One dimensional motion of a viscous fluid

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Summary. — The effect of the friction has been studied on the one dimensional motion of a viscous fluid. This friction is usually schematized in various semiempirical formulae. In this work the different schematizations of the friction were not studied separately but it was shown that a solution exists for the fluid motion. The results give information on the damping of the fluid motion in the case of the seiches.

Riassunto. — Gli effetti dell'attrito nel moto di un fluido non possono essere trascurati nello studio di molti fenomeni geofisici. Le origini di tali attriti sono riconducibili a vari effetti fisici che normalmente vengono schematizzati in diverse formule semiempiriche. Sulla base di una schematizzazione del moto di un fluido lungo una sola dimensione si è determinata la soluzione dell'equazione del moto, considerando le varie forme di attrito agenti contemporaneamente. I risultati danno informazione sullo smorzamento del moto del fluido e sul peso relativo dei vari attriti.

The effect of the friction in the motion of a fluid, e.g. the sea, cannot be neglected in many geophysical phenomena. Its origin can be related to various physical effects, so one usually prefers to schematize it in different semiempirical formulae. In a previous work (6) some of the possibilities of schematization for the one dimensional motion of a fluid have been studied. Particularity of this work has been that these effects were not studied separately but it was shown

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that a solution exists if one considers also a different type of schematization.

In formulae the effect was related to the non-linear differential equation:

$$\partial^2_{tt} u - g\, \partial^2_{xx} [u \cdot h(x)] = \nu\, \partial_t\, \partial^2_{xx} u - \sum_{\alpha=0}^{1} \chi_\alpha\, \partial_t\, u|u|^\alpha + \partial_t f(t, x) \quad [1]$$

where the symbol u denotes the velocity of the fluid at x = const. section of the channel, the surface of which is $h(x)$ the coefficient ν is characteristic of the $\nu\Delta u$ type friction while χ_α is the one related to $\chi_\alpha\, u|u|^\alpha$ friction, f is the symbol of an external force, in our case it could be the wind. In [1] the boundary condition are those of an open and semi-closed basin. We have also considered, on the basis of common sense, that the roughness of the bottom may be considered as a origin of a friction the which is not possible to be seen through the usual numerical studies that utilize computers. So we have preferred to treat a flat bottom case which allows an analytical treatment of the formulae. In this way we have supposed that the surface of the section $x=x_0$ has the value:

$$h(x_0) = \bar{h}(x_0) + \delta \sin\gamma x_0 \quad [2]$$

with two small values of δ and γ. One can intuitively say that this roughness increases the turbulence and then it renormalizes the above mentioned coefficient.

The purpose of this work is to put into a comparison these effects. We use analytical methods which in some cases are richer with information than the purely ones. Therefore we limit ourselves to a flat schematization h_0 of the Adriatic Sea. The method used, because of the smallness of the ν, χ_α, δ coefficients, can be the first order perturbation of the free motion. Obviously this method gives some information for not a long period of time. We will discuss the results in the conclusion.

1. – The problem and its elementary solutions

In order to simulate at our best the Adriatic Sea, despite our flat hypothesis $\bar{h}(x) = h_0$, we use the Defant's boundary conditions. It implies that for the first and third seiches the channel is closed at Venice and is opened at Otranto. For the second seiche, at the contrary, the channel has to be considered closed both at Venice and at Otranto. We then consider two main cases: the effect of the friction on the free motion of the water and the effect of a strong wind on a quiet sea. As for the wind, we have assumed, in this case, also an "ad hoc" shape for semplifying calculation. We suppose that the wind starts suddenly at $t = 0$ and the force is more relevant at Otranto than in the northern part of the basin:

$$f(x, t) = \begin{cases} 0 & t < 0 \\ \Phi \sin \dfrac{x}{\lambda} & t > 0 \end{cases} \qquad \lambda = \frac{2L}{\pi}$$

Having considered these schematization, we start by discussing the first case.

1.1. – Motion and friction of the seiches

We repeat here the equation

$$\partial^2_{tt}\, u - g\, \partial^2_{xx} [h(x) \cdot u] = \nu\, \partial_t\, \partial_{xx}\, u - \sum_{0}^{1}{}_a\, \chi_a\, \partial_t\, u\, |u|^a \qquad [1]$$

If we impose that the basin is closed at Venice and opened at Otranto, the surelevation is zero, we obtain:

$$u(0, t) = \partial_x\, u\, (x, t)_{x=L} = \partial_x\, u\, (L, t) = 0$$

for the odd order seiches. In the second seiche case we have:

$$u(0, t) = u(L, t) = 0$$

We then proceed by supposing, as described in the first paragraph, that δ, ν and χ_a are small parameters, of order ε. We then develop our equation in power of this ε parameter. For the solution, we have:

$$u = u_0 + \varepsilon\, u_1 + \varepsilon^2 u_2 + \ldots$$

For $\varepsilon = 0$ we have for the odd order seiches:

$$\partial^2_{tt}\, u_0 - g h_0\, \partial^2_{xx}\, u_0 = 0; \qquad u_0(0, t) = \partial_x u_0(L, t)$$

which solution is also for the second seiche case:

$$u_0 = A \sin \frac{n}{\lambda}\, x \cos \left\{ n \frac{\sqrt{g\, h_0}}{\lambda}\, t \right\} \qquad n = 1, 2, 3 \qquad [2]$$

We now schematize the frictions as an effect that starts at $t=0$ in order to make clearer the difference between the frictions and the free motion. For $u = u_o + \varepsilon u_1$, we have to take into account the properties of u_o:

$$\partial^2_{tt} u_1 - g h_o \partial^2_{xx} u_1 = F(u_o);\ u_1(x, 0) = \partial_x u_1(x, 0) = 0$$

where

$$F(u_o) = g\, \partial^2_{xx} (u_o \delta \sin \gamma x) + \nu\, \partial_t\, \partial^2_{xx} u_o - \sum_0^1{}_a\, \chi_a\, \partial_t \left\{ u_o \cdot |u_o|^a \right\}$$

In the following we put $a = 1$. The case $a = 0$, which is also among the most interesting ones, can be easily deduced from calculation. We deal now with the well known inhomogeneous wave problem. We will follow the Courant-Hilbert's classical scheme ([4]). We develop at the first

$$F(u_o) = \sum_k A_x \sin \beta x \cdot f_k(t)$$

Then we make easier our problem by remarking that we can say

$$u = \sum_k T_k(t) \sin \beta_k x$$

where the unknown $T_k(t)$ are determined by

$$\begin{cases} \partial^2_{tt} T_k(t) + g\, h_o\, \beta_k^2\, T_k(t) = f_k(t) \\ T_k(0) = \partial_k T_k(0) = 0 \end{cases}$$

We then know that

$$T_k(t) = \frac{1}{\beta_k \sqrt{g h_o}} \int_0^t \sin \sqrt{g h_o}\, \beta_k (t - s)\, f_k(s)\, ds$$

and it implies that

$$u = \sum_k \sin \beta_k x \cdot T_k(t)$$

1.2. – Effect of the friction when the wind moves the water

We study the case of a semiopen channel in rest at $t=0$, when suddenly the wind produces an external force:

$$f(x, t) = \begin{cases} 0 & t < 0 \\ \Phi \sin \dfrac{x}{\lambda} & t > 0 \end{cases}$$

also in this case we will start by leaving aside the friction.

The motion equation is given by

$$\partial^2_{tt} u_0 - g h_0 \, \partial^2_{xx} \, u_0 = \Phi \sin \frac{x}{\lambda} \, \delta(t)$$

the solution of which is

$$u_0 \, (x, t) = \begin{cases} 0 & t \leq 0 \\ \tau \, \Phi \sin \dfrac{x}{\lambda} \sin \dfrac{t}{\tau} & t \geq 0 \end{cases}$$

Because of the remarkable similarity with the previous case, we will use also in this case the above described method to solve the equation at first order in ε.

2. – The analytical results

For the first seiche we have

$$F(u_0) = \frac{A}{2} \, g \, \delta \left\{ \left(\gamma + \frac{1}{\lambda} \right)^2 \cos \left[\left(\gamma + \frac{1}{\lambda} \right) x \right] + \right.$$

$$\left. - \left(\gamma - \frac{1}{\lambda} \right)^2 \cos \left[\left(\gamma - \frac{1}{\lambda} \right) x \right] \right\} \cos \frac{t}{\tau} + \frac{\nu A}{\lambda^2 \tau} \sin \frac{x}{\lambda} \sin \frac{t}{\tau} +$$

$$- \chi_1 \frac{A^2}{\tau} \left\{ \cos \frac{2x}{\lambda} - 1 \right\} \sin \frac{2t}{\tau} \frac{\cos t/\tau}{|\cos t/\tau|} + \frac{\chi_0 A}{\tau} \cdot \sin \frac{x}{\lambda} \cos \frac{t}{\tau}$$

One may remark that the first term is related to the roughness of the bottom. It is proportional to δ and it contains the γ coefficient. The second part is related to the laminar friction in the $\alpha = 0$ and $\alpha = 1$ cases and provides the last two terms. We remark here that in this case the $\alpha = 0$ part of the friction is proportional to the laminar friction, we will then not discuss it any more. We now describe $F(u_0)$ as $\sum_k A_k \sin \, (\beta_k x + \varphi_k) \cdot f_k(t)$ from which it follows:

$$A_1 = \frac{A}{2} \, g \, \delta \left(\gamma + \frac{1}{\lambda} \right)^2 \quad ; \quad \beta_1 = \gamma + \frac{1}{\lambda} \quad ; \quad \varphi_1 = \frac{\pi}{2} \quad ; \quad f_1 = \cos \frac{t}{\tau}$$

$$A_2 = - \frac{A}{2} \, g \, \delta \left(\gamma - \frac{1}{\lambda} \right)^2 \quad ; \quad \beta_2 = \gamma - \frac{1}{\lambda} \quad ; \quad \varphi_2 = \frac{\pi}{2} \quad ; \quad f_2 = \cos \frac{t}{\tau}$$

$$A_3 = \frac{\nu A}{\lambda^2 \tau} \qquad \beta_3 = \frac{1}{\lambda} \qquad \varphi_3 = 0 \qquad f_3 = \sin\frac{t}{\tau}$$

$$A_4 = -\chi_1 \frac{A^2}{\tau} \qquad \beta_4 = \frac{2}{\lambda} \qquad \varphi_4 = \frac{\pi}{2} \qquad f_4 = \sin\frac{2t}{\tau}\,\frac{\cos t/\tau}{|\cos t/\tau|}$$

$$A_5 = \chi_1 \frac{A^2}{\tau} \qquad \beta_5 = 0 \qquad \varphi_5 = \frac{\pi}{2} \qquad f_5 = \sin\frac{2t}{\tau}\,\frac{\cos t/\tau}{|\cos t/\tau|}$$

$$A_6 = \chi_0 \frac{A}{\tau} \qquad \beta_6 = \frac{1}{\lambda} \qquad \varphi_6 = 0 \qquad f_6 = \sin\frac{t}{\tau}$$

Then

$$u = A \sin\frac{x}{\lambda}\sin\frac{t}{\tau} + \Sigma_k A_k \sin(\beta_k x + \varphi_k)\, T_k(t)$$

where

$$T_1 = \left[g h_0 \left(\gamma + \frac{1}{\lambda}\right)^2 - \frac{1}{\tau^2}\right]\left\{\cos\frac{t}{\tau} - \cos\left[\sqrt{g h_0}\left(\gamma + \frac{1}{\lambda}\right) t\right]\right\}$$

$$T_2 = \left[g h_0 \left(\gamma - \frac{1}{\lambda}\right)^2 - \frac{1}{\tau^2}\right]\left\{\cos\frac{t}{\tau} - \cos\left[\sqrt{g h_0}\left(\gamma - \frac{1}{\lambda}\right) t\right]\right\}$$

$$T_3 = -\frac{\tau}{2}\, t \cos\frac{t}{\tau} + \frac{\tau^2}{2}\sin\frac{t}{\tau}$$

This last result is proportional to the effect of χ_0. The function T_4 and T_5 are more complicated to describe in order to elementary functions. We must introduce the S_k time intervals:

$$S_0 \quad 0 \leqslant \frac{t}{\tau} \leqslant \frac{\pi}{2} \qquad\qquad S_1 \quad \frac{\pi}{2} \leqslant \frac{t}{\tau} \leqslant \frac{3}{2}\pi$$

$$S_2 \quad \frac{3}{2}\pi \leqslant \frac{t}{\tau} \leqslant \frac{5}{2}\pi \qquad\qquad S_2 \quad \frac{5}{2}\pi \leqslant \frac{t}{\tau} \leqslant \frac{7}{2}\pi \quad \text{etc.}$$

Then in the K^{th} interval we have

$$T_4 = \frac{\tau^2}{4}(-1)^k\left\{\frac{1}{2}\sin\frac{2t}{\tau} - \frac{t}{\tau}\cos\frac{2t}{\tau} - k\pi\right\}$$

$$T_5 = -\frac{\tau^2}{4}\left\{-\frac{4t}{\tau} + (-1)^k\left[\frac{2t}{\tau} + \sin\frac{2t}{\tau} - 2k\pi\right]\right\}$$

For the second seiche, we can easily calculate the result by putting $\lambda \to \frac{\lambda}{2}$ and $\tau \to \frac{\tau}{2}$ in the $T_k(t)$ expressions.

In the case of a wind, in analogy with the free motion, the result is:

$$F\,(u_0) = \delta\,\frac{\Phi\tau}{2}\left(\gamma+\frac{1}{\lambda}\right)^2 \cos\left(\gamma+\frac{1}{\lambda}\right) x \sin\frac{t}{\tau} +$$

$$-\,\delta\,\frac{\Phi\tau}{2}\left(\gamma-\frac{1}{\lambda}\right)^2 \cos\left(\gamma-\frac{1}{\lambda}\right) x \sin\frac{t}{\tau} - \frac{\Phi\nu}{\lambda^2}\sin\frac{x}{\lambda}\cos\frac{t}{\tau} +$$

$$-\,\frac{\Phi^2\chi_1}{\tau}\sin\frac{2t}{\tau}\,\frac{\sin t/\tau}{|\sin t/\tau|} + \frac{\chi_1\Phi^2}{\tau^2}\cos\frac{2x}{\lambda}\sin\frac{2t}{\tau}\,\frac{\sin t/\tau}{|\sin t/\tau|}$$

from which we obtain:

$$A_1 = g\,\delta\,\Phi\,\frac{\tau}{2}\left(\gamma+\frac{1}{\lambda}\right)^2;\ \beta_1 = \gamma+\frac{1}{\lambda};\ \varphi_1 = \frac{\pi}{2};\ f_1 = \sin\frac{t}{\tau}$$

$$A_2 = g\,\delta\,\Phi\,\frac{\tau}{2}\left(\gamma-\frac{1}{\lambda}\right)^2;\ \beta_2 = \gamma-\frac{1}{\lambda};\ \varphi_2 = \frac{\pi}{2};\ f_2 = \sin\frac{t}{\tau}$$

$$A_3 = -\,\frac{\Phi}{\lambda^2}\,\nu \qquad ;\ \beta_3 = \frac{1}{\lambda} \qquad ;\ \varphi_3 = 0 \quad ;\ f_3 = \cos\frac{t}{\tau}$$

$$A_4 = -\,\Phi^2\,\chi_1\,\tau \qquad ;\ \beta_4 = 0 \qquad ;\ \varphi_4 = \frac{\pi}{2};\ f_4 = \sin\frac{2t}{\tau}\,\frac{\sin t/\tau}{|\sin t/\tau|}$$

$$A_5 = \Phi^2\,\chi_1\,\tau \qquad ,\ \beta_5 = \frac{2}{\lambda} \qquad ;\ \varphi_5 = \frac{\pi}{2};\ f_5 = \sin\frac{2t}{\tau}\,\frac{\sin t/\tau}{|\sin t/\tau|}$$

and the $T_k(t)$ functions are

$$T_1\,(t) = \left[\sqrt{g\,h_0}\left(\gamma+\frac{1}{\lambda}\right)+\frac{1}{\tau}\right]^{-1}\sin\frac{t}{\tau} + \frac{2\,\sqrt{g\,h_0}\left(\gamma-\frac{1}{\lambda}\right)}{g h_0\left(\gamma+\frac{1}{\lambda}\right)^2-\frac{1}{\tau^2}}$$

$$\sin\sqrt{g\,h_0}\left(\gamma+\frac{1}{\lambda}\right)t + \left[\sqrt{g h_0}\left(\gamma+\frac{1}{\lambda}\right)-\frac{1}{\tau}\right]$$

$$\sin\left[2\,\sqrt{g\,h_0}\left(\gamma+\frac{1}{\lambda}\right)-\frac{1}{\tau}\right]t$$

$$T_2(t) = T_1(t) \left\{ \frac{1}{\lambda} => -\frac{1}{\lambda} \right\}$$

$$T_3(t) = \frac{\tau}{2}\, t \sin \frac{t}{\tau}$$

$$T_4(t) = \frac{\tau^2}{2} \left\{ \frac{t}{\tau} - \frac{1}{2} \sin \frac{2t}{\tau} + \left[k + \frac{1}{2}\,(1 - (-1)^k) \right] \pi \right\} (-1)^k$$

$$T_5(t) = \frac{\tau^2}{4} \left\{ \frac{1}{2} \sin \frac{2t}{\tau} - \frac{t}{\tau} \cos \frac{2t}{\tau} + \left[k + \frac{1}{2}\,(1 - (-1)^k) \right] \pi \right\} (-1)^k$$

We note here a difference between the effect of the roughness of the bottom (which gives results limited when t increases) and the two other cases, where the velocity increases in norm as $t \to \infty$. In our opinion this effect is due to the main effect of the roughness: it gives many small movement but it doesn't decrease the kinetic energy. So we are encouraged to try and see the effect of a laminar friction $\nu \Delta u$ on this perturbation of the velocity. This friction should decrease the kinetic energy but not the amplitude of the effect, because the wave-length of the perturbation is $1/\gamma$ which is very large comparated with λ. We study therefore the part of u_1 which is proportional to δ, u_1 to which we apply the $\nu\Delta$ operator. We obtain:

$$u_1 = A\, g\, \frac{\delta}{2} \left\{ \left[\left(\gamma + \frac{1}{\lambda} \right)^2 \cos \left(\gamma + \frac{1}{\lambda} \right) x \cdot \frac{\cos \frac{t}{\tau} - \cos \left(\gamma + \frac{1}{\lambda} \right) \sqrt{g\, h_0}\; t}{g\, h_a \left(\gamma + \frac{1}{\lambda} \right)^2 - \frac{1}{\tau^2}} + \right.\right.$$

$$\left. - \left[\left(\gamma - \frac{1}{\lambda} \right)^2 \cos \left(\gamma - \frac{1}{\lambda} \right) x \; \frac{\cos \frac{t}{\tau} - \cos \sqrt{g\, h_0} \left(\gamma - \frac{1}{\lambda} \right) t}{g\, h_0 \left(\gamma - \frac{1}{\lambda} \right)^2 - \frac{1}{\tau^2}} \right] \right.$$

thence

$$u_2 =: \beta \cos \gamma\, x\, \Phi_1(t) + c \sin \frac{x}{\lambda} \sin \gamma\, x\, \Phi_2(t)$$

$$\Phi_1 = \frac{A\, a\, \delta\, \nu}{\sqrt{g\, h_0}\, \beta}\, \gamma^3 \left\{ \frac{1}{2\sqrt{g\, h_0}\, h} \sin^3 \sqrt{g\, h_0}\, \gamma\, t - \frac{\sin 2\sqrt{g\, h_0}\, t}{4\,(\sqrt{g\, h_0}\, \gamma)^3\, \tau} + \right.$$

$$\left. + \frac{t}{g\, h\, \gamma^3\, \tau} \cos 2\sqrt{g\, h_0}\, \gamma\, t + \frac{t^2}{2\, \tau \sqrt{g\, h_0}\, \gamma} \sin \sqrt{g\, h_0}\, t \right\}$$

$$\Phi_2 = - \frac{A\, g\, \delta\, \nu\, \gamma^3}{c\, \sqrt{g\, h_0}} \left\{ \frac{t}{2\sqrt{g\, h_0}\, \gamma} \cos \sqrt{g\, h_0}\, \gamma\, t - \frac{1}{2\, g\, h_0\, \gamma^2} \sin \sqrt{g\, h_0}\, \gamma t \right\}$$

We remark that we have obtained velocities which decrease with time. Their numerical importance will be discussed in the following part. We would like to add that one could intuitively assume that this linear first part of a negative exponential, at least for the laminar friction term and perhaps for the others as well.

3. – Numerical evaluation and final discussion

In order to evaluate numerically the relative weight of these results we have assumed

$$\begin{array}{llll} L = 10^6 \text{ m} & \nu = 1.4\, 10^{-2} \text{ m}^2 \text{ s}^{-1}; & \gamma = 10^{-2} \text{ m}^{-1} \\ h_0 = 3 \cdot 10^2 \text{ m} & A = 10^{-1} \text{ m s}^{-1} & \\ g = 10 \text{ m s}^{-2} & \chi_1 = 6 \cdot 10^{-6} \text{ m}^{-1} & \\ \chi_0 = 10^{-5} \text{ s}^{-1} & \delta = 1 \text{ m} & \end{array}$$

We then have

$$u_1 = \frac{A\delta}{2 h_0}\left(\gamma + \frac{1}{\lambda}\right)^2 \frac{1}{\left(\gamma + \dfrac{1}{\lambda}\right)^2 - \dfrac{1}{\tau^2}} \cos\left(\gamma + \frac{1}{\lambda}\right) x \left\{ \cos \frac{t}{\tau} + \right.$$

$$\left. - \cos \sqrt{g\, h_0}\left(\gamma + \frac{1}{\lambda}\right) t \right\} - \frac{A\delta}{2h_0} \frac{\left(\gamma - \dfrac{1}{\lambda}\right)^2}{\left(\gamma - \dfrac{1}{\lambda}\right)^2 - \dfrac{1}{\tau^2}} \cos\left(\gamma + \right.$$

$$\left. - \frac{1}{\lambda}\right) x \left\{ \cos \frac{t}{\tau} - \cos \sqrt{g\, h_0}\left(\gamma + \frac{1}{\lambda}\right) t \right\} - \frac{\nu\, A\, \tau}{2\, \lambda^2} \left\{ \frac{t}{\tau} \cos \frac{t}{\tau} + \right.$$

$$\left. - \sin \frac{t}{\tau} \right\} \sin \frac{x}{\lambda} - \chi_1 \frac{A^2\, \tau}{4} (-1)^k \left\{ \frac{1}{2} \sin \frac{2t}{\tau} - \frac{t}{\tau} \cos \frac{2t}{\tau} + \right.$$

$$\left. - k\pi \right\} \cos \frac{2\, x}{\lambda} - \chi_1 \frac{A^2\, \tau}{4} \left\{ - \frac{4\, t}{\tau} + (-1)^k \left\{ \frac{2\, t}{\tau} + \right. \right.$$

$$\left. \left. + \sin \frac{2\, t}{\tau} - 2\, k\, \pi \right\} \right. - \chi_0 \frac{A\, \tau}{2} \left[\frac{t}{\tau} \cos \frac{t}{\tau} \sin \frac{x}{\lambda} \right]$$

This relation becomes, if $\gamma \gg \dfrac{1}{\lambda}$,

$$u_1 = \frac{A\,g\,\delta}{h \cdot 2}[\ldots.] - \frac{\nu\,A\,\tau}{\lambda^2}[\ldots] - \psi_1\,A\,^2\tau\,[\ldots\ +\ \ldots] - \chi_0\,A\,\tau =$$

$$= 2 \cdot 10^{-4}\,[\ldots] - 4.4 \cdot 10^{-15}\,[\ldots] - 2 \cdot 10^{-4}\,[\ldots] - 6 \cdot 10^{3}\,[\ldots]$$

$$u_2 = 3 \cdot 10^{-11}\,[\ldots]$$

In conclusion we can say that this simple analysis can give some information on these processes (see fig. 1). Remembering the expression

$$\partial_x\,(h \cdot u) + \partial_t\,\xi = 0$$

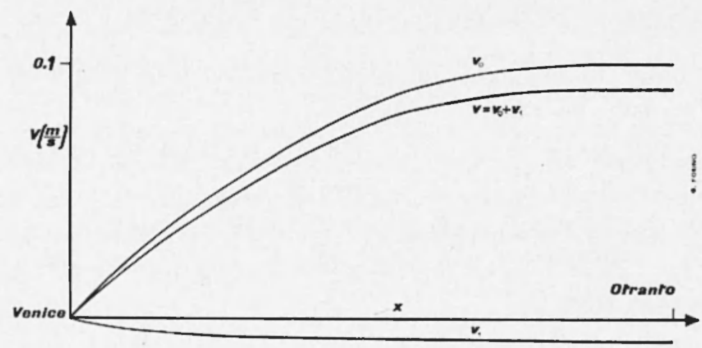

Fig. 1 – The distribution of the velocity v, versus x, at $t = \tau = 3^h 20^m$. v_0 is velocity calculate without friction, v_1 is the velocity determined by various frictions, v is the resulting velocity.

we obtain an other representation of friction whose effects are drawn in fig. 2.

The roughness of the bottom is strongly dependent on the ratio of γ and $1/\lambda$. In our schematization of the Adriatic Sea, this ratio is small and the bottom roughness doesn't influence the motion, at least at the first order. We may say, in a different way, that this interaction can describe the energy transfer between motions of large scale length and turbulence type motions, which can give an important contribution to the dissipation of energy. We have called "*turbulence*" the smaller scale motions which are strongly related to the bottom irregularities.

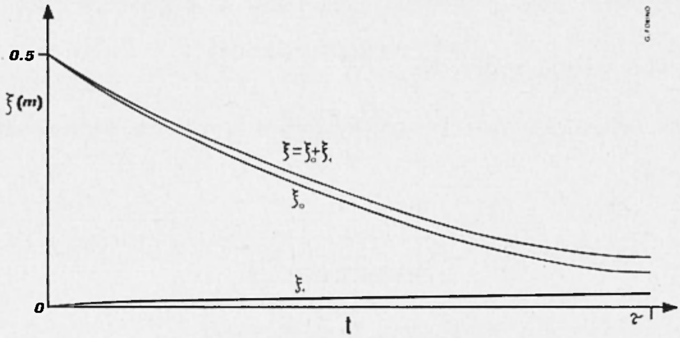

Fig. 2 – The position at Venice of the free surface ξ during $t = \tau = 3^h 20^m$. ξ_0 is the position corresponding to the velocity v_0, ξ_1, to v_1 and ξ to v.

The preceding statement is stressed by the fact that at the second order, the bottom roughness gives an internal friction the value of which can be larger than the corresponding first order quantity. The part of the friction which is proportional to ν, the laminar part, gives a very small contribution to the general form of the friction, because the coupling constant is very small. It has to remarked, however, that its analytical form is very closed to the part proportional to χ_0. This analytical similarity could give origin a bit of confusion in the interplay of these quantities, in any case we consider here the values usually assumed on an experimental field. We can also say that the term proportional to χ_0, usually called linear bottom friction, describes also the effect of the friction on the boundary layer. On these same ground, moreover, we can say that in the term proportional to χ_1 the higher velocities give the more remarkable effect. If, however, one has not a flat bottom, one could expect that this kind of friction will give different effect for different depth h_0. We expect this phenomenon to be important for a study of the real Adriatic Sea which is now in progress. We can also say that all the frictions by us dealt have a linear effect, because we have analyzed the first order of the perturbation expansion of the real friction. If we assume on physical field that these effects are exponential, this hypothesis has some experimental evidence and we could also say that these terms are proportional to

$$\chi_1 \Lambda^2 \tau, \quad \chi_0 A \tau, \quad \frac{\nu A \tau}{\lambda^2}, \quad \frac{A \delta}{2 h_0}$$

and at the second order, to

$$\frac{A g \delta \nu \gamma^3}{g h_0}$$

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