Long term components in polar motion (*)

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Summary. — The analysis of a long series of latitude observations carried out in the International Latitude Station has allowed a very careful study of forced motion due to meteorological effects and of the remaining motion ascribed to the free nutation of an elastic Earth. The discussion of these observations has, however, led to the discovery of the existence of long term components in polar motion (6, 8). These results have recently been confirmed by Proverbio et al. (11).

In this work, the latitudes observed in the period 1900.0 - 1969.0 reduced to a single homogeneous system are analyzed separately for the five stations of the ILS. The analysis of this quantity points up the existence of two different long term components in polar motion of about 16 and 26 years. One possible interpretation of these phenomena is based on " beat " phenomena between the nutation of the mantle and the inner core of the Earth (2).

Riassunto. — L'analisi della lunga serie di osservazioni di latitudine effettuata nelle stazioni internazionali ha permesso uno studio molto accurato delle componenti annuali dovute a cause meteorologiche ed a quella chandleriana del moto polare.

La discussione di queste osservazioni ha tuttavia rilevato l'esistenza di componenti a lungo termine nel moto del polo (6, 8). Questi risultati sono stati confermati recentemente da Proverbio et al. (11).

In questo lavoro sono analizzate, separatamente per le cinque stazioni del SIL, le latitudini osservate nel periodo 1900.0-1969.0 ridotte ad un unico sistema omogeneo. L'analisi di questa quantità mette in evidenza l'esistenza di due differenti termini a lungo periodo nel moto del polo di circa 16 e 26 anni. Una possibile interpretazione di questi fenomeni è basata su fenomeni di battimento fra la nutazione del mantello e del nucleo interno della Terra (2).

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1. - INTRODUCTION

1.1 - Analysis of the latitude observations made in the International Latitude Service Stations starting from 1899 has supplied important knowledge on polar motion and on the causes of this motion, particularly on the causes of the Chandler component owing to the free nutation of the elastic Earth. Although the true nature of this motion still constitutes a problem of difficult solution, at least three models have been proposed as an explanation of the Chandler wobble:

a) the so-called linear damped model introduced by Jeffreys (1940) and developed by Walker and Young (1957);

b) the time variable model proposed by Melchior ([9, 10]) based on a resonance phenomenon of the variable frequency of the free nutation;

c) the two frequency model proposed in order to explain the bifurcation of the Chandler wobble recently confirmed also by the power spectrum analysis of Gaposchkin ([5]).

According to Colombo and Shapiro ([4]) this phenomenon can be explained by viscous coupling between two layers in the upper mantle that would give rise to two distinct frequencies in the Chandler motion.

The possibility prospected by the two frequency model of a hypothetical coupling between the two different layers of the mantle or core as a basis of beat phenomena opens up notable prospects for explaining not only the mechanism of the Chandler wobble, but also to explain the existence of long term polar motion variations recently reported on.

1.2 - The existence of long term polar motion variations reported by Hattori ([6]) was pointed up by Markowitz ([8]). From the analysis of polar motion Markowitz finds the existence of an empirical libration of the Earth's axis with a 24 year period in the direction 132° W or 48° E. More recently, Proverbio et al. ([11]) in analysing the amplitude spectrum as a function of the periods, confirmed the existence of this long term polar motion component, which can be represented by the equations

$$\begin{aligned} x &= 0''.023 \sin\left(\frac{2\pi t}{26} + 170\right) \\ y &= 0''.024 \cos\left(\frac{2\pi t}{24} + 210\right) \end{aligned} \qquad [1]$$

2. – LONG TERM COMPONENTS IN LATITUDE OBSERVATIONS.

2.1 – The analysis carried out by Proverbio et al. (¹¹) utilizing the values of the polar coordinates x and y for the period 1900.0-1961.9, calculated starting from the homogeneous latitude system $\Phi_{\rm II}$ (¹³) led to the singling out of two distinct periods of about 26 and 24 years in the coordinates x and y respectively, as can be seen from equation [1]. However, with this analysis, as with that of Markowitz, polar variations are not deduced directly starting from the latitudes observed, but indirectly by using, as has been said, the values of the polar coordinates x and y. In the case of long term variations of a non-polar nature in the latitude of one or more stations, this could somehow alter the results of the analysis itself.

In this work, the analysis was extended to the latitudes observed in the five stations of the ILS in the period 1900.0 – 1969.0. For this reason, the latitude values in the $\Phi_{\rm II}$ system published by Proverbio et al. (¹³) for the period 1900.0 – 1962.0 were used, while for the interval from 1962.0 to 1969.0 the values of the latitudes observed published in the Annual Report of the International Polar Motion Service from 1962 to 1969 were averaged, reduced to the system $\Phi_{\rm II}$ and interpolated for every tenth of a year (³) by means of the equation

$$\Phi_{\rm II}^{(i)} = \Phi_{\rm obs}^{(i)} - Z \qquad (i = 1, 2, \ldots, 5)$$

where Z represents the international Z term calculated by the ILS.

In Table I the original values of the latitude $\Phi_{\rm II}^{i}$ for the five stations of the ILS for the period 1962.0 – 1969.0 are given. From these values, using the F60 filter ($s_{59/2}/60$ in the notation of Labrouste) the filtered latitudes by tenths of a year were obtained. These are given in Table II. On the other hand, the latitude values given in Table III represent the annual means of the filtered latitudes and, together with those calculated and published previously by Proverbio et al. (¹²) for the period 1903.5 – 1958.5, represent the observational material used here.

2.2 – Spectrum in amplitude of the frequency with the period T included between 7 and 40 years had been calculated by the least squares method utilizing the observational equation in the form

$$\Delta\Phi_t^{(i)} = \Phi_t^{(i)} - \Phi_o^{(i)} = a^{(i)} + b^{(i)}t + A^{(i)} \sin\left[\frac{2\pi t}{T} + B^{(i)}\right] \qquad (i = 1,2,\ldots 5)$$

Table I

	MIZ	UKI	CAR	GAI	KIT		MIZ	UKI	CAR	GAI	KIT
1962.0	3.386	12.376	8.924	13.450	1.524	1965.5	3.210	12.513	8.906	13.575	1.558
1962.1	3.368	12.356	8.928	13.479	1.580	1965.6	3.206	12.344	9.034	13.561	1.670
1962.2	3.312	12.334	8.984	13.476	1.696	1965.7	3.244	12.245	9.118	13.416	1.718
1962.3	3.286	12.284	9.058	13.389	1.701	1965.8	3.273	12.180	9.130	13.372	1.768
1962.4	3.320	12.204	9.074	13.346	1.776	1965.9	3.376	12.244	9.006	13.289	1.852
1962.5	3.358	12.133	9.048	13.288	1.878	1966.0	3.418	12.278	8.962	13.270	1.861
1962.6	3.462	12.140	9.034	13.240	1.854	1966.1	3.500	12.275	8.916	13.264	1.808
1962.7	3.500	12.236	8.952	13.246	1.812	1966.2	3.530	12.320	8.796	13.277	1.770
1962.8	3.530	12.298	8.862	13.294	1.760	1966.3	3.494	12.390	8.764	13.324	1.680
1962.9	3.524	12.379	8.800	13.396	1.654	1966.4	3.329	12.408	8.759	13.367	1.610
1963.0	3.477	12.468	8.792	13.459	1.562	1966.5	3.320	12.498	8.780	13.442	1.592
1963.1	3.434	12.478	8.830	13.507	1.516	1966.6	3.370	12.467	8.876	13.492	1.590
1963.2	3.316	12.444	8.902	13.545	1.521	1966.7	3.308	12.358	9.012	13.492	1.594
1963.3	3.208	12.349	9.018	13.550	1.589	1966.8	3.288	12.314	9.020	13.460	1.646
1963.4	3.174	12.273	9.120	13.500	1.718	1966.9	3.310	12.269	8.975	13.452	1.696
1963.5	3.220	12.140	9.166	13.440	1.824	1967.0	3.360	12.285	8.984	13.390	1.714
1963.6	3.310	12.067	9.164	13.294	1.850	1967.1	3.456	12.280	8.962	13.292	1.724
1963.7	3.504	12.076	9.070	13.220	1.920	1967.2	3.460	12.294	8.942	13.274	1.705
1963.8	3.586	12.166	8.940	13.172	1.840	1967.3	3.454	12.324	8.932	13.269	1.724
1963.9	3.584	12.266	8.772	13.240	1.730	1967.4	3.398	12.314	8.912	13.252	1.750
1964.0	3.590	12.350	8.710	13.318	1.620	1967.5	3.382	12.334	8.984	13.286	1.742
1964.1	3.480	12.502	8.698	13.421	1.554	1967.6	3.394	12.356	8.962	13.310	1.698
1964.2	3.350	12.596	8.746	13.527	1.507	1967.7	3.402	12.398	8.912	13.300	1.686
1964.3	3.210	12.571	8.764	13.615	1.503	1967.8	3.420	12.430	8.776	13.317	1.650
1964.4	3.120	12.410	8.990	13.591	1.578	1967.9	3.418	12.500	8.869	13.390	1.585
1964.5	3.119	12.270	9.082	13.530	1.596	1968.0	3.390	12.517	8.926	13.393	1.575
1964.6	3.220	12.154	9.146	13.465	1.784	1968.1	3.330	12.473	8.957	13.341	1.585
1964.7	3.296	12.075	9.148	13.346	1.860	1968.2	3.300	12.476	8.988	13.333	1.676
1964.8	3.406	12.100	9.036	13.295	1.921	1968.3	3.296	12.421	8.968	13.326	1.724
1964.9	3.523	12.149	8.940	13.260	1.910	1968.4	3.324	12.373	8.960	13.285	1.719
1965.0	3.576	12.268	8.826	13.260	1.820	1968.5	3.320	12.311	9.016	13.281	1.726
1965.1	3.562	12.380	8.740	13.266	1.674	1968.6	3.400	12.325	9.022	13.263	1.779
1965.2	3.482	12.528	8.676	13.385	1.592	1968.7	3.420	12.375	8.940	13.230	1.748
1965.3	3.402	12.575	8.662	13.487	1.530	1968.8	3.460	12.464	8.854	13.238	1.646
1965.4	3.264	12.553	8.750	13.545	1.492	1968.9	3.496	12.518	8.826	13.295	1.576

where $t = \tau_n$ represents a generic epoch included in an interval N. By means of this procedure in the calculated spectrum of the amplitudes the spectral line corresponding to the apparent periods $T' = T + \Delta T$ is represented by broadened main peaks accompanied by a series of other yearly peaks (⁷). The band width of the main peaks can be deduced from the quality factor Q corresponding to each peak and calculated by means of the expression

$$1/Q_{T'} = 8.85\,\frac{T'\tau'}{N\tau}\cdot 10^{-2} \qquad [2]$$

where τ, τ' represent the units with which the intervals t (or T) and N are expressed.

Since the apparent period T' can be held to be very close to the real period T for $N \gg 1$, this procedure can be considered to be, as a first approximation, sufficiently capable of pointing up the existence of long term polar motion components with periods on the order of those already singled out.

Since the interval N considered here results as 62 years ($\tau = 1$) putting into [2] successively T'_{16} = 16 years and T'_{26} = 26 years

Table II

MIZUSAWA

	.05	.15	.25	.35	.45	.55	.65	.75	.85	.95
1958										3.4107
1959	3.4108	3.4131	3.4143	3.4148	3.4151	3.4146	3.4144	3.4134	3.4113	3.4086
1960	3.4062	3.4052	3.4045	3.4047	3.4056	3.4072	3.4083	3.4102	3.4108	3.4090
1961	3.4071	3.4042	3.4008	3.3970	3.3942	3.3938	3.3957	3.3985	3.4005	3.4018
1962	3.4009	3.3988	3.3954	3.3921	3.3887	3.3866	3.3869	3.3886	3.3903	3.3914
1963	3.3903	3.3896	3.3891	3.3880	3.3844	3.3813	3.3806	3.3796	3.3793	3.3793
1964	3.3797	3.3811	3.3822	3.3827	3.3820	3.3809	3.3801	3.3795	3.3794	3.3790
1965	3.3797	3.3790	3.3788	3.3790	3.3791	3.3784	3.3774	3.3761	3.3749	3.3744

UKIAH

	.05	.15	.25	.35	.45	.55	.65	.75	.85	.95
1958										12.2357
1959	12.2361	12.2367	12.2381	12.2400	12.2423	12.2448	12.2477	12.2518	12.2550	12.2573
1960	12.2583	12.2576	12.2551	12.2533	12.2512	12.2499	12.2515	12.2540	12.2578	12.2619
1961	12.2640	12.2653	12.2665	12.2672	12.2654	12.2638	12.2629	12.2632	12.2655	12.2689
1962	12.2741	12.2799	12.2857	12.2899	12.2928	12.2949	12.2946	12.2942	12.2941	12.2968
1963	12.3003	12.3036	12.3065	12.3096	12.3115	12.3141	12.3158	12.3156	12.3148	12.3137
1964	12.3132	12.3132	12.3141	12.3158	12.3174	12.3191	12.3207	12.3223	12.3238	12.3261
1965	12.3284	12.3304	12.3327	12.3350	12.3378	12.3408	12.3439	12.3462	12.3490	12.3513

CARLOFORTE

	.05	.15	.25	.35	.45	.55	.65	.75	.85	.95
1958										8.9930
1959	8.9928	8.9924	8.9899	8.9865	8.9837	8.9808	8.9793	8.9781	8.9773	8.9775
1960	8.9793	8.9814	8.9831	8.9845	8.9851	8.9842	8.9822	8.9790	8.9762	8.9730
1961	8.9713	8.9710	8.9715	8.9712	8.9730	8.9737	8.9725	8.9705	8.9667	8.9629
1962	8.9597	8.9574	8.9555	8.9538	8.9535	8.9545	8.9552	8.9547	8.9532	8.9496
1963	8.9464	8.9449	8.9426	8.9410	8.9404	8.9399	8.9395	8.9363	8.9385	8.9363
1964	8.9342	8.9324	8.9309	8.9304	8.9305	8.9319	8.9329	8.9327	8.9296	8.9273
1965	8.9273	8.9278	8.9279	8.9262	8.9245	8.9239	8.9237	8.9233	8.9234	8.9235

GAITHERSBURG

	.05	.15	.25	.35	.45	.55	.65	.75	.85	.95
1958										13.3888
1959	13.3871	13.3857	13.3848	13.3838	13.3834	13.3837	13.3849	13.3859	13.3877	13.3903
1960	13.3921	13.3915	13.3896	13.3873	13.3848	13.3829	13.3806	13.3799	13.3805	13.3823
1961	13.3853	13.3881	13.3898	13.3906	13.3893	13.3868	13.3837	13.3802	13.3779	13.3776
1962	13.3798	13.3820	13.3854	13.3878	13.3878	13.3867	13.3850	13.3813	13.3787	13.3779
1963	13.3781	13.3792	13.3806	13.3821	13.3832	13.3844	13.3853	13.3859	13.3861	13.3867
1964	13.3870	13.3865	13.3861	13.3859	13.3853	13.3852	13.3850	13.3842	13.3832	13.3829
1965	13.3819	13.3796	13.3772	13.3762	13.3752	13.3750	13.3754	13.3752	13.3742	13.3725

KITAB

	.05	.15	.25	.35	.45	.55	.65	.75	.85	.95
1958										1.7148
1959	1.7141	1.7140	1.7151	1.7148	1.7138	1.7127	1.7100	1.7069	1.7046	1.7035
1960	1.7033	1.7040	1.7055	1.7077	1.7106	1.7131	1.7133	1.7131	1.7113	1.7081
1961	1.7057	1.7047	1.7046	1.7058	1.7086	1.7101	1.7117	1.7120	1.7122	1.7120
1962	1.7104	1.7071	1.7038	1.7023	1.7019	1.7026	1.7057	1.7082	1.7097	1.7106
1963	1.7103	1.7088	1.7067	1.7033	1.7010	1.7001	1.6997	1.6993	1.6997	1.7000
1964	1.6998	1.6992	1.6982	1.6978	1.6978	1.6975	1.6964	1.6953	1.6945	1.6939
1965	1.6947	1.6948	1.6945	1.6949	1.6939	1.6914	1.6901	1.6891	1.6872	1.6859

Table III

DATE	MIZ	KIT	CAR	GAI	UKI	DATE	MIZ	KIT	CAR	GAI	UKI
1903.5	3.591		8.947	13.204	12.089	1935.5	3.464	1.728	8.929		12.207
1904.5	3.589		8.947	13.220	12.098	1936.5	3.484	1.730		13.344	12.208
1905.5	3.582		8.941	13.229	12.105	1937.5	3.479	1.741		13.349	12.209
1906.5	3.581		8.935	13.230	12.104	1938.5	3.473	1.751		13.344	12.205
1907.5	3.577		8.932	13.232	12.102	1939.5	3.464	1.758		13.337	12.198
1908.5	3.568		8.923	13.233	12.097	1940.5	3.462	1.755		13.331	12.193
1909.5	3.550		8.914	13.238	12.096	1941.5	3.456	1.746		13.329	12.187
1910.5	3.538		8.914	13.240	12.097	1942.5	3.455	1.776		13.321	12.186
1911.5	3.531		8.912	13.234	12.102	1943.5	3.456	1.783		13.313	12.181
1912.5	3.521		8.912		12.113	1944.5	3.449	1.787		13.307	12.184
1913.5	3.512		8.910		12.119	1945.5	3.443	1.784		13.304	12.188
1914.5	3.505		8.915		12.122	1946.5	3.432	1.789		13.305	12.187
1915.5	3.510		8.919		12.125	1947.5	3.428	1.785		13.323	12.188
1916.5	3.508		8.925		12.128	1948.5	3.434	1.777		13.332	12.185
1917.5	3.510		8.937		12.129	1949.5	3.425	1.768		13.339	12.183
1918.5	3.518		8.961		12.115	1950.5	3.436	1.757	9.017	13.338	12.176
1919.5	3.522		8.983		12.110	1951.5	3.451	1.752	9.002	13.346	12.171
1920.5	3.523		8.993		12.110	1952.5	3.462	1.742	9.004	13.343	12.173
1921.5	3.515		8.999		12.108	1953.5	3.464	1.737	9.004	13.335	12.175
1922.5	3.512		9.000		12.104	1954.5	3.453	1.729	9.000	13.344	12.186
1923.5	3.512		8.994		12.101	1955.5	3.455	1.719	8.993	13.357	12.200
1924.5	3.503		8.985		12.115	1956.5	3.445	1.717	8.990	13.377	12.204
1925.5	3.504		8.973		12.122	1957.5	3.433	1.714	8.995	13.382	12.214
1926.5	3.507		8.965		12.137	1958.5	3.421	1.715	8.992	13.387	12.230
1927.5	3.515		8.966		12.145	1959.5	3.413	1.711	8.984	13.386	12.245
1928.5	3.509		8.960		12.153	1960.5	3.407	1.709	8.981	13.385	12.255
1929.5	3.495		8.952		12.163	1961.5	3.389	1.709	8.970	13.385	12.265
1930.5	3.490		8.947		12.171	1962.5	3.397	1.706	8.955	13.383	12.289
1931.5	3.485		8.942		12.181	1963.5	3.384	1.703	8.945	13.383	12.311
1932.5	3.480		8.936		12.184	1964.5	3.381	1.697	8.931	13.385	12.319
1933.5	3.475		8.925		12.194	1965.5	3.378	1.691	8.925	13.376	12.340
1934.5	3.480	1.728	8.923		12.200						

($\tau' = 1$) the following values calculated from the quantities $\frac{1}{\varphi} : \frac{1}{\varphi_{16}} = 2.3$ years, $\frac{1}{\varphi_{26}} = 3.7$ years are obtained. These values constitute an index of the theoretical separating power of the method used here.

2.3 – Using the latitude values given in Table III, the values of the quantities a, b, C_1 and C_2 as well as the amplitudes A and the phases F as a function of the period T, for T variable from 7 to 40 years with steps of one year were calculated by means of equations of condition of the type

$$\Delta\Phi = a + bt + C_1 \sin \frac{2\pi t}{T} + C_2 \cos \frac{2\pi t}{T}, \qquad [3]$$

where

$$C_1 = A \cos F$$
$$C_2 = A \sin F$$

The calculated values of C_1, C_2, A and F with the respective m.s.e. are given in Table IV. The values of the amplitudes are represented in

Table IV

MIZUSAWA

T	C1	C2	AMP	E(AMP)	FAS	E(FAS)	T
7	-0.7153E-03	-0.2861E-03	0.7704E-03	0.282E-02	0.2190E 02	0.161E 00	7
8	-0.2218E-03	-0.5042E-02	0.5047E-02	0.273E-02	0.8748E 02	0.158E 00	8
9	-0.1051E-02	0.2701E-02	0.2898E-02	0.280E-02	-0.6873E 02	0.160E 00	9
10	-0.2159E-02	-0.2397E-02	0.3227E-02	0.279E-02	0.4799E 02	0.160E 00	10
11	-0.2468E-02	-0.8989E-03	0.2627E-02	0.279E-02	0.2000E 02	0.161E 00	11
12	-0.1263E-02	-0.7988E-02	0.8088E-02	0.265E-02	0.8101E 02	0.149E 00	12
13	0.5488E-02	-0.1690E-02	0.5742E-02	0.273E-02	-0.1712E 02	0.156E 00	13
14	-0.3163E-02	0.7657E-04	0.3164E-02	0.279E-02	-0.1386E 01	0.160E 00	14
15	-0.2180E-02	-0.1149E-01	0.1170E-01	0.246E-02	0.7926E 02	0.135E 00	15
16	0.1038E-01	-0.1167E-01	0.1562E-01	0.199E-02	-0.4835E 02	0.114E 00	16
17	0.1443E-01	-0.8616E-03	0.1446E-01	0.205E-02	-0.3414E 01	0.122E 00	17
18	0.8631E-02	0.6510E-02	0.1081E-01	0.245E-02	0.3702E 02	0.140E 00	18
19	0.1380E-02	0.5208E-02	0.5388E-02	0.280E-02	0.7515E 02	0.155E 00	19
20	-0.2441E-03	-0.1097E-02	0.1124E-02	0.292E-02	0.7746E 02	0.159E 00	20
21	0.3936E-02	-0.5444E-02	0.6718E-02	0.273E-02	-0.5413E 02	0.156E 00	21
22	0.9450E-02	-0.4972E-02	0.1067E-01	0.250E-02	-0.2775E 02	0.144E 00	22
23	0.1306E-01	-0.1052E-02	0.1311E-01	0.223E-02	-0.4605E 01	0.133E 00	23
24	0.1390E-01	0.4194E-02	0.1452E-01	0.208E-02	0.1678E 02	0.121E 00	24
25	0.1221E-01	0.9161E-02	0.1526E-01	0.201E-02	0.3687E 02	0.115E 00	25
26	0.8723E-02	0.1277E-01	0.1547E-01	0.202E-02	0.5567E 02	0.115E 00	26
27	0.4419E-02	0.1449E-01	0.1515E-01	0.215E-02	0.7304E 02	0.117E 00	27
28	0.2836E-03	0.1428E-01	0.1429E-01	0.233E-02	0.8886E 02	0.123E 00	28
29	-0.2909E-02	0.1256E-01	0.1290E-01	0.247E-02	-0.7696E 02	0.134E 00	29
30	-0.4770E-02	0.1001E-01	0.1109E-01	0.258E-02	-0.6453E 02	0.146E 00	30
31	-0.5327E-02	0.7337E-02	0.9067E-02	0.269E-02	-0.5401E 02	0.155E 00	31
32	-0.4897E-02	0.5030E-02	0.7020E-02	0.280E-02	-0.4576E 02	0.161E 00	32
33	-0.3888E-02	0.3330E-02	0.5119E-02	0.288E-02	-0.4058E 02	0.164E 00	33
34	-0.2651E-02	0.2260E-02	0.3484E-02	0.292E-02	-0.4043E 02	0.167E 00	34
35	-0.1421E-02	0.1725E-02	0.2235E-02	0.294E-02	-0.5051E 02	0.168E 00	35
36	-0.3247E-03	0.1602E-02	0.1634E-02	0.298E-02	-0.7854E 02	0.166E 00	36
37	0.5836E-03	0.1776E-02	0.1870E-02	0.297E-02	0.7181E 02	0.166E 00	37
38	0.1266E-02	0.2160E-02	0.2514E-02	0.293E-02	0.5922E 02	0.166E 00	38
39	0.1784E-02	0.2686E-02	0.3225E-02	0.289E-02	0.5640E 02	0.165E 00	39
40	0.2065E-02	0.3304E-02	0.3907E-02	0.285E-02	0.5774E 02	0.165E 00	40

GAITHERSBURG

T	C1	C2	AMP	E(AMP)	FAS	E(FAS)	T
7	0.2291E-02	0.2776E-02	0.3599E-02	0.377E-02	0.5047E 02	0.216E 00	7
8	0.4331E-04	0.2096E-02	0.2097E-02	0.375E-02	0.8881E 02	0.219E 00	8
9	0.3295E-02	-0.4638E-02	0.5690E-02	0.368E-02	-0.5460E 02	0.211E 00	9
10	-0.9582E-02	0.5946E-02	0.1127E-01	0.332E-02	-0.3182E 02	0.187E 00	10
11	0.2206E-02	-0.1025E-01	0.1049E-01	0.343E-02	-0.7785E 02	0.193E 00	11
12	0.4585E-02	0.6816E-02	0.8215E-02	0.363E-02	0.5607E 02	0.205E 00	12
13	-0.4489E-02	0.3241E-02	0.5537E-02	0.371E-02	-0.3582E 02	0.216E 00	13
14	-0.5429E-02	-0.1030E-02	0.5526E-02	0.362E-02	0.1074E 02	0.221E 00	14
15	-0.4009E-02	-0.8102E-02	0.9040E-02	0.383E-02	0.6366E 02	0.203E 00	15
16	0.5032E-02	-0.1170E-01	0.1273E-01	0.359E-02	-0.6672E 02	0.196E 00	16
17	0.1262E-01	-0.4558E-02	0.1342E-01	0.330E-02	-0.1985E 02	0.201E 00	17
18	0.1019E-01	0.7810E-02	0.1284E-01	0.335E-02	0.3745E 02	0.187E 00	18
19	-0.8647E-03	0.1485E-01	0.1488E-01	0.294E-02	-0.8666E 02	0.179E 00	19
20	-0.1313E-01	0.1256E-01	0.1817E-01	0.269E-02	-0.4373E 02	0.154E 00	20
21	-0.1965E-01	0.2490E-02	0.1981E-01	0.255E-02	-0.7219E 01	0.138E 00	21
22	-0.1704E-01	-0.9080E-02	0.1931E-01	0.253E-02	0.2804E 02	0.142E 00	22
23	-0.8379E-02	-0.1619E-01	0.1823E-01	0.263E-02	0.6263E 02	0.149E 00	23
24	0.1329E-02	-0.1741E-01	0.1746E-01	0.268E-02	-0.8563E 02	0.158E 00	24
25	0.8864E-02	-0.1430E-01	0.1683E-01	0.276E-02	-0.5821E 02	0.164E 00	25
26	0.1326E-01	-0.9167E-02	0.1612E-01	0.294E-02	-0.3465E 02	0.165E 00	26
27	0.1493E-01	-0.3662E-02	0.1538E-01	0.303E-02	-0.1377E 02	0.175E 00	27
28	0.1464E-01	0.1407E-02	0.1471E-01	0.312E-02	0.5489E 01	0.186E 00	28
29	0.1296E-01	0.5755E-02	0.1418E-01	0.336E-02	0.2394E 02	0.192E 00	29
30	0.1022E-01	0.9268E-02	0.1380E-01	0.356E-02	0.4217E 02	0.202E 00	30
31	0.6683E-02	0.1183E-01	0.1359E-01	0.354E-02	0.6055E 02	0.224E 00	31
32	0.2572E-02	0.1335E-01	0.1359E-01	0.345E-02	0.7909E 02	0.245E 00	32
33	-0.1788E-02	0.1374E-01	0.1385E-01	0.357E-02	-0.8258E 02	0.252E 00	33
34	-0.6053E-02	0.1305E-01	0.1439E-01	0.366E-02	-0.6513E 02	0.244E 00	34
35	-0.9933E-02	0.1147E-01	0.1517E-01	0.406E-02	-0.4910E 02	0.236E 00	35
36	-0.1324E-01	0.9185E-02	0.1612E-01	0.413E-02	-0.3473E 02	0.231E 00	36
37	-0.1591E-01	0.6399E-02	0.1715E-01	0.417E-02	-0.2190E 02	0.227E 00	37
38	-0.1790E-01	0.3272E-02	0.1820E-01	0.424E-02	-0.1035E 02	0.220E 00	38
39	-0.1921E-01	-0.6144E-04	0.1921E-01	0.431E-02	0.1831E 00	0.213E 00	39
40	-0.1986E-01	-0.3478E-02	0.2016E-01	0.434E-02	0.9933E 01	0.211E 00	40

continuation Table IV

UKIAH

T	C1	C2	AMP	E(AMP)	FAS	E(FAS)	T
7	0.1243E-02	0.2924E-02	0.3177E-02	0.488E-02	0.6696E 02	0.279E 00	7
8	-0.2426E-02	0.3631E-02	0.4367E-02	0.485E-02	-0.5625E 02	0.279E 00	8
9	0.2014E-02	0.3866E-02	0.4359E-02	0.487E-02	0.6247E 02	0.278E 00	9
10	0.8954E-04	-0.6499E-02	0.6500E-02	0.480E-02	-0.8921E 02	0.277E 00	10
11	-0.1510E-02	-0.4516E-02	0.4762E-02	0.488E-02	0.7151E 02	0.277E 00	11
12	0.7930E-02	-0.3364E-03	0.7937E-02	0.475E-02	-0.2429E 01	0.277E 00	12
13	-0.3020E-02	0.1056E-02	0.3199E-02	0.489E-02	-0.1927E 02	0.280E 00	13
14	0.6305E-02	-0.7526E-02	0.9818E-02	0.473E-02	-0.5004E 02	0.270E 00	14
15	0.1158E-01	0.8006E-02	0.1407E-01	0.452E-02	0.3465E 02	0.263E 00	15
16	-0.3549E-02	0.1372E-01	0.1417E-01	0.457E-02	-0.7550E 02	0.262E 00	16
17	-0.1088E-01	0.1454E-02	0.1098E-01	0.462E-02	-0.7612E 01	0.274E 00	17
18	-0.3498E-02	-0.6858E-02	0.7699E-02	0.480E-02	0.6297E 02	0.275E 00	18
19	0.3881E-02	-0.2494E-02	0.4613E-02	0.484E-02	-0.3273E 02	0.282E 00	19
20	0.1438E-02	0.5702E-02	0.5881E-02	0.500E-02	0.7584E 02	0.274E 00	20
21	-0.7782E-02	0.6983E-02	0.1045E-01	0.479E-02	-0.4190E 02	0.274E 00	21
22	-0.1491E-01	-0.9390E-03	0.1494E-01	0.455E-02	0.3601E 01	0.263E 00	22
23	-0.1445E-01	-0.1289E-01	0.1937E-01	0.425E-02	0.4173E 02	0.245E 00	23
24	-0.6786E-02	-0.2253E-01	0.2353E-01	0.391E-02	0.7324E 02	0.220E 00	24
25	0.4401E-02	-0.2648E-01	0.2685E-01	0.347E-02	-0.8056E 02	0.199E 00	25
26	0.1544E-01	-0.2463E-01	0.2907E-01	0.315E-02	-0.5791E 02	0.179E 00	26
27	0.2412E-01	-0.1840E-01	0.3034E-01	0.289E-02	-0.3733E 02	0.168E 00	27
28	0.2939E-01	-0.9474E-02	0.3088E-01	0.272E-02	-0.1786E 02	0.166E 00	28
29	0.3094E-01	0.5597E-03	0.3094E-01	0.272E-02	0.1036E 01	0.165E 00	29
30	0.2898E-01	0.1027E-01	0.3075E-01	0.285E-02	0.1952E 02	0.165E 00	30
31	0.2419E-01	0.1853E-01	0.3047E-01	0.298E-02	0.3744E 02	0.169E 00	31
32	0.1750E-01	0.2456E-01	0.3016E-01	0.307E-02	0.5452E 02	0.179E 00	32
33	0.9916E-02	0.2807E-01	0.2977E-01	0.320E-02	0.7054E 02	0.188E 00	33
34	0.2332E-02	0.2910E-01	0.2919E-01	0.339E-02	0.8541E 02	0.196E 00	34
35	-0.4543E-02	0.2797E-01	0.2834E-01	0.361E-02	-0.8077E 02	0.203E 00	35
36	-0.1023E-01	0.2516E-01	0.2716E-01	0.380E-02	-0.6786E 02	0.213E 00	36
37	-0.1449E-01	0.2119E-01	0.2567E-01	0.396E-02	-0.5563E 02	0.224E 00	37
38	-0.1725E-01	0.1658E-01	0.2393E-01	0.409E-02	-0.4387E 02	0.234E 00	38
39	-0.1862E-01	0.1181E-01	0.2205E-01	0.424E-02	-0.3238E 02	0.242E 00	39
40	-0.1879E-01	0.7209E-02	0.2013E-01	0.439E-02	-0.2098E 02	0.248E 00	40

KITAB

T	C1	C2	AMP	E(AMP)	FAS	E(FAS)	T
7	-0.1571E-02	-0.5875E-02	0.6082E-02	0.605E-02	0.7502E 02	0.343E 00	7
8	-0.3183E-02	0.1592E-02	0.3559E-02	0.610E-02	-0.2657E 02	0.353E 00	8
9	0.7394E-03	-0.5214E-02	0.5266E-02	0.612E-02	-0.8192E 02	0.343E 00	9
10	0.9497E-03	0.5172E-02	0.5258E-02	0.594E-02	0.7959E 02	0.358E 00	10
11	-0.2861E-02	0.5317E-03	0.2910E-02	0.671E-02	-0.1052E 02	0.354E 00	11
12	-0.4240E-02	-0.4800E-02	0.6405E-02	0.608E-02	0.4854E 02	0.348E 00	12
13	0.7056E-02	-0.6452E-02	0.9561E-02	0.589E-02	-0.4244E 02	0.337E 00	13
14	0.1015E-01	0.6127E-02	0.1185E-01	0.588E-02	0.3110E 02	0.326E 00	14
15	-0.1465E-02	0.1469E-01	0.1477E-01	0.594E-02	-0.8430E 02	0.312E 00	15
16	-0.1552E-01	0.7385E-02	0.1719E-01	0.561E-02	-0.2544E 02	0.315E 00	16
17	-0.1688E-01	-0.8119E-02	0.1873E-01	0.538E-02	0.2568E 02	0.306E 00	17
18	-0.7067E-02	-0.1904E-01	0.2031E-01	0.502E-02	0.6964E 02	0.294E 00	18
19	0.7603E-02	-0.2113E-01	0.2246E-01	0.472E-02	-0.7021E 02	0.267E 00	19
20	0.2113E-01	-0.1295E-01	0.2479E-01	0.415E-02	-0.3151E 02	0.245E 00	20
21	0.2662E-01	0.2823E-02	0.2677E-01	0.375E-02	0.6053E 01	0.212E 00	21
22	0.2097E-01	0.1892E-01	0.2825E-01	0.328E-02	0.4205E 02	0.187E 00	22
23	0.7026E-02	0.2846E-01	0.2931E-01	0.291E-02	0.7613E 02	0.165E 00	23
24	-0.9391E-02	0.2864E-01	0.3014E-01	0.254E-02	-0.7184E 02	0.153E 00	24
25	-0.2298E-01	0.2060E-01	0.3086E-01	0.241E-02	-0.4187E 02	0.136E 00	25
26	-0.3066E-01	0.7572E-02	0.3158E-01	0.231E-02	-0.1387E 02	0.125E 00	26
27	-0.3162E-01	-0.6891E-02	0.3236E-01	0.204E-02	0.1229E 02	0.130E 00	27
28	-0.2662E-01	-0.1988E-01	0.3322E-01	0.203E-02	0.3675E 02	0.126E 00	28
29	-0.1727E-01	-0.2951E-01	0.3420E-01	0.226E-02	0.5965E 02	0.112E 00	29
30	-0.5447E-02	-0.3486E-01	0.3529E-01	0.226E-02	0.8112E 02	0.116E 00	30
31	0.7132E-02	-0.3579E-01	0.3649E-01	0.206E-02	-0.7873E 02	0.134E 00	31
32	0.1903E-01	-0.3269E-01	0.3782E-01	0.207E-02	-0.5978E 02	0.142E 00	32
33	0.2921E-01	-0.2625E-01	0.3928E-01	0.245E-02	-0.4195E 02	0.133E 00	33
34	0.3698E-01	-0.1734E-01	0.4085E-01	0.283E-02	-0.2512E 02	0.122E 00	34
35	0.4199E-01	-0.6831E-02	0.4254E-01	0.294E-02	-0.9239E 01	0.132E 00	35
36	0.4413E-01	0.4476E-02	0.4436E-01	0.279E-02	0.5790E 01	0.160E 00	36
37	0.4350E-01	0.1585E-01	0.4630E-01	0.264E-02	0.2002E 02	0.185E 00	37
38	0.4030E-01	0.2671E-01	0.4835E-01	0.283E-02	0.3353E 02	0.195E 00	38
39	0.3487E-01	0.3657E-01	0.5053E-01	0.337E-02	0.4635E 02	0.187E 00	39
40	0.2756E-01	0.4507E-01	0.5283E-01	0.399E-02	0.5855E 02	0.170E 00	40

continuation Table IV

CARLOFORTE

T	C1	C2	AMP	E(AMP)	FAS	E(FAS)	T
7	-0.1015E-03	-0.3559E-02	0.3561E-02	0.582E-02	0.8836E 02	0.334E 00	7
8	-0.1440E-02	-0.1028E-01	0.1038E-01	0.565E-02	0.8202E 02	0.322E 00	8
9	0.9153E-03	-0.6694E-02	0.6756E-02	0.583E-02	-0.8221E 02	0.329E 00	9
10	-0.2561E-02	0.7833E-02	0.8241E-02	0.566E-02	-0.7189E 02	0.336E 00	10
11	0.7942E-03	-0.3122E-02	0.3221E-02	0.586E-02	-0.7572E 02	0.336E 00	11
12	-0.3939E-02	-0.2114E-02	0.4470E-02	0.576E-02	0.2823E 02	0.340E 00	12
13	0.2835E-02	-0.4183E-02	0.5053E-02	0.583E-02	-0.5587E 02	0.335E 00	13
14	-0.3857E-02	-0.2766E-03	0.3867E-02	0.582E-02	0.4100E 01	0.335E 00	14
15	-0.4148E-02	-0.1676E-01	0.1726E-01	0.546E-02	0.7609E 02	0.297E 00	15
16	0.1696E-01	-0.1977E-01	0.2605E-01	0.448E-02	-0.4937E 02	0.256E 00	16
17	0.2775E-01	-0.1036E-02	0.2777E-01	0.415E-02	-0.2139E 01	0.237E 00	17
18	0.2036E-01	0.1595E-01	0.2586E-01	0.448E-02	0.3807E 02	0.257E 00	18
19	0.7147E-02	0.1890E-01	0.2021E-01	0.544E-02	0.6928E 02	0.282E 00	19
20	0.2234E-02	0.1132E-01	0.1154E-01	0.593E-02	0.7884E 02	0.321E 00	20
21	0.7964E-02	0.7440E-02	0.1089E-01	0.586E-02	0.4305E 02	0.333E 00	21
22	0.1353E-01	0.1400E-01	0.1947E-01	0.543E-02	0.4597E 02	0.311E 00	22
23	0.1099E-01	0.2487E-01	0.2719E-01	0.492E-02	0.6614E 02	0.261E 00	23
24	0.1841E-02	0.3197E-01	0.3202E-01	0.409E-02	0.8670E 02	0.221E 00	24
25	-0.8483E-02	0.3308E-01	0.3415E-01	0.326E-02	-0.7561E 02	0.197E 00	25
26	-0.1688E-01	0.3050E-01	0.3486E-01	0.291E-02	-0.6103E 02	0.172E 00	26
27	-0.2334E-01	0.2660E-01	0.3539E-01	0.280E-02	-0.4872E 02	0.159E 00	27
28	-0.2876E-01	0.2212E-01	0.3629E-01	0.273E-02	-0.3756E 02	0.165E 00	28
29	-0.3349E-01	0.1670E-01	0.3743E-01	0.276E-02	-0.2650E 02	0.183E 00	29
30	-0.3701E-01	0.9827E-02	0.3829E-01	0.309E-02	-0.1486E 02	0.202E 00	30
31	-0.3831E-01	0.1592E-02	0.3834E-01	0.380E-02	-0.2379E 01	0.213E 00	31
32	-0.3671E-01	-0.7045E-02	0.3738E-01	0.461E-02	0.1086E 02	0.221E 00	32
33	-0.3235E-01	-0.1476E-01	0.3556E-01	0.515E-02	0.2452E 02	0.243E 00	33
34	-0.2614E-01	-0.2052E-01	0.3323E-01	0.532E-02	0.3814E 02	0.284E 00	34
35	-0.1916E-01	-0.2392E-01	0.3065E-01	0.537E-02	0.5131E 02	0.322E 00	35
36	-0.1235E-01	-0.2505E-01	0.2793E-01	0.566E-02	0.6373E 02	0.342E 00	36
37	-0.6372E-02	-0.2427E-01	0.2509E-01	0.621E-02	0.7528E 02	0.341E 00	37
38	-0.1560E-02	-0.2209E-01	0.2214E-01	0.681E-02	0.8595E 02	0.331E 00	38
39	0.1938E-02	-0.1899E-01	0.1909E-01	0.725E-02	-0.8417E 02	0.324E 00	39
40	0.4149E-02	-0.1545E-01	0.1600E-01	0.745E-02	-0.7497E 02	0.330E 00	40

the diagrams in figs. 1 and 2 together with the respective errors $E(A)$. The values thus calculated are, however, still affected by a systematic variation depending on the F60 filter used on the observed latitude data. Actually, because of the application of the F60 filter on initial data (the latitudes given in Table II), every sinusoidal component contained in them will have its amplitude multiplied by the factor

$$\varrho = \frac{1}{60} \frac{\sin \frac{60\pi}{10T}}{\sin \frac{\pi}{10T}} .$$

The amplitudes found through the procedure described must therefore be "normalized" by dividing them by the factor ϱ, which belongs to the period of each of them. The "normalized" values of the amplitudes are given in the same figs. 1 and 2.

Given the type of filter used, which presents a zero for periods of 6 years, it is opportune to consider that around this period the ampli-

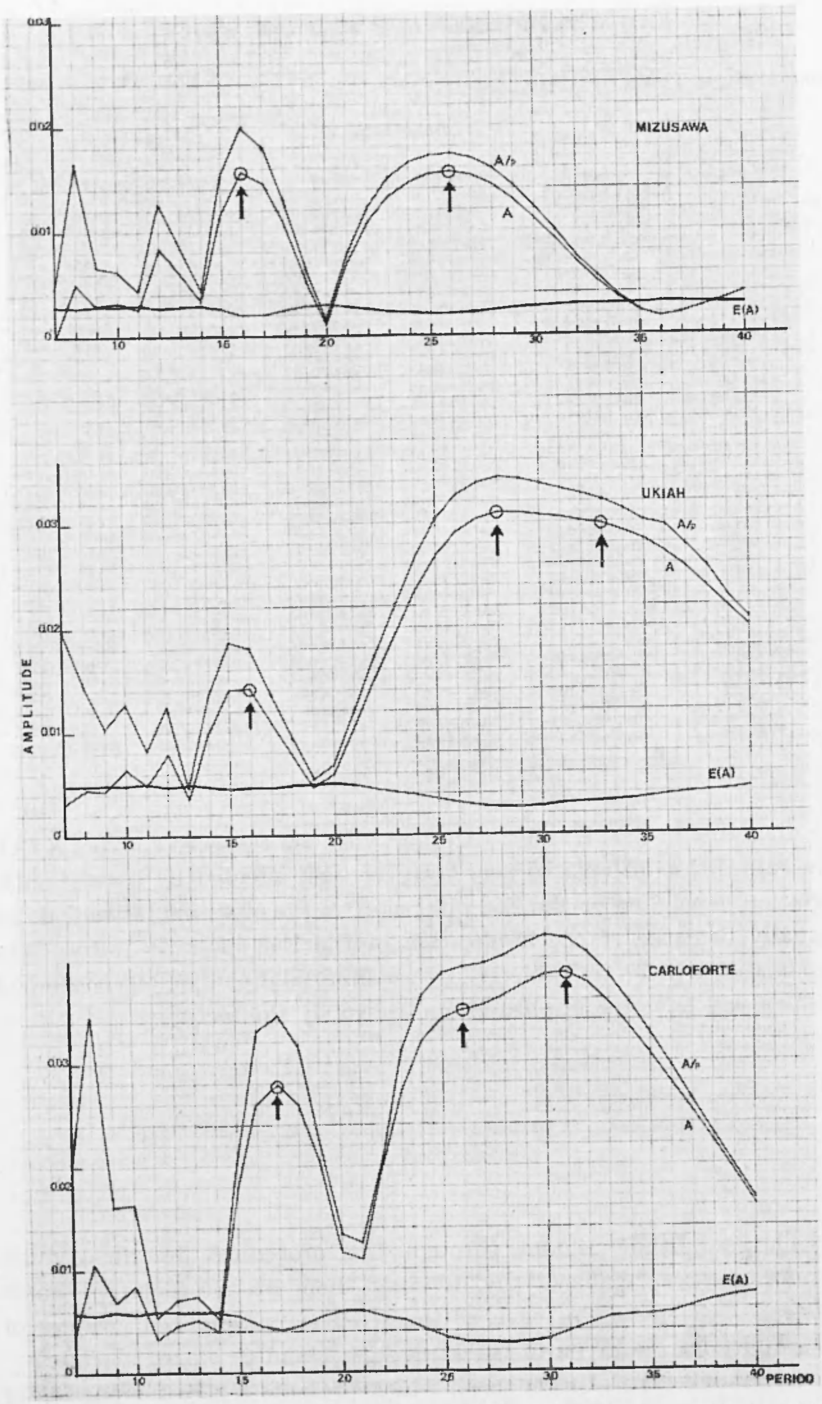

Fig. 1 – Spectrum in amplitude against period P for various values of P.

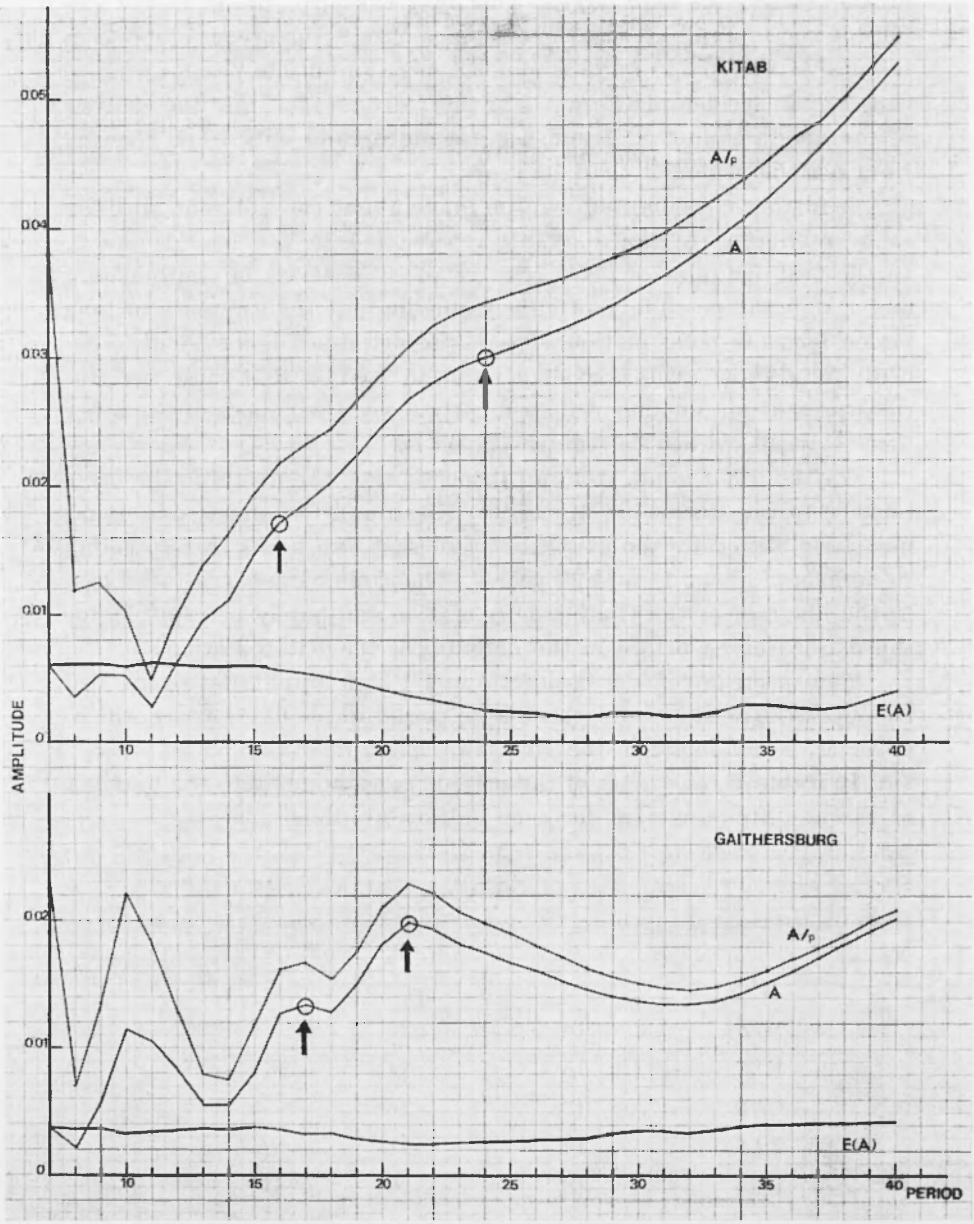

Fig. 2 – Spectrum in amplitude against period P for various values of P.

fication factors are very large. That may cause to appear, in the amplitude spectrum diagrams, the existence of only apparent components because of the effect of the amplification of amplitude peaks owing to causes of an accidental nature. For this reason we feel that the analysis of the diagrams in figs. 1 and 2 is certainly valid only when starting from periods of longer than 10 years.

A further consideration can be made about the different meaning to be attributed to the diagrams represented in the figs. 1 and 2. In the two diagrams of fig. 2 relative to the stations of Gaithersburg and Ukiah the possibility of clearly singling out the existence of long period terms in polar motion appears, in fact, much less evident, especially for Ukiah. This is evidently due to the fact that these two stations present an amount of data greatly lower than those of the other three stations, as can be seen in Table III.

On the other hand, from an inspection, of the diagrams in fig. 1 relative to the stations of Mizusawa, Kitab and Carloforte, one is immediately struck by the existence of at least two peaks corresponding to periods of about 16 and 26 years. The same periods are to be seen in the diagram of the Gaithersburg station and barely perceptible, for the reasons given before in the diagram of the Kitab station.

The spectrum of the Carloforte and Ukiah amplitudes seems also to hint at the existence of a third large period of about 31 years which, however, is not found in the Mizusawa spectrum.

In Table V the values of the periods corresponding to the maxima of the amplitude spectra for each station are given.

Table V

	T_1	T_2	T_3
MIZ	16	26	—
KIT	16	24	—
CAR	17	26	31
GAI	17	21	—
UKI	16	28	33

If, as it is logical, more weight is attributed to the spectra relative to the stations of fig. 1, it can be concluded, overlooking the period indicated as T_3 in Table V, that polar motion seems to be affected by periodical variations around 16 and 26 years. These two periods seem to be confirmed by the trend of the errors $E\ (A)$ of the amplitudes represented in figs. 1 and 2 and of the errors in the phases given in Table IV. In correspondence to them, in fact, both the calculated amplitude and phase errors present minimum values.

The order of approximation with which these periods are observed can thus be directly deduced from the diagrams in fig. 1.

In Table VI the values calculated by means of [2] and observed by the measurement of band width of the quantity $1/Q$ expressed in years are given for the stations of Mizusawa, Carloforte and Ukiah.

Table VI

	$(1/Q_{26})_{\mathrm{cal}}$	$(1/Q_{16})_{\mathrm{cal}}$	$(1/Q_{26})_{\mathrm{obs}}$	$(1/Q_{\ 6})_{\mathrm{obs}}$
MIZ	3.7	2.3	4.8	1.9
CAR	3.7	2.3	4.0	2.2
UKI	3.7	2.3	6.0	2.0

The data in this Table point up the good agreement between the observed and calculated values of $1/Q$ and an uncertainty calculated in the observed values of T_1 and T_2 of ± 2 and ± 5 years respectively.

In Table VII we find instead the phases expressed in degrees corresponding to the periods T_1, T_2 and T_3 for each of the five stations of the ILS.

Table VII

	F_1	F_2	F_3
MIZ	— 48	+ 56	—
KIT	+155	+108	—
CAR	— 21	+119	+156
GAI	— 20	+108	—
UKI	+104	— 18	+ 71

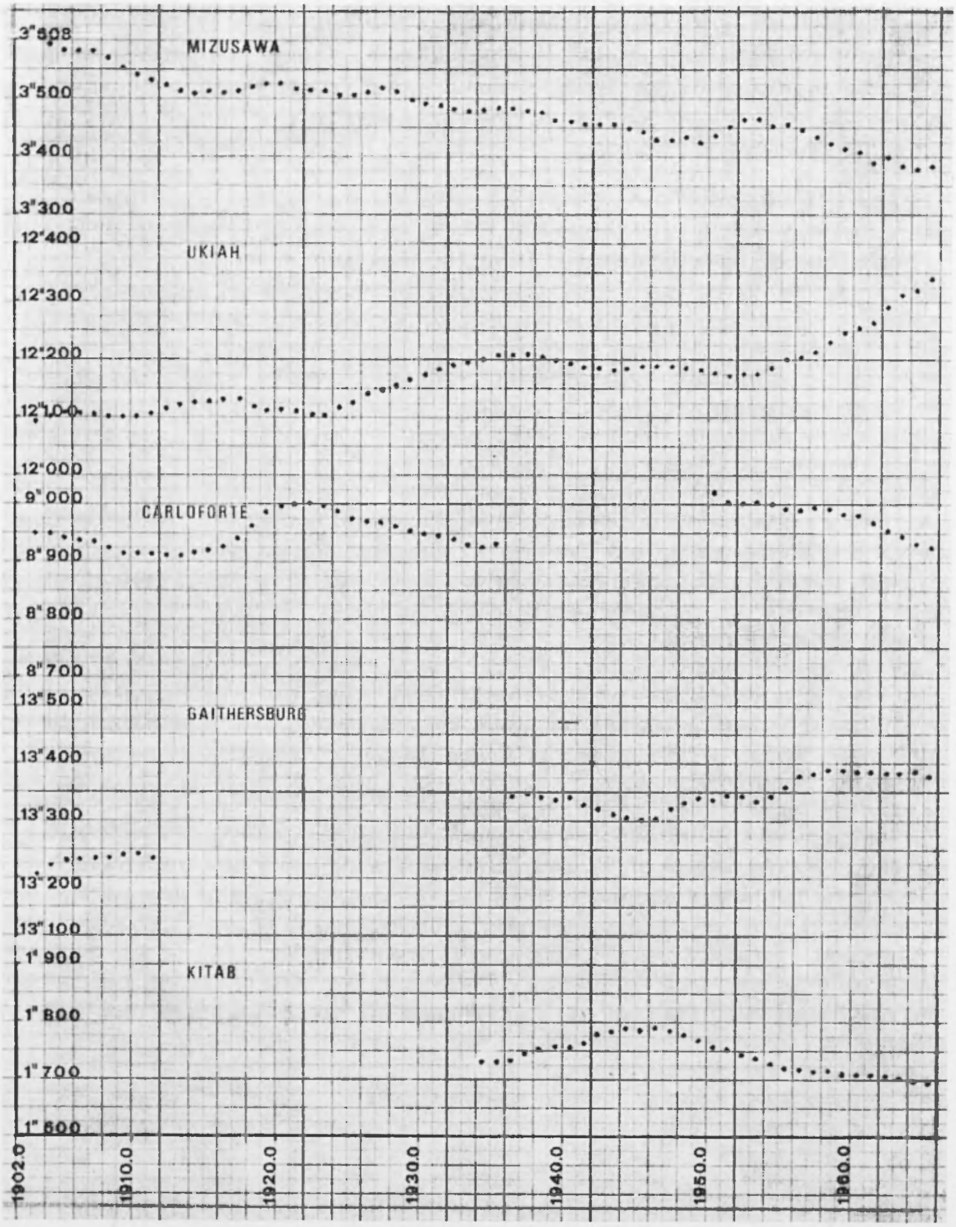

Fig. 3 – Long period and secular variations observed in latitudes for the period 1903.5-1965.5 for the I.L.S. Stations.

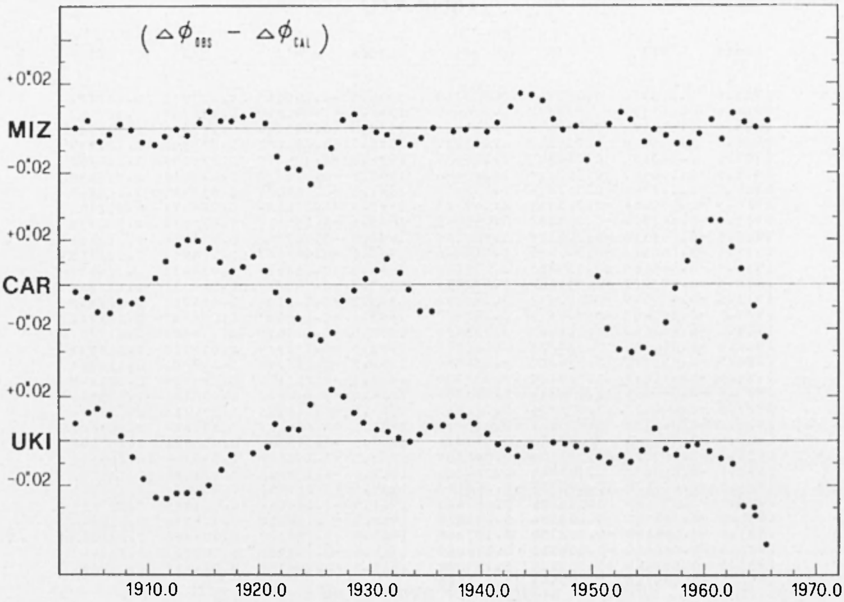

Fig. 4 – Residuals between observed and calculated values of the latitude variations $\Delta\Phi$ for the stations of Mizusawa, Carloforte and Ukiah.

2.4 – In fig. 4 are represented the residuals (0 — *C*) of the differences of the quantities $\Delta\Phi$ observed (which trend is given in fig. 3) and calculated by means of the relation

$$\Delta\Phi^{(i)} = a^{(i)} + b^{(i)}t + A_1^{(i)} \sin\left(\frac{2\pi t}{T_1} + F_1^{(i)}\right) + A_2^{(i)} \sin\left(\frac{2\pi t}{T_2} + F_2^{(i)}\right) \quad [4]$$

in which the values of the amplitudes A and the phases F are taken from Table IV in correspondence to the periods $T_1 = 16$ years, $T_2 = 26$ years. The values of the latitudes $\Delta\Phi^{(i)}$ calculated by means of the relation [4] are given, at less of one constant, in Table VIII.

The amplitude and the distributions of these residuals seems put in evidence the existence of mean period components in the polar motion of the order of 10 years. For the stations of Carloforte and Ukiah there is also evidence for very long fluctuation closed to period of about 30 years pointed out by the diagrams of the fig. 1.

Table VIII

EPOCA	MIZ	CAR	UKI	EPOCA	MIZ	CAR	UKI
1903.5	0.012311	0.022347	0.002268	1935.5	-0.090153	0.007335	0.127722
1904.5	0.013937	0.023872	0.006491	1936.5	-0.091316	0.014258	0.127007
1905.5	0.013144	0.022614	0.011328	1937.5	-0.093840	0.020576	0.125166
1906.5	0.009474	0.018109	0.017132	1938.5	-0.098082	0.025381	0.122939
1907.5	0.002836	0.010409	0.024077	1939.5	-0.104099	0.028188	0.120966
1908.5	-0.006461	0.000128	0.032094	1940.5	-0.111620	0.028998	0.119698
1909.5	-0.017746	-0.011616	0.040853	1941.5	-0.120075	0.028285	0.119343
1910.5	-0.030064	-0.023339	0.049783	1942.5	-0.128674	0.026893	0.119861
1911.5	-0.042301	-0.033387	0.058142	1943.5	-0.136521	0.025874	0.120998
1912.5	-0.053336	-0.040175	0.065116	1944.5	-0.142748	0.026289	0.122361
1913.5	-0.062192	-0.042425	0.069946	1945.5	-0.146657	0.028997	0.123515
1914.5	-0.068165	-0.039365	0.072046	1946.5	-0.147834	0.034483	0.124089
1915.5	-0.070921	-0.030664	0.071112	1947.5	-0.146222	0.042735	0.123868
1916.5	-0.070539	-0.017491	0.067192	1948.5	-0.142154	0.053210	0.122858
1917.5	-0.067487	-0.000412	0.060707	1949.5	-0.136313	0.064880	0.121311
1918.5	-0.062560	0.018647	0.052425	1950.5	-0.129655	0.076366	0.119707
1919.5	-0.056756	0.037721	0.043379	1951.5	-0.123279	0.086128	0.118693
1920.5	-0.051140	0.054800	0.034752	1952.5	-0.118284	0.092701	0.118983
1921.5	-0.046690	0.068109	0.027728	1953.5	-0.115611	0.094918	0.121245
1922.5	-0.044169	0.076334	0.023350	1954.5	-0.115920	0.092112	0.125980
1923.5	-0.044027	0.078791	0.022382	1955.5	-0.119496	0.084238	0.133424
1924.5	-0.046351	0.075501	0.025216	1956.5	-0.126211	0.071919	0.143480
1925.5	-0.050873	0.067176	0.031824	1957.5	-0.135541	0.056391	0.155699
1926.5	-0.057032	0.055103	0.041759	1958.5	-0.146646	0.039359	0.169310
1927.5	-0.064072	0.040955	0.054220	1959.5	-0.158483	0.022786	0.183297
1928.5	-0.071174	0.026555	0.068157	1960.5	-0.169953	0.008634	0.196521
1929.5	-0.077589	0.013624	0.082403	1961.5	-0.180054	-0.001388	0.207658
1930.5	-0.082756	0.003554	0.095828	1962.5	-0.188010	-0.006058	0.216344
1931.5	-0.086388	-0.002759	0.107468	1963.5	-0.193373	-0.004798	0.221306
1932.5	-0.088510	-0.005015	0.116640	1964.5	-0.196067	0.002257	0.222447
1933.5	-0.089449	-0.003501	0.123006	1965.5	-0.196385	0.014278	0.219895
1934.5	-0.089765	0.000993	0.126585				

3. – LONG TERM PERTURBATIONS IN POLAR MOTION

3.1 – Because of the existence of the long period terms shown up by the analysis of the series of latitudes observed in the ILS stations, polar motion presents long term fluctuations. These fluctuations may be expressed as a function of the periods $T_1 = 16$ and $T_2 = 26$ years, knowing the amplitudes and the phases of the two components x and y of polar motion by means of the relations

$$\begin{aligned} x &= A_x \sin\left(\frac{2\pi t}{T} + B_x\right), \\ y &= A_y \cos\left(\frac{2\pi t}{T} + B_y\right). \end{aligned} \qquad [6]$$

Substituting these latter in the well-known relation of Kostinsky

$$\Delta\Phi^{(i)} = A^{(i)} \sin\left(\frac{2\pi t}{T} + F^{(i)}\right) = x \cos\lambda^{(i)} + y \sin\lambda^{(i)}$$

it is possible to calculate the quantities A_x, A_y, B_x and B_y contained in [6]. By doing the calculations, the following expressions are arrived at:

$$x_{26} = 0''.030 \sin\left(\frac{2\pi t}{26} + 123\right)$$

$$y_{26} = 0''.026 \cos\left(\frac{2\pi t}{26} + 197\right)$$

$$x_{16} = 0''.021 \sin\left(\frac{2\pi t}{16} - 49\right)$$

$$y_{16} = 0''.008 \cos\left(\frac{2\pi t}{16} + 78\right)$$

These represent the long term components of polar motion. The comparison of these relations with those previously calculated (11) and given by [1] better shows up the fact that the explicitation of the term with period T_1 leads to a variation in the amplitude and the phases in the other term.

4.2 – The existence of long term variations in polar motion revealed, as has been said, by Markowitz (8) has led various theoreticians to attempt a plausible explanation of this phenomenon.

This is not the best place to enter into details on these theories. We feel, however, that it is useful to point out that Abraham (1), and later Busse (2) proposed two theories based on a "beat" phenomenon. In particular, the theory proposed by Busse provides for a long term polar motion (24 years) caused by precessional motion in the inner core on an eigen-frequency. This motion is supposedly characterized by the existence of a long term libration of positive circular polarization. The polar motion expressed by equations [6] may be held to be in agreement with what is expected according to this theory.

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