase-velocity method for determination of crustal thickness is here applied to Fennoscandia. Two earthquakes were selected for the study, one in Mexico with wave propagation perpendicular to the west coast of Norway, another in the Kurile Islands with wave propagation parallel to the Norwegian coast. Because of heterogeneous wave paths the wave fronts are deformed. The actual curvature of the wave fronts was determined and taken into account in the velocity determination. The direction of the wave fronts, as determined from the arrival times at different stations, was compared with horizontal particle movements of the same waves at Kirnua *and Uppsala, and considerable deviation was found. New phase-velocity curves were calculated on base of observed group velocities across Africa. The crustal thickness in Fennoscandia was found to be nearly constant inside the area, about 35 km.*

RIASSUNTO

In questa nota viene applicato alla Fennoscandia, il metodo basato sulla velocità di fase, per la determinazione dello spessore della crosta.

Per tale studio, e allo scopo di ovviare alla deformazione dei fronti d'onda causata da tragitti eterogenei, sono stati scelti due terremoti, uno avvenuto nel Messico, con direzione di propagazione normale alla costa della Norvegia, l'altro avvenuto nelle Isole Curili, con direzione di propagazione parallela alla costa stessa. È stata ricavata e tenuta in debito conto, nella determinazione della velocità, la curvatura dei fronti d'onda. La direzione di questi ultimi, quale è stata determinata dai tempi di arrivo nelle diverse stazioni, è stata confrontata con i movimenti orizzontali delle particelle interessate dalle stesse onde, a Kiruna e Uppsala; è stata trovata una notevole divergenza. Le nuove curve della velocità di fase, sono state calcolate sulla base delle velocità di gruppo, osservate attraverso l'Africa. Per lo spessore della crosta in Fennoscandia, entro i limiti dell'area considerata, è stato trovato un valore quasi costante di circa 35 km.

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