The rheology of the Earth in the long time range

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SUMMARY. — The available evidence bearing upon the rheological behavior of the tectonically significant layers of the Earth (" tectonosphere ") in the long (> 1000 years) time range is analyzed. The phenomena from which data were gleaned are isostatic uplift and sinking, the nonhydrostatic bulge, the theoretical behavior of rocks at high temperature and pressure, and the stability of mountains. The main result of the study is that the commonly assumed viscous model for the rheological behavior of the tectonosphere does, in fact, not fit the evidence; the latter indicates *logarithmic*, not viscous creep. In addition, there is probably a low strength threshold of 10^6 dynes/cm^2 present. This rheological behavior is very similar to that of the tectonosphere in the intermediate time range.

RIASSUNTO. — Vengono analizzate le prove disponibili nel campo del comportamento reologico degli strati tettonicamente significativi della Terra (" tectonosphere "), riferite a lunghe scadenze, (oltre i mille anni). I fenomeni dai quali sono stati prelevati dei dati sono i sollevamenti e gli abbassamenti isostatici, il gonfiamento non idrostatico, il comportamento teorico delle rocce ad alte temperature e pressioni, e la stabilità delle montagne. Il risultato principale cui giunge lo studio è che il modello viscoso, normalmente adottato per il comportamento reologico della tectonosfera, non collima con le prove le quali indicano scorrimenti *logaritmici*, non viscosi. Probabilmente, vi è presente anche una soglia di forza minima di 10^6 dine/cm^2. Questo comportamento reologico rassomiglia molto a quello manifestato dalla tectonosfera negli intervalli di tempo medi.

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1. – Introduction.

It is well known that the time-scale of geological phenomena reaches from fractions of a second to hundreds of millions of years. Regarding the rheological properties of the tectonically significant layers (crust and upper mantle; "tectonosphere"), it was proposed by the writer [38] to split this vast time range into three, termed "short" (up to 4 hours), intermediate (4 hours to 15,000 years) and long (longer than 15,000 years). In each of these time ranges, the rheological response of the Earth to applied stresses appeared to be very different. The then available evidence on this rheological response was reviewed by this writer some fourteen years ago [38].

Recently, the writer [41] has begun to re-evaluate the available evidence bearing upon the rheology of the Earth, starting with the "intermediate" time range. The present paper continues this re-evaluation, concerning itself with the "long" time range.

The largest body of data bearing upon the rheological behavior of the tectonosphere is provided by secular vertical motions of the Earth's crust. The data on such vertical motions have recently been collected by Gopwani and Scheidegger [21]. The most important of these data are connected with postglacial isostatic recovery. The end of the last ice age is now usually put at around 11,000 years ago, but it culminated about 18,000 years ago [19]. Thus, according to our terminology, glacial recovery would barely enter into the "long" time range. However, the natural division point between "long" and "intermediate" time ranges should probably be put closer to 1000 years, inasmuch as there are typical "intermediate" phenomena whose characteristic times are of the order of days and years (Chandler wobble, seismic aftershock sequences, etc.), and then there is a time-range gap until one comes to the "secular" phenomena such as slow tectonic motions. The "natural" (rather than logarithmic) division of the geological time ranges would therefore be: Short, up to 4 hours; intermediate, 4 hours to 1000 years; long, longer than 1000 years. We shall adhere to this terminology in the present paper.

In addition to isostatic glacial recovery, there may be loading-unloading features of origin other than glacial, such as the loading of land by water when a lake is formed or by the positioning of guyots. The nonhydrostatic bulge of the Earth, commonly believed to be due to a delay in adjustment of the shape of the Earth to the changing rate of rotation, represents a load on a rather large scale.

Attempts have also been made to deduce the long-term rheology of the tectonosphere from other than vertical motion data. Thus, the recently postulated plate-motion hypothesis of crustal blocks has been used for this purpose. However, it seems too early to deduce reliably the mechanical properties of the Earth from such a hypothesis.

A further attempt is due to a theoretical discussion of the rheology of rocks at high temperatures and pressures in the long time range, such as might prevail in the tectonosphere, based upon solid state physics. Such attempts are tied up with estimates of the temperature and pressure in the Earth's interior, which introduces a measures of uncertainty.

The lay-out of the paper will be that all the possible rheological models suggested for the long-term behavior of the tectonosphere will be discussed in individual chapters; in each of these chapters the diverse available observational material will be presented. Since the latter is mostly based upon observations of vertical land-motions, an initial section will deal with the time-analysis of such motions, before the above-mentioned rheological models are studied.

It will be seen that the evidence is overwhelming that some type of nonlinear (such as logarithmic) creep is the dominant phenomenon in the tectonosphere. The generally proposed viscous behavior is obviated by the shape of the postglacial rebound curves.

2. – Empirical data.

As noted, most of our knowledge of the long-term rheology comes from measurements of the contemporary and recent rates of vertical motions. Herein, one understands under "contemporary" the rates observed over the last 60 years or so (usually by making repeated precise levellings), and under "recent" the rates deduced from geomorphical changes over the last 100-500 years or so. A world-wide picture of the presently known rates has been recently presented by Gopwani and Scheidegger (21).

An interesting extension of the study of present motions is the study of past vertical motions. It is, of course, very difficult to do this on a geological scale, but attempts to construct uplift rate-curves for the times since the end of the last ice-age have been reported.

These are based on radiocarbon datings of strandlines; corrections have to be made for the eustatic changes of the sea-level.

Thus, Schofield ([43]) presented uplift curves for the Fennoscandian and Canadian Shields, shown here in Fig. 1.

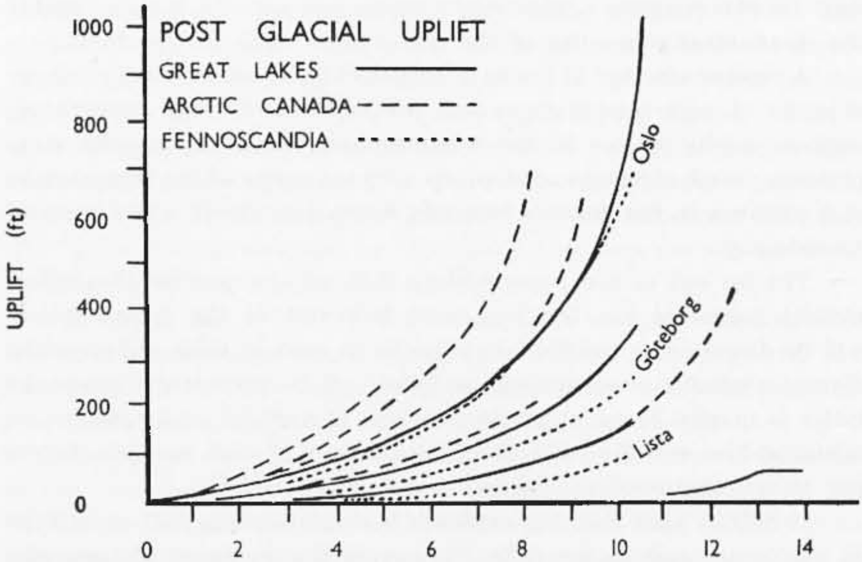

Fig. 1 – Postglacial uplift curves; after Schofield ([43]).

Much of the recent evidence on uplift curves has been summarized by Andrews ([3,4]). Of particular interest is the form of these uplift curves. Thus, empirically, several equations have been postulated to describe uplift. The first possibility is an exponential " decay ".

$$W = C\, e^{-kt} \qquad [1]$$

where W is the " remaining uplift ", that is the amount of uplift that has to be accomplished at time t after deglaciation to attain complete isostatic equilibrium, and k a " decay constant " for which Andrews ([3]) gives values between 0.34 and 0.56 (mean 0.45) if W is measured in meters and t in millennia. The constant C represents the total uplift at a site from the time of deglaciation. Writing equation [1] in terms of uplift accomplished, U, one obtains

$$U = C - W = C\,(1-e^{-kt})\ . \qquad [2]$$

Andrews ([3]) noted that the uplift equation [1] does not necessarily fit the observational data all too well; there is a marked decrease in k as time proceeds. In fact, this indicates that the exponential form of [1] is perhaps a poor approximation of the observed facts. Thus, Walcott ([47]) suggested that the uplift might in actuality be given by the sum of two exponentials and postulated

$$W = C_1\, e^{-k_1 t} + C_2\, e^{-k_2 t}\ . \qquad [3]$$

For Canadian conditions, Walcott found

$$C_1 = 150 \text{ m}$$

$$C_2 = 450 \text{ m}$$

$$k_1 = 1 \text{ millennium}^{-1}$$

$$k_2 = 0.02 \text{ millenia}^{-1}$$

Because of the poor fit of the uplift data with Eq. [1], Andrews ([3]) abandoned the hypothesis of an exponential decay altogether and suggested a logarithmic equation instead:

$$U = a + r \ln t\ . \qquad [4]$$

In terms of rates, Eq. [4] yields upon differentiation

$$\frac{dU}{dt} = \dot{U} = \frac{r}{t} \qquad [5[$$

Andrews ([3]) found a much better agreement of equation [4] with natural data than of equation [1]. He ([4]) made later a specific comparison of the two equations for 58 uplift curves and found that the exponential dependence gives consistently present-day uplift rates that are too low in comparison with observations, whereas the logarithmic dependence gives correct values.

Equation [4], being logarithmic, is evidently not valid for $t = 0$. Thus, a cutoff must be applied near $t = 0$. This can easily be done by modifying [4] to read

$$U = a + r \ln (t + 1)\ . \qquad [6]$$

Regarding the constants r and a in equation [4], Andrews (4) found some interesting relationships. From a comparison of 21 uplift rates he deduced by a least squares procedure

$$-a = 5.6\,r + 10 \text{ meters}$$

(U in meters, t in years), and thus, using [4], and setting $t = 1000$, one finds

$$r = \frac{A + 10}{1.3}$$

where A is the uplift in meters that occurred during the first 1000 years of deglaciation. A direct correlation analysis of observational data gives a similar result, viz.

$$r = \frac{A - 9.8}{1.02} \; .$$

In approximation, r is thus simply equal to the uplift A that occurred during the first thousand years.

Thus, there is a strong indication that the form [4] is the best-fitting representation of the observational evidence, and that the exponential from [1] is actually contradicted by the data.

In addition to the above exponential and logarithmic dependences, a simple power law has also been suggested

$$U = at^{\beta} \; . \qquad [7]$$

In this case, one finds from a statistical evaluation of observational material (4)

$$\beta \sim 0.5 \; .$$

Any discussion of the rheological response of the tectonosphere to isostatic stresses will have to account for the above empirical evidence.

3. – The available rheological models.

If the data mentioned in the Introduction are to be interpreted in terms of the rheology of the tectonosphere, it is necessary to set up hypothetical mechanical *models* of the phenomena in question.

Naturally, these models are built up from basic rheological elements, embodying features of viscous, elastic or ideally plastic materials. In addition, one might think of the possibility that logarithmic creep, phase change, or a breaking threshold might occur. The possible rheological equations describing such phenomena were discussed recently by the writer ([41]) in connection with the intermediate-time rheology of the Earth. The basic physical possibilities, of course, are the same in all time ranges.

The specific models that can be constructed for the interpretation of the long-term rheology of the tectonosphere will be discussed one by one below.

1. – Viscous model.

The most common view of the rheological properties of the "tectonosphere" in the "long" time range is that the latter exhibits the behavior of a viscoelastic (Maxwell) material. Not only has this been postulated with regard to the interpretation of postglacial rebound, but also with regard to the explanation of global plate movements ([27]), polar wandering [([35]) pag. 265], and the nonhydrostatic bulge of the Earth ([33]).

The customary way of deducing viscosity values for the model-viscous material representing the tectonosphere is based on isostatic uplift and sinking observations. In general terms, this can be done as follows.

Let a block of material (continent, crust) float upon a viscous fluid (substratum). If the weight of the block is exactly equalled by the weight of the displaced fluid (Archimedes' principle), we have an equilibrium situation. If in such an equilibrium situation, material is taken away (melting ice) or added (filling of a reservoir with water) on top, the equilibrium is destroyed and a new equilibrium situation must be found. If we denote the vertical deviation from the equilibrium situation by x, then the restoring force F is proportional to x

$$F = -kx$$

where k is some constant.

The resistance R to restoration, in a viscous material, is proportional to the velocity $\dot{x}$ and the viscosity η, viz.

$$R = - c\eta\dot{x}$$

where c is a constant, so that the equation of motion reads

$$m\ddot{x} = - kx - c\eta\dot{x}$$

where m is the mass of the block.

If one assumes that the restoring motion is highly damped, so that the acceleration term can be neglected, the solution of the above differential equation is

$$x = x_o \, e^{-K/\eta t} \qquad [8]$$

where $K = k/c$. This shows that the equilibrium position will be reached by an exponential time function.

The connection between the viscosity and the "relaxation time" of the uplift (or sinking, as the case may be) data depends on the specific model that is adopted. One of the most rigorous discussions of this problem has been given by Haskell [23, 24] assuming a homogeneous "liquid" in which a piece of crust is rising or sinking. Applying the general Haskell theory to a cylindrical crustal block (radius $1/b$), one obtains for the viscosity η

$$\eta = \frac{\varrho \, g \, T_r}{2 \, b \sqrt{\pi}} \qquad [9a]$$

where T_r is the relaxation time of the exponential in [8], ϱ the density of the fluid and g the gravity acceleration [cf. e.g. Scheidegger [39], p. 349]. If a rectangular block is used instead (sides l, m), one obtains [cf. e.g. Heiskanen & Vening Meinesz [25], p. 364]

$$\eta = \frac{\varrho \, g \, T_r}{2 \, \pi \, f} \qquad [9b]$$

where

$$\frac{1}{f} = \frac{l \, m}{(l^2 + m^2)^{1/2}} \, .$$

In either case, if one applies the above formulas to the uplift of Fennoscandia ($T_r = 5000$ years; $1/b = 750$ km), one obtains viscosities of the order of

$$\eta \sim 10^{22} \text{ Poise} \, .$$

A similar application of the above formulas to the observed rise of the land around Pleistocene Lake Bonneville in Utah ([14,15]) yields a viscosity of the order of

$$\eta = 10^{21} \text{ Poise}$$

i.e. an order of magnitude less than from the postglacial rise of Fennoscandia.

The submergence of guyots ([37,40]) can be considered as the exact opposite of the postglacial rise of land; they are assumed to be volcanic eruptions which sink subsequently until isostatic equilibrium is reached. Depending on the sinking rates assumed, one obtains for the order of magnitude of the viscosity

$$\eta \sim 10^{22} \text{ to } 10^{23} \text{ Poise}$$

with the lower value from Saito ([37]), the higher one from Scheidegger [([40]) p. 345].

A similar analysis of rebound processes in linear and terrestrial meteor craters by Daneš ([17]) yielded the result that the viscosity η would have to be even much higher.

One thus sees that the lowest viscosity value is required by features of the size of Lake Bonneville. For both, smaller (craters, guyots) and larger (Fennoscandia) features, the required viscosity increases.

However, it should be noted that the model upon which the above viscosity calculations are based, envisages a *homogeneous* viscous body for the pertinent tectonically active layers of the Earth. This is evidently an oversimplification.

It may be expected that a *layered* viscosity-model may exhibit features which may explain the observed viscosity values as a function of the size ("wavelength") of the isostatically rebounding features ([31,44]).

The simplest layer-assumption is the hypothesis that just below the crust, there exists a low-viscosity layer. Thus, Takeuchi and Hasegawa ([45]) made some calculations of isostatic rebound in a viscous layer of finite thickness and showed that the discrepancy in viscosity values arising from taking a homogeneous, infinitely downward extending viscous substance, as a model, can thereby indeed be reconciled. The idea of layers with different viscosities was also elaborated by Artyushkov ([5,6]) and by Anderson and O'Connell ([1]).

Calculations of the result in isostatic adjustment assuming a variable viscosity in the tectonosphere have also been made by Danes([18]), but his dependences (exponential and linear) with depth are too simple to have much practical significance.

The layered-viscous model of isostatic rebound can be made even more realistic by assuming a thin elastic layer on top. The effects of some such models were calculated by McConnell ([32]) who arrived at postulating a viscosity distribution in the Earth as shown here in Figure 2.

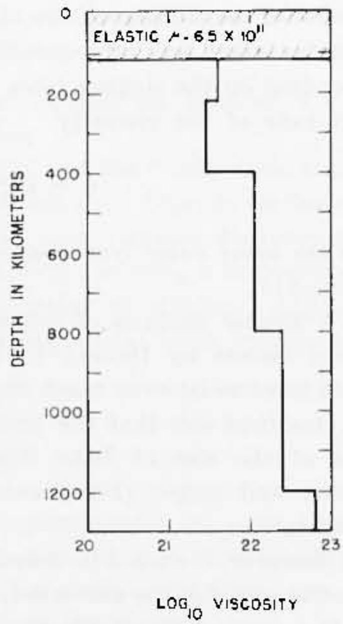

Fig. 2 – Viscosity distribution in the Earth obtained by assuming a viscous model of the tectonosphere. After McConnell ([32]).

Attempts have been made to correlate the crustal motions off the sides of uplifting or sinking blocks, based on viscosity models. Thus, Broecker ([8]) computed the deformation of the shoreline of proglacial lakes caused by the curvature of the uplifted land-portion. Alternatively, Krass and Ushakov ([29]) calculated the free surface of a viscous fluid near a subsiding solid material. It should be noted, however, that such calculations may yield a qualitative agreement with observed phenomena, but do not help much with regard to an elucidation of the rheological behavior of the tectonosphere.

The data obtained from an analysis of the non-hydrostatic bulge of the Earth seem to confirm the viscosity distribution with depth, obtained above. According to Anderson and O'Connell ([1]) the non-equilibrium bulge is due to the changing rate of rotation which requires a mantle viscosity of 10^{26} Poise. Similarly high values were obtained by McKenzie ([34]). These values can be explained by noting that the non-hydrostatic bulge of the Earth is a feature which is of even much larger extent than the ones previously discussed, thus being affected by the viscosity of the *deepest* layers of the mantle. The general course of the viscosity-depth curve is thus confirmed.

The postulate of a rheologically viscous behavior of the " tectonosphere " has significance with regard to the possibility of existence of such phenomena as global plate movements ([27]), convection currents ([31]) and polar wandering [([35]) p. 265 ff.]. In turn, viscosity values can be deduced for the tectonosphere from a study of these phenomena. Thus, Cramer ([13]) obtained, on this basis, viscosity values that were close to those calculated from postglacial uplift. However, the very existence of these phenomena is too hypothetical to get viscosity values that would be at all reliable.

Attempts have been made to justify the assumption of a viscous rheological behavior of the tectonosphere not only from a phenomenological-observational standpoint, but also from considerations of solid-state theory. Accordingly, it is usually assumed that the viscosity is due to diffusion creep which appears macroscopically as viscous behavior. The viscosity η is then estimated from the Nabarro ([36])-Herring ([26]) equation

$$\eta = A \frac{kTl^2}{a^3 D}; \quad \text{with } D = D_o e^{-E/RT} \qquad [10]$$

where A is a constant (about 1/30), k is Boltzmann's constant, T the absolute temperature, a an atomic lattice dimension, l the grain size, D the coefficient of self-diffusion, E the activation energy for self-diffusion and R the gas constant. Upon the above theory, estimates of the viscosity in the mantle using reasonable values for the parameters, have been made by Zharkov ([50]), Gordon ([22]), and McKenzie ([34]). Zharkov ([50]) found an incredibly low viscosity (10^{14} — 10^{17} Poise); the main feature of all calculations is an increase of viscosity with depth, confirming qualitatively the findings from uplift data.

In conclusion of our discussion of the viscous model for the rheology of the tectonosphere, one can thus say that this model qualitatively fits some of the observed data, if a low-viscosity layer is assumed in the upper mantle.

However, it should be noted that viscosity always yields exponential relaxation patterns; these do not seem to be borne out by the uplift curves discussed in Sec. 3. Furthermore, there are theoretical as the mantle is assumed to consist of) is not viscous ([10]). Thus, the treatment of the rheological behavior as " viscous " is probably not justified. The reason that partly satisfactory results were obtained by its use lies in the fact that many (non-linear!) types of relaxation can be approximated by a " pseudo "-viscosity, using some fictitious value for η.

5. – Elastic model.

The theory of isostasy does seem to account for many of the observed vertical displacements on the Earth's crust. However, it may be that the final equilibrium configuration is not entirely determined by buoyancy forces, but that there is a residual effect due to elastic bending of the Earth's crust. Thus, the final equilibrium deformation of the Earth's crust under a surface load may be considered as the equilibrium configuration of a loaded elastic plate on top of a viscous fluid below. The "tectonosphere" would thus consist of the elastic crustal plate plus the fluid below to a certain depth.

This model has been used to calculate hypothetical configurations of the Earth's crust under various types of loads for a long time. A recent summary of work of this type has been given, *e.g.* by Walcott ([48]). Tables of the deformation of an Earth model as discussed here under surface loads have been given by Caputo ([11]).

For an elucidation of the rheology of the "tectonosphere", the reverse problem is of interest: What are the elastic constants that can be inferred from an analysis of deformations?

This problem has been attacked by Walcott ([48]). Accordingly, the quantity that can be determined is the flexural rigidity D

$$D = \frac{E\,T^3}{12\,(1 - \sigma^2)} \qquad [11]$$

where E is Young's modulus, T the thickness and σ Poisson's ratio of the plate. Beyond the edge of an areal load, the elastostatic equations for a horizontal elastic plate have solutions containing terms of the type $e^{-x/a}\cos(x/a)$ which indicate a damped harmonic wave with wavelength $\lambda = 2\pi a$ with

$$a^4 = \frac{4D}{\varrho_f\, g} \qquad [12]$$

where ϱ_f is the density of the fluid below the elastic plate and g the gravity acceleration. The quantity a is called " flexural parameter "; it has the dimension of a length.

Using appropriate solutions of the elastostatic equations, it is possible to obtain values for D by a comparison with observations. Walcott ([48]) obtained values varying from 5×10^{22} to 9×10^{24} Newton-metres (1 Newton-metre $= 10^7$ dyne-cm). This is evidently quite a range; the individual values do not correlate well with crustal thickness, etc. One must therefore conclude, as admitted also by Walcott ([48]), that the elastic model is too simple and that relaxation effects corresponding to some other rheological behavior of the *whole* tectonosphere are of paramount importance.

Elastic models of the tectonosphere of an even more extreme kind have also been proposed: The whole tectonosphere has been taken as purely elastic ([20]). Usually, the short-term values for the rigidity and Poisson's ratio are taken in this case.

Because of the proven relaxational character of uplift and sinking phenomena, any purely or even mainly elastic model of the long-term rheological behavior of the tectonosphere would seem to be inappropriate.

6. – Kelvin-type models.

The elastic models discussed in the last section can be improved somewhat if an elastic aftereffect is introduced to account for the observed relaxation phenomena. If this idea is applied to the interpretation of glacial rebound data ([38]) the mental picture is that elastic afterworking (not delayed buoyancy) supplies the gradual readjustment. Of course, the attainment of equilibrium would again be in an exponential form, with the same relaxation time (probably 5000 years) as that for the viscous model. This, again, does not agree with the shape of the uplift curves.

There is no confirmation of a Kelvin-type model from solid state theory either, inasmuch as materials at high temperatures and pressures tend to approximate a viscous or creep-type behavior rather than an elastic one.

7. – Plastic creep model.

Slow motions seem to be a characteristic feature of the long-term rheological behavior of the " tectonosphere ". In this instance, it would appear that some sort of viscous model would describe the observed facts most appropriately. However, as noted on various previous instances in this paper, objections against viscous models have been raised on various occasions. Thus, in order to retain the slow-motion aspect of viscous models without having to be tied to linear viscosity, one might think of some other form of " creep ".

Most of the arguments *against* viscous and *for* some other form of creep adduced heretofore in the literature are based on considerations of the molecular behavior of polycrystalline solids, of which the lower crust and upper mantle are presumably composed, under the pressure and temperature conditions that are assumed to prevail in the "tectonosphere ".

A recent summary of these considerations has been given by Weertman ([49]) who presented the theoretical results in the form of calculations of " effective " viscosities with depth: For a given strain rate, it is of course obvious that even a non linear rheologial equation can be characterized by an " effective ' (or " pseudo "-) viscosity. These attempts at deducing the rheological behavior of the tectonosphere from microscopic considerations are very pertinent, but they are unfortunately affected by all the uncertainties in our knowledge of the composition and of the conditions in the Earth's interior. More than qualitative indications of the rheological behavior of the Earth's interior can therefore not be expected from them.

Much more hopeful is the direct observation of the response of the " tectonosphere " to changes of stress, and in this instance, Schofield's ([43]) postglacial uplift curves (see Fig. 1) provide an important clue.

We have noted in Sec. 2 of this paper that the isostatic rebound is not exponential, but logarithmic (cf. Eq. 4). This can be explained by introducing a rheological model which exhibits logarithmic creep recovery, rather than exponential creep recovery. This type of behavior has been found in many materials long ago by Andrade ([2]); it is common for the laboratory response of rocks to stress-changes [42] and has recently also been postulated by the writer [41] as the rheological behavior of the tectonosphere in the *intermediate* time range. It thus appears that this type of behavior is also characteristic of the Earth in the *long* time range.

Thus, it is clear that a logarithmically creeping material could satisfactorily describe the glacial rebound curves. Such a material has the following strain (ε) – time (t) relation

$$\varepsilon = \frac{2\eta}{\beta} \ln (a + bt) \qquad [13]$$

where η is a viscosity, β a creep factor. The quantities a and b are constant. In terms of stress σ and strain ε, the above rheological equation [42] has the following form:

$$\dot{\sigma} = 2\eta\ddot{\varepsilon} + \beta\dot{\varepsilon}^2 . \qquad [14]$$

Unfortunately, it is not possible in a simple fashion to deduce the values of η and β in the basic rheological equation from data obtained from postglacial uplift curves at this time. It does appear, however, that a logarithmic creep model is the only one that does not contradict the uplift data.

8. – The long-term strength of the tectonosphere.

An even cursory inspection of the Earth indicates that creep behavior cannot possibly be the whole character of the long-term behavior of the "tectonosphere". The fact that mountains, the non-hydrostatic bulge and gravity anomalies exist, indicates that there may be a strength-threshold. Of course, it may be that some or all of these features are the expression of a dynamic steady state rather than of a static equilibrium; in that case the strength of the Earth might in fact be very low. Nevertheless, values for the long-term strength of the tectonosphere have been deduced from all of these features.

Turning first to the information that can be deduced from mountain ranges, we note that from simple stability considerations one obtains for the shearing strength

$$\theta = 4 \times 10^8 \text{ dynes/cm}^2 .$$

This comes out of the well-known stability formula of Terzaghi ([45])

$$H = \frac{\theta}{\varrho g} N \qquad [15]$$

for a mount of height H consisting of a material of density ϱ. N is a stability factor depending on the slope angle. For a slope angle of about 45°, one has $N = 6$. Choosing for the other variables $h = 8$ km (roughly the height of Mt. Everest), $\varrho = 3$ g/cm³, $g = 980$/sec², yields the above quoted value for θ. This deduction assumes, of course, that mountains are static and not dynamic-steady state phenomena.

Similar values for θ are obtained if the non-hydrostatic bulge of the Earth is interpreted as a static (and not as a lag-exhibiting transient creep) phenomenon. Munk and McDonald [([35]), p. 281] obtain

$$\theta = 10^8 \text{ dynes/cm}^2 .$$

The same order of magnitude, viz.

$$\theta = 3 \times 10^8 \text{ dynes/cm}^2$$

was obtained by Jeffreys [([28]), p. 209 ff.] for the tectonosphere below 50 km depth (up to $\theta \sim 1.5 \times 10^9$ for the region above 50 km depth) from an analysis of the effect of gravity anomalies of large horizontal extent.

Again a similar value ($\theta < 2 \times 10^8$ dynes/cm²) has an upper limit of the shear strength and was obtained by Brune et al ([9]) by noting the absence of a heat flow anomaly greater than about 0.3 cal sec⁻¹ cm⁻² on the San Andreas Fault.

Perusing data on the gravity field of the Earth that have been obtained from satellite measurements, Caputo ([11]) arrived at an improved strength value on this basis of

$$\theta \sim 3 \times 10^7 \text{ dynes/cm}^2 .$$

The consideration of rotational features of the Earth, other than the non hydrostatic bulge also leads to strength values [([35]) p. 280]. Thus, an Earth without strength would be completely unstable with regard to polar wandering. The fact that the present north pole does not move towards the pole of the continent-ocean system, if assumed due to strength properties, leads to a lower limit of θ, viz.

$$\theta > 10^7 \text{ dynes/cm}^2 .$$

Similar strength values were obtained by Chinnery [12] when estimating the stresses that are released in an earthquake, assuming that the latter corresponds roughly to the strength-threshold of the material. Generally, he finds

$$\theta \sim 10^7 \text{ dynes/cm}^2 .$$

The lowest strength values are indicated from isostatic rebound data. The mere fact that rebound *does* seem to occur, puts an upper limit on the strength of the "tectonosphere". Thus, Crittenden [16] finds maximally

$$\theta \sim 10^6 \text{ dynes/cm}^2 .$$

This low value comes from the observation that the Earth yields to ice surface loads that can be calculated.

In summary, one finds that values of the shearing strength θ for the "tectonosphere" have been estimated as between 10^6 to 10^8 dynes/cm². A reconciliation between the various values can only be achieved if some of the theoretical models, upon which the calculations were based, are rejected. Thus it is quite conceivable that neither mountains nor the bulge of the Earth are static equilibrium features, but rather dynamic steady-state or even transient features that are dynamically supported.

9. – Phase transition models.

The uplift data mentioned in the earlier part of this paper have also been interpreted in terms of an entirely different type of "rheology". Thus, Broecker [7] estimated the contribution to land-rebound that might be caused by load-induced phase changes in the material of the "tectonosphere". He found that the contribution of phase transitions would probably not be the dominant mechanism in glacial rebound. On the other hand, Magnitsky and Kalashnikova [30] claim that the hypothesis of a "phase-transitional" origin of vertical movements of the Earth's crust is justified.

Unfortunately, calculations of this type are tied up with the nature of the Mohorovicic discontinuity and of other discontinuities in the Earth. It is a fact that very little is really known about the occurrence of such phase changes inside the Earth, and therefore phase transition models of the rheology of the tectonosphere are highly speculative.

Furthermore, what is of interest in a mechanical context is only the overall rheological behavior of the tectonosphere. The question of whether this behavior is due to phase transitions or due to some other process is actually of very little phenomenological significance. The justification or rejection of phase-transition models, therefore, does not affect the mechanical results which we have adduced in the earlier sections of this paper.

10. – Conclusion.

In conclusion, it may be well to summarize the main results of the present study.

The rheological models of any merit that were investigated are based upon viscosity, elastic flexure, elastic afterworking, logarithmic creep and the idea of a strength-threshold. The phenomena from which data were gleaned are isostatic uplift and sinking, the non-hydrostatic bulge, the theoretical behavior of rocks at high temperature and pressure ("solid-state theory") and the stability of mountains. The results are summarized in Table I. There are other phenomena, like the plate motions of crustal blocks, and other models, like the phase-change theory; however they are not included in Table I because not enough information can be deduced from them.

The main result of the study is, thus, that the commonly assumed viscous model for the rheological behavior of the tectonosphere does, in fact, *not* agree with the observed postglacial uplift curves. The latter indicate *logarithmic*, not viscous, "creep".

There is probably a low strength-threshold in the tectonosphere. However, one obtains conflicting values from uplift, bulge and mountain-stability data. The discrepancy can be removed if the lower value ($\theta = 10^6$ cgs) is adopted, and it is assumed that the mountain-stability and the bulge are dynamic, not static phenomena.

Table I – Long-term Rheology of the Tectonosphere.

Phenomenon / Model	Isostatic Uplift	Bulge	Solid state theory	Stability of mountains
Viscous substance	Tectonosphere must be layered. Poor fit with uplift curves $\eta = 10^{21}$ to 10^{23} cgs	Possible $\eta = 10^{26}$ cgs	Poor (polycristalline substances do not quite behave in this fashion) $\eta \approx 10^{14} - 10^{17}$ increasing with depth.	
Elastic flexure	Poor $D = 5\times10^{22} \sim 9\times10^{24}$ cgs			
Kelvin	Poor fit with uplift curves. $\eta/\mu \sim 5000$ years		Poor	
Logarithmic creep	Good. Uplift curves fit.		Good. Fits predicted behavior of polycrystalline substances.	
Strength	Possible $\theta = 10^6$ cgs	Possible $\theta = 10^8$ cgs		Possible $\theta = 4\times10^8$ cgs

It is worthy of note that the above-deduced rheological behavior of the tectonosphere in the *long* time range is very similar to that found in the *intermediate* time range by the writer ([11]). The recent evidence, therefore, does no longer bear out a dichotomy between intermediate and long time ranges.

REFERENCES

(1) ANDERSON D. L. and O'CONNELL R., *Viscosity of the Earth.* " Geophys. J. Roy. Astr. Soc. ", **14**, 287-295 (1967).

(2) ANDRADE E. N. da C., *On the viscous flow of metals and allied phenomena.* " Proc. Roy. Soc. London ", **A 84**, 1-12 (1910).

(3) ANDREWS J. T., *Postglacial rebound in Arctic Canada: Similarity and prediction of uplift curves.* " Canad. J. Earth. Sci. ", **5**, 39-47 (1968).

(4) ANDREWS J. T., *Present and postglacial rates of uplift for glaciated northern and eastern North America derived from postglacial uplift curves.* " Canad. J. Earth Sci. ", **7**, 703-715 (1970).

(5) ARTYUSHKOV E. V., *On the depth-dependence of the viscosity of the upper mantle.* " Bull. (Izv.) Acad. Sci. U.S.S.R., Earth Physics " No. **8**, 8-21 (1966).

(6) ARTYUSHKOV E. V., *On the isostatic equilibrium in the Earth's crust.* "Geophys. J. Roy. Astron. Soc. ", **14**, 251-260 (1967).

(7) BROECKER W. S., *The contribution of pressure-induced phase changes to glacial rebound.* " J. Geophys. Res. ", **67**, 4837-4842 (1962).

(8) BROECKER W. S., *Glacial rebound of the shorelines of proglacial lakes.* " J. Geophys. Res. ", **71**, 4777-4783 (1965).

(9) BRUNE J. N., HENYEY T. L. and ROY R. F., *Heat flow, stress and rate of slip along the San Andreas Fault, California.* " J. Geophys. Res. ", **74**, 3821-3827 (1969).

(10) CAPUTO M., *Tables for the deformation of an Earth model by surface mass distribution.* " J. Geophys. Res. ", **67**, 1611-1616 (1962).

(11) CAPUTO M., *The minimum strength of the Earth.* " J. Geophys. Res. ", **70**, 955-963 (1965).

(12) CHINNERY M. A., *The strength of the Earth's crust under horizontal shear stress.* " J. Geophys. Res. ", **96**, 2085-2089 (1964).

(13) CRAMER C. H., *Viscosity of the Atlantic Ocean bottom.* "Science", **67**, 1123-1124 (1970).

(14) CRITTENDEN M. D., *Effective viscosity of the Earth derived from isostatic loading of Pleistocene Lake Bonneville.* "J. Geophys. Res.", **68**, 5517-5530 (1962).

(15) CRITTENDEN M. D., *New data on the isostatic deformation of Lake Bonneville.* " U.S. Geol. Surv. Prof. Pap. " 454-E (1963).

(16) Crittenden M. D., *Viscosity and finite strength of the mantle as determined from water and ice loads.* "Geophys. J. Roy. Astron. Soc.", **14**, 261-279 (1967).

(17) Danes Z. F., U.S. Geol. Surv. Ann. Progr. Rept., Astrogeol. Studies, A81-A100 (1965).

(18) Danes Z. F., *Isostatic process in media of variable viscosity.* "Icarus", **9**, 1-11 (1968).

(19) Donn W. L. and Ewing M., *Pollen from Alaska and the origin of ice ages.* "Science", **147**, 632 (1965).

(20) Durney B., *Stresses induced in a purely elastic Earth model under various tectonic loads.* "Geophys. J. Roy. Astr. Soc.", **10**, 163-173 (1965).

(21) Gopwani M. and Scheidegger A. E., *Contemporary and recent vertical crustal movements and their interpretation.* "Annali di Geofisica", **XXIII**, 4 (1970)

(22) Gordon R. B., *Diffusion creep in the Earth's mantle.* "J. Geophys. Res.", **70**, 2413-2418 (1965).

(23) Haskell N. A., *Motion of viscous fluid under a surface load-I.* "Physics", **6**, 265-269 (1935).

(24) Haskell N. A., *Motion of viscous fluid under a surface load-II*, "Physics", **7**, 56-61 (1936).

(25) Heiskanen W. A. and Vening Meinesz F. A., *The Earth and its Gravity Field.* New York, McGraw-Hill Book Co. (1958).

(26) Herring C., *Diffusional viscosity of a polycrystalline solid.* "J. Appl. Phys." **21**, 437-445 (1950).

(27) Jacoby W. R., *Instability in the upper mantle and global plate movements.* "J. Geophys. Res." **75**, 5671-5680 (1970).

(28) Jeffreys H., *The Earth.* 4th Ed., Cambridge (1959).

(29) Krass M. S. and Ushakov S. A., *Some effects of the dynamics of glacial isostasy of the upper mantle in the peripheral zone of regions of continental glaciation.* "Bull. (Izv.) Acad. Sci. U.S.S.R., Earth Physics", N. **1**, 48-52 (1969).

(30) Magnitsky V. A. and Kalashnikova I. V., *Problem of phase transitions in the upper mantle and its connection with the Earth's crustal structure.* "J. Geophys. Res.", **75**, 877-885 (1970).

(31) McConnell R. K., Comments on Letter by H. Takeuchi, "*Time Scales of isostatic compensation*". "J. Geophys. Res.", **68**, 4397 (1963).

(32) McConnell R. K., *Isostatic adjustment in a layered Earth.* "J. Geophys. Res.", **70**, 5171-5188 (1965).

(33) McConnell R. K., *Viscosity of the mantle from relaxation time spectra of isostatic adjustment.* "J. Geophys. Res.", **73**, 7089-7105 (1968).

(34) McKenzie D. P., *The viscosity of the lower mantle.* "J. Geophys. Res.", **71**, 3995-4010 (1966).

(35) Munk W. and MacDonald G., *The Rotation of the Earth.* Cambridge U. Press (1960).

(36) NABARRO F. R. N., *Deformation of crystals by the motion of single ions.* In " Report of a Conference on Strenght of Solids ", Physical Soc., London, 75-90 (1948).

(37) SAITO Y., *The Seamounts.* " J. Oceanogr. Soc. Japan 20th Anniv. Vol. ", **25-41** (1962).

(38) SCHEIDEGGER A. E., *Rheology of the Earth, the basic problem of geodynamics.* " Canad. J. Phys. ", **35**, 383-397 (1957).

(39) SCHEIDEGGER A. E., *Principles of Geodynamics.* Second Ed. Berlin, Springer (1963).

(40) SCHEIDEGGER A. E., *Theoretical Geomorphology.* Second Ed. New York, Springer (1970).

(41) SCHEIDEGGER A. E., *On the rheology of the Earth in the intermediate time range.* " Annali di Geofisica ", **XXIII**, 1, (1970).

(42) SCHEIDEGGER A. E., *On the rheology of rock creep.* " Rock Mech ", **2**, 138-145 (1970).

(43) SCHOFIELD J. C., *Postglacial sea levels and isostatic uplift.* " New Zealand J. Geol. Geophys. ", **7**, 359-370 (1964).

(44) TAKEUCHI H., *Time scales of isostatic compensations.* "J. Geophys. Res.", **68**, 2357 (1963).

(45) TAKEUCHI H. and HASEGAWA Y., *Viscosity distribution within the Earth.* " Geophys. J. Roy. Astron. Soc. ", **9**, 503-508 (1965).

(46) TERZAGHI K., *Theoretical soil mechanics.* London, Chapman and Hall, (1943).

(47) WALCOTT R. I., *Isostatic response to loading of the crust in Canada.* " Canad. J. Earth Sci. ", **7**, 716-729 (1970).

(48) WALCOTT R. I., *Flexural rigidity, thickness and viscosity of the lithosphere.* " J. Geophys. Res. ", **75**, 3941-3954 (1970).

(49) WEERTMAN J., *The creep strength of the Earth's mantle.* " Revs. Geophys. Space Phys. ", **8**, 145-168 (1970).

(50) ZHARKOV V. N., *Vyazkost' nedr zemli.* " Trans. (Trudy) Inst. Phys. Earth, Acad. Sci. U.S.S.R. ", 11, (1960).