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Progressive estimation of hypocentres and three dimensional crustal model

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ABSTRACT

The arrival times of a set of earthquakes at a seismic station network contain information both on the hypocentres of earthquakes as well as on the physical parameters of the area crossed by seismic ray paths. This paper discusses an evaluation that renders possible the contemporary reckoning of both the hypocentres and the model by means of a set-up of two computer program types which have separate characteristics but are both capable of reckoning the arrival times at the stations with three-dimensional variable speed models and the employment of a routine edited by the CERN COMPUTER CENTRE so as to minimize residuals.

To do this, the work is carried out following a sequece of approximations: on the basis of an initial model the hypocentres are worked out, and on the basis of the latter a more detailed model than the former is subsequently worked out; this process is repeated until a satisfactory solution is obtained.

In this paper, in order to determine the efficiency of the method employed, a theoretical simulation is used which gives encouraging results.

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Riassunto

I tempi di arrivo delle onde sismiche di un insieme di terremoti a una rete di stazioni contengono informazioni sia sugli ipocentri dei terremoti che sul modello della regione attraversata dai raggi sismici. In questo lavoro, tramite la messa a punto di due tipi di programmi per calcolatore a caratteristiche diverse ma entrambi in grado di calcolare i tempi di arrivo alle stazioni con modelli a velocità variabile in tre dimensioni e l'utilizzo di una routine edita dal « CERN COMPUTER CENTRE » per minimizzare residui, si è cercato di valutare la capacità di calcolare simultaneamente gli ipocentri e il modello.

Per fare questo, il lavoro si è svolto per approssimazioni successive; sulla base di un modello iniziale si calcolano gli ipocentri, e sulla base di questi ultimi si ricalcola successivamente il modello che sarà più dettagliato del precedente; si ripete questo processo fino a raggiungere una soluzione soddisfacente.

In questo lavoro per valutare la bontà del metodo è stata fatta una simulazione teorica che ha fornito dei risultati confortanti.

Introduction

The problem of determining hypocentres is closely linked to the knowledge of the behaviour of speed in the area near the seismic network. The errors attributed to the hypocentres are as a rule well below the real error; that is also due to the source of the earthquake never being pinpointed but being usually something that covers a certain surface or volume of rock, whence the point that we should be looking for would be either the barycentre of the area in question or the point of balance rupture that will later affect a whole volume.

But the main indeterminate cause of the error linked to a hypocentre is the great difference that is usually found between reality and the model used by us for evaluating the hypocentres.

Since, at present, we are not even able to evaluate to what extent the model used by us in a given area differs from reality, the idea of error associated with a hypocentre does not seem to make sense.

The problem however of determining a model that approaches reality is extremely important not merely for determining the hypocentres, but also for a whole series of other problems linked with more general model-making associated with earthquakes: it is linked to the origin of earthquakes, to the study of tectonics and also to the discovery of layers of various types. The accuracy of the solution of the model will be greater the more hypocentres there are, the better they are distributed throughout the area in question and the greater the area covered by a good local seismic network.

The overall idea behind the paper is to find, through a sequence of approximations starting from the arrival times, taken as a whole, of a set of earthquakes at seismic network, the best model evaluated in three dimensions and relative hypocentres.

From the paper of Crosson (1976) and Aki Lee (1976) a method has been worked out that enables the evaluation of model and hypocentres by successive stages. Thus, starting from an initial model chosen on the basis of former studies, prospecting or other means of seismic research, the hypocentres are determined. Thereafter the process is inverted; on the basis of the hypocentres thus determined a search is made for the model having the above hypocentres which best reproduces the arrival times of the waves to the stations and so on, complicating the model by degrees until the process becomes convergent. Of course, if would be absurd to expect to reach a level of detail comparable to that of real conditions. The latter may only be approached but never reached.

This was done using a computer minimizing technique, i.e. the « Minuit » edited by CERN COMPUTER CENTRE. The function to be minimized is made up of the sum of squares of residuals between the real arrival times at the stations of seismic ray paths and theoretical times worked out on the basis of the model chosen; it follows that the unknowns will at times be parameters of the hypocentre and at other times those of model, depending on the stage being investigated.

The decision to use this program is not, of course, the only possible one, although it is based on the program's reliability and on its adaptability to different situations. The next important feature is that it does not stop short at the relative minima and continues to seek the absolute minimum.

It involves using different minimizing criteria, none of which is based on the linearization of the equations and the subsequent employment of square minima or improvements of the method; the first « Simplex » according to Nelder and Mead (1965) starting from a point in n-dimensional space, where on represents the number of parameters to be sought, rectifying it on subsequent attempts following strictly geometrical criteria down to the most satisfactory situation, of great utility when one is relatively distant from the solution; the second, « Migrad » criterion, which is more rigorous from a theoretical viewpoint, based on the algorithm of Fletcher (1970), can be used in proximity to the solution. It works using as its starting point the solution supplied by « Simplex ». There also exists a very useful criterion for very difficult cases involving the search for a large number of parameters: in our case the latter amounted to about 15-20, though the predicted maximum is 55. It is known as « Seek » (James 1968) and works on a statistical criterion sorting out a certain number of positions where the value of the function can be worked out near the fixed starting point, and chooses among these the point where the function is at its least; there are also other methods that make the program extremely handy.

The main phase of the work was however the setting of two routines in a position to work out the travel rate employed by the seismic ray paths in a model with variable speeds in three dimensions, from a possible point source to a surface station. In the two processes the model is described in different ways; by means of either polygonals or parallelepipeds. In the first the area in question is divided up into polygonals of any shape and each section is assigned a certain model with an N number of layers with varying depths and speeds; the other routine separates the volume underlying the network into parallelepipeds with a constant base and variable depth, assigning a specific speed to each of them.

At the start of the process, when the information on the model is poor, the first routine is used because of its greater adaptation to any existing geological formations.

It supplies a more detailed description of the model, allowing the parallelepiped routine to be used, in which the dimension of the latter will at first be relatively large and then, possibly on the basis of the data, they will gradually become smaller.

Description of the poligonals routine

The area considered over which the station network is distributed is split into polygonals of any shape that separate geologically different zones. In each polygonal, which is described by the coordinates of the vertices, there will be a different crustal model for N parallel layers with layer depths and speeds that are typical of the polygonal.

A variable three-dimension model can thus be found within the area considered, which is quite suitable for simulating real conditions. The time that the seismic wave (only upgoing rays are considered) takes to travel from a given hypocentre to any station will be determined by taking into account refraction laws and considering the speeds through the layers actually crossed. It was noticed that it was worthwhile adopting different speeds among the different polygonals only for the first layer, i.e. the most superficial one, whereas for the deeper ones it was better to vary only the tickness within each polygonal. It must be mentioned that simplifying in this way does not render the program less general, since in a polygonal, a determination may, for instance, have a depth equal to zero, which means the exclusion of a given layer from a given area.

The program that works out the travel periods of seismic waves can basically be divided into two parts: a first part works out the polygonals affected by the sweep of the wave as well as the distance in kilometres on a horizontal plane; a second part works out the sweep of the wave and its travel rate, taking the polygonals involved into account.

Fig. 1 shows an area split up into five polygonals, E representing the surface projection of the hypocentre and S the station.

Each polygonal is represented by a different number; the intersections between the line AF joining E to S and the polygonals, are ABCDF. The first part of the program works out the distances AB, BC, CD, DF, and connects the corresponding polygonal to each of these distances.

Let us now consider a section corresponding to AF (see fig. 2). Each polygonal has its own different model. The second part of the program now computes the time taken by the wave to travel from the hypocentre I to the surface station S.

The problem of determining the start angle of the wave is

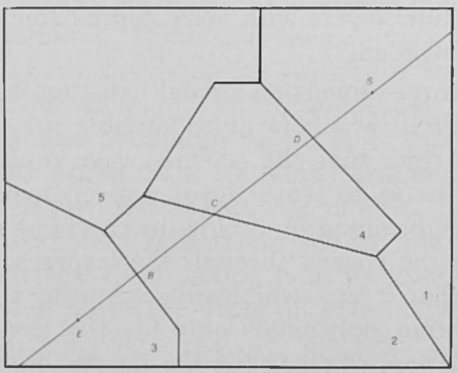

Fig. 1 - Example of region divided into five polygonals; E: epicenter S: seismic station.

resolved by trial and error; it is the program itself that selects the correct value of Θ by considering as valid the angle that enables the ray path to surface at a distance of less than Δ from the station S. The layers between one polygonal and another do not end suddenly, forming corners but, (as can be observed in fig. 2), run into planes; this is done so as to avoid sudden gaps and also in an attempt to follow natural layers as closely as possible.

One limitation of the program is that the run of the seismic wave, as can be seen in fig. 1, can only be straight in plan view, i.e. it does not take into account any angles in the run due to the model not being homogeneous; however this is a generally accettable approximation. (Generally the seismic wave speed does not vary suddenly on the horizontal plane).

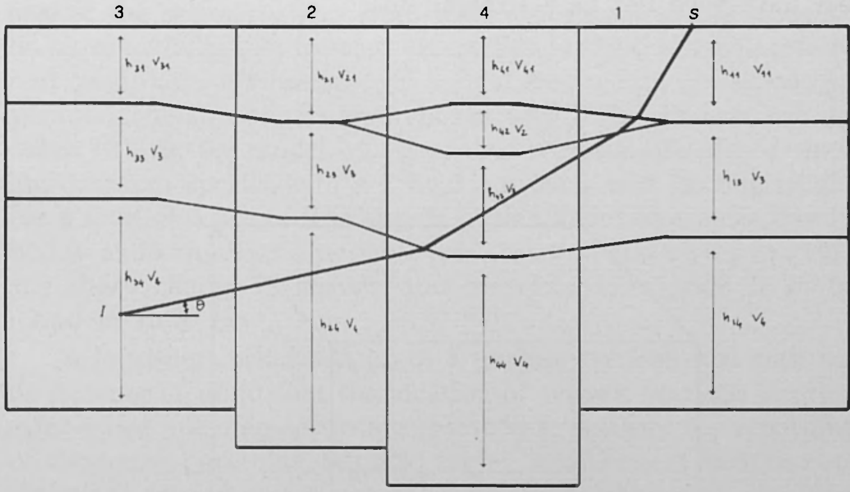

Fig. 2 - Two-dimensional speed model of a longitudinal section on the ES path of fig. 1.

Description of the parallelepiped routine

In this second case the area under investigation is split up into N parallelepipeds with equivalent bases and variable heights, see fig. 3.

A specific seismic wave speed is given to each parallelepiped, thus giving shape to a model variable in three dimensions; in its journey the seismic ray path undergoes refraction each time it leaves one parallelepiped and enters another at a different speed.

The direction at the start of the seismic ray path travelling from the hypocentre to the station, depending on two parameters, is determined by following a sequence of trials till the one that anables the ray path to get quite close to the station is found. The time employed to travel will then be found by adding up the travel period of each parallelepiped.

Also in this routine only upgoing rays are considered.

One advantage of this reckoning program compared to that of the polygonals is that the surface projection of the seismic ray path need not be a straight line.

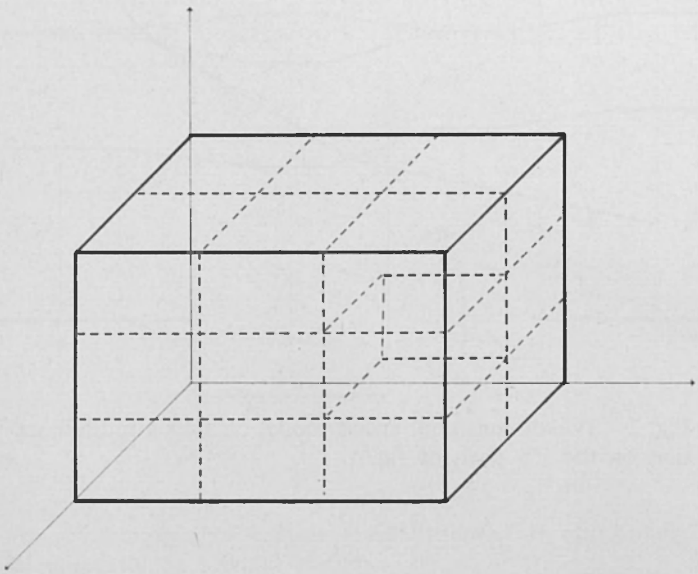

Fig. 3 - Example of blocks describing a crustal structure: given speed corresponds to each block.

The program is, however, subject to a limitation due to the sudden shifts in speed near the corners of the parallelepipeds; but this effect will be smaller, the smaller the size and the greater the number of parallelepipeds: at most, if the parallelepipeds were to have zero volume, we should have a continuous description of the model, which would be the ideal situation.

Simulation described

To test the above method a fully theoretical simulation was set up; a model variable in three dimensions was fixed, a network of stations hypothesized and a series of hypocentres was set out.

The first stage of the research was to compute the arrival times of the seismic waves from the hypocentres to the stations. So as to actually test to what extent this method is applicable to real conditions, a situation was sought that would not be too favorable. The size of the area chosen was 10 by 15 Km, and its depth 10 Km; the model was described with parallelepipeds having different speeds with a 2 by 3 Km base and varying height for a total of 5 layers. The speeds of the model have been described so as to simulate a possible real situation and trying to avoid any sharp change in speeds; the complete description is to be found in (tab. 1).

A local network made up of 8 seismic stations was then set up bearing in mind that the location of seismic stations is often established not only according to the best position for recording on theoretical grounds, but also taking into account various contingent requirements; it was thus decided to choose the position of 4 stations at random and to place the others in such a way as to render the structure of the network efficient. (tab. 2).

It was endeavoured to distribute the hypocentres throughout the area at varying depths, see fig. 4.

The times of travel of the seismic waves were determined using the parallelepiped routine and allowing for an error of 10 m, between the wave emergence point and the station, i.e. an error

(*) It must be underlined that, on the basis of the consideration made in the introduction, in this paper the errors in determining the hypocentres and the model are no longer taken into account.
Furthermore no values are reported for residuals as their value generally does not depend on the accuracy of the solution but on the threshold we have selected to decide when the process is convergent.

in the times of travel of the order of a thousandth of a second. It was here that the real work began: the first stage was to determine the hypocentres. To do this we supplied the program with a first approximation model made up of three parallel and

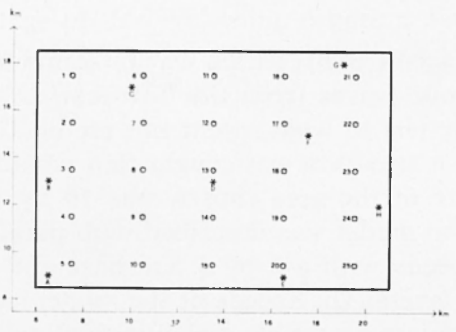

Fig. 4 - Map showing seismic stations and simulated epicentres.

homogeneous layers the thickness of which was arbitrarily fixed on the grounds of a typical distribution (fig. 5).

As can be seen (fig. 6, tab. 7), from this first hypocentral determination, there is a mean epicentral error of about 2.6 km,

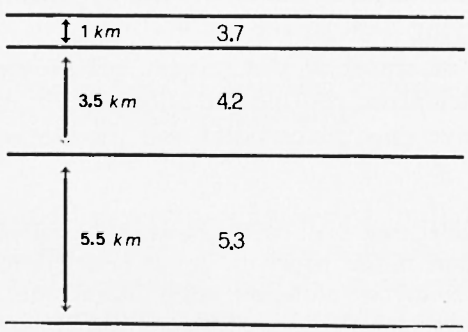

Fig. 5 - Three-layer model employed for the first hypocentre location.

and the epicentres tend to shift to the centre of the area; this is probably due to an initial underrating of our speeds. The first determination of the model was made dividing the area observed into 4 equal rectangular polygonals (fig. 7, tab. 3).

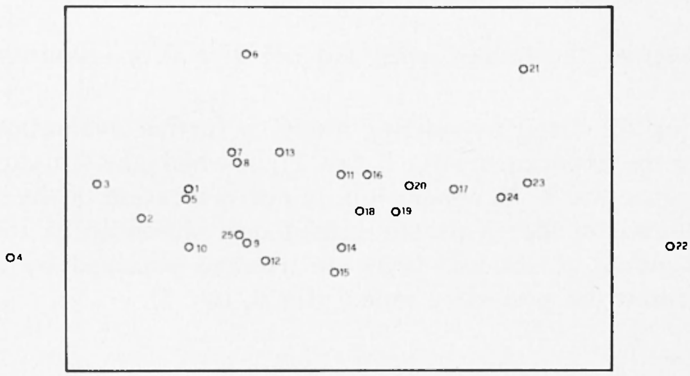

Fig. 6 - Map showing the first hypocentre location.

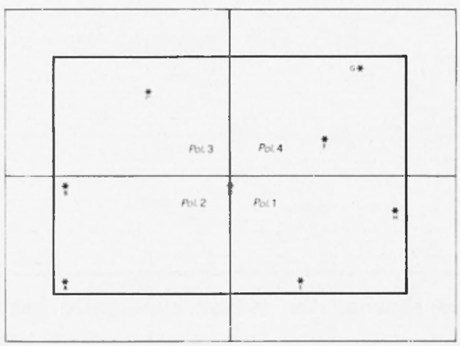

Fig. 7 - Map showing the adopted subdivision into polygons.

In this stage of the work the input consists of the hypocentres worked out previously and the starting model is the one used in the first estimate of the hypocentres, in which, however, an attempt was made to insert a fourth slightly thicker layer.

At this stage we would like to mention that the use of polygonals during the first stage of the work is to be preferred since it enables one to vary the thickness of the layers even if, as in our case, they are chosen rectangularly shaped and not otherwise, as is generally the case.

The results of this first model determination are set out in tab. 4.

Inserting the fourth layer did not give very favourable results.

Using the latter as starting model, a further evaluation was made of the hypocentres (fig. 8, tab. 7), in which the situation did not improve much. In conclusion, in our evaluation of the model let us now consider a parallelepiped description for a total of 24 and giving as starting point an average obtained by taking into account the preceding model (fig. 9, tab. 5).

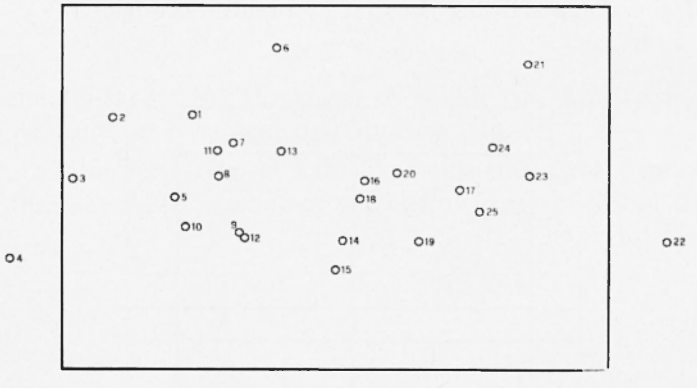

Fig. 8 - Map showing the second hypocentre location.

It should be noted that at this stage there is a steep rise (tab. 6) in all the speeds of the surface layers compared to the preceding model and above all compared to the 3.7 km./sec. start. There are however some areas still far from the solution but it should de underlined that such areas are not sufficiently covered by seismic stations.

Finally on the basis of the latter model we have determined the hypocentres which, as can be seen (fig 10, tab. 7), are markedly improved.

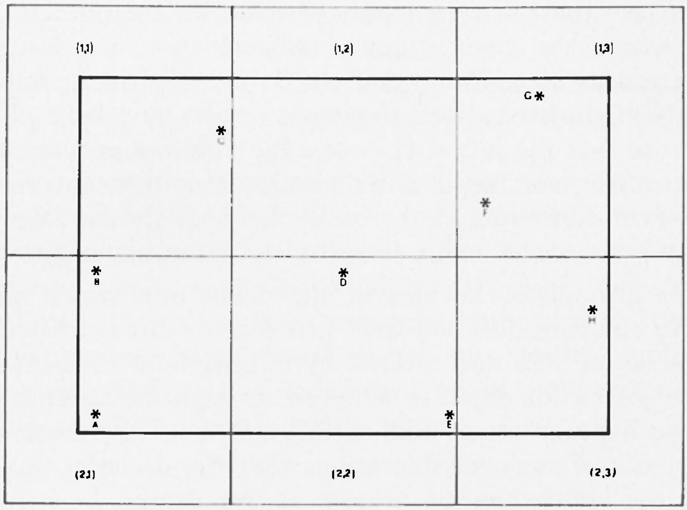

Fig. 9 - Map showing the earth under the seismograph network. The adopted subdivision into blocks.

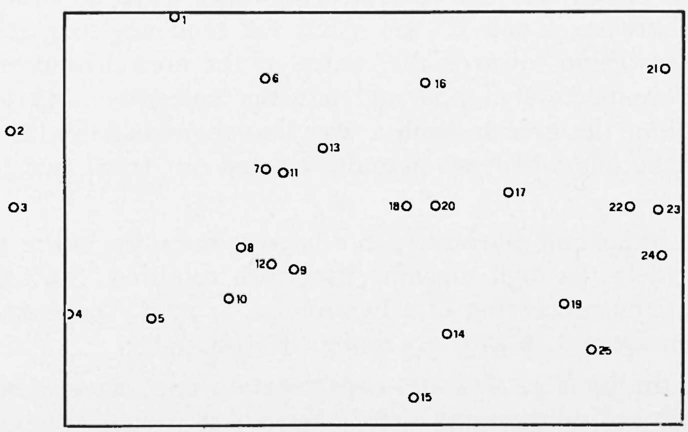

Fig. 10 - Map showing the third hypocentre location.

Conclusion

The final results can be said to be accettable if we take into account various causes of disturbance. Essential elements to this end are the scanty amounts of data, i.e. the number of hypocentres and the distribution of network that should surround the area under observation and not be included in it; actually if we pinpoint the area where the worst results have been obtained we can see that the latter, (1,2) (see fig. 4) is not covered by any station whatsoever, while if we observe the hypocentres of the second determination, on the basis of which the model was deduced, we can see that this area is also without epicentres.

This also gives the idea of the detail into which one can delve: in our consideration with just 6 areas differentiated there came to be an area not covered by information. It is thus clear that the search for detail is achieved through the number of stations and hypocentres as well as through their distribution. It is on the basis of such considerations that the decision was taken to stop the approximation process at this degree.

However the general behaviour of the whereabouts of the hypocentre, following insertion of the non-homogenous model, is actually found to be markedly improved. Not only in the majority of cases did the epicentral distance have an acceptable value (between 1 and 2.5 km.), but the tendency to gather the points of origin towards the centre of the area disappeared. It is now immediately apparent that the epicentres are located throughout the area in such a way that their distribution much recalls the one which we actually started out from (see fig. 10, fig. 4).

An important element with a bearing upon the entire technique used is the high machine time rate required. Suffice it to say that the processing of a hypocentre or model determination requires about 1.30 CPU hrs using UNIVAC 1108.

In the light of this last consideration one can evaluate the possibility of determining simultaneously in a single elaboration both the hpocentres and speed of model; in fact while it is possible in this instance to obtain more satisfactory results it is also true that the machine time rate would be about equivalent and fully concentrated into one single run. Added to this is the need for using a minimizing program with a good stability notwithstanding the elevated number of parameters to be determined (4 for every hypocentre plus all the speeds).

As regards the real applications of such a process for the hypocentres and the model, it is evident than once a satisfactory model is found, it can be directly used for subsequently hypocentres determination.

Lastly it must be stressed that the problem of evaluating the model and hypocentres on the basis of the arrival times at the stations alone cannot be determined at the start: e.g. there may be different solutions (relative minimals) which may have no connection with reality. It is thus very important to start from a well determined model based on previous studies; such a technique must therefore be understood as an improvement of a situation which is already partly known.

TABLE 1

A diagram of the artificial simulation model.

1° layer	3.86	4.28	4.57	4.74	4.78	Thickness 0.4 km
	3.86	4.25	4.35	4.64	4.74	
	3.86	4.35	4.35	4.50	4.57	
	3.64	3.86	4.07	4.28	4.35	
	3.50	3.64	3.86	4.07	4.07	
2° layer	4.78	4.78	4.78	4.82	4.82	Thickness 1.0 km
	4.74	4.78	4.78	4.78	4.82	
	4.71	4.74	4.78	4.78	4.78	
	4.67	4.71	4.74	4.78	4.78	
	4.67	4.67	4.71	4.74	4.74	
3° layer	4.50	4.35	4.28	4.21	4.21	Thickness 1.6 km
	4.57	4.42	4.35	4.28	4.21	
	4.71	4.50	4.39	4.35	4.28	
	4.78	4.64	4.50	4.39	4.35	
	4.78	4.71	4.64	4.57	4.39	
4° layer	4.57	4.74	4.85	4.92	4.92	Thickness 2.0
	4.50	4.57	4.74	4.85	4.85	
	4.35	4.42	4.57	4.74	4.74	
	4.28	4.28	4.24	4.57	4.64	
	4.28	4.28	4.35	4.50	4.57	
			5.1 km/sec throught			thickness 5.0 km.

TABLE 2

Coordinates of the artificial seismic stations

station	X	Y
A	6.5	9.0
B	6.5	13.0
C	10.0	17.0
D	13.5	13.0
E	16.5	9.0
F	17.5	15.0
G	19.0	18.0
H	20.5	12.0

TABLE 3

Starting point for the first model determination.

Thickness	velocity	layer
1.0	3.7	1
0.2	3.9	2
3.5	4.2	3
5.3	5.3	4

TABLE 4

First results of model determination.

layer	pol. n. 3			pol. n. 4		
	thickness	v	δv	thickness	v	δv
1	.84	3.66	.81	1.0	3.80	.98
2	.2	3.92	.85	.2	3.92	.88
3	4.0	4.22	.34	4.0	4.22	.40
4	4.96	5.4	—.30	4.8	5.4	—.30
layer	**pol. n. 2**			**pol. n. 1**		
	thickness	v	δv	thickness	v	δv
1	.96	3.66	.67	1.1	3.86	.70
2	.15	3.92	.78	.2	3.92	.84
3	4.0	4.22	.30	4.0	4.22	.34
4	4.80	5.4	—.30	4.76	5.4	—.30

TABLE 5

First results of model determination.

layer	blocks					
	(1,1)	(1,2)	(1,3)	(2,1)	(2,2)	(2,3)
1	3.66	3.75	3.80	3.66	3.76	3.86
2	4.00	4.00	4.00	4.00	4.00	4.00
3	4.25	4.25	4.25	4.25	4.25	4.25
4	5.40	5.40	5.40	5.40	5.40	5.40

TABLE 6

Starting point for the second model determination.

layer	blocks (1,1) v	δ v	(1,2) v	δ v	(1,3) v	δ v	(2,1) v	δ v	(2,2) v	δ v	(2,3) v	δ v	thickness km
1	4.59	−.45	3.86	.66	4.10	.62	4.20	—.29	4.24	—.06	4.20	.17	0.5
2	4.66	0.00	4.18	.42	4.31	.27	4.51	.18	4.18	.49	4.04	.58	1.5
3	4.74	—.21	4.33	.28	4.33	.32	4.33	.11	4.33	.14	4.33	.21	3.0
4	5.40	—.30	5.40	—.30	5.40	—.30	5.40	—.30	5.40	—.30	5.40	—.30	5.0

TABLE 7

Location results for different structure model.

z_0	$\delta\Delta_1$	δz_1	δt_{01}	$\delta\Delta_2$	δz_2	δt_{02}	$\delta\Delta_3$	δz_3	δt_{03}
6.0	4.4	3.0	.8	2.9	2.6	.6	1.5	5.0	.4
2.5	2.9	—1.8	—.3	0.2	—2.0	—.5	2.8	—4.7	—.9
1.0	0.6	—2.1	—.4	1.2	—2.2	—.4	2.7	—2.7	—.6
0.5	3.0	—0.6	—.0	2.9	0.3	.0	1.4	0.4	.5
5.0	4.1	1.8	—.3	4.0	1.8	.3	1.7	—4.1	—.4
2.5	0.6	0.0	—.1	1.4	2.0	—.2	0.7	0.4	.1
4.0	1.0	2.3	.2	0.8	2.2	.2	0.9	2.2	.2
3.5	0.4	—1.5	—.3	0.4	—1.8	—.4	0.7	—3.6	—.5
1.0	0.7	—1.5	—.2	0.7	—1.6	—.2	1.4	—1.4	—.1
2.5	2.6	—0.4	—.1	3.1	—0.5	.0	2.1	1.6	.4
4.0	3.6	—5.0	—.6	4.4	—5.8	—.8	3.6	—4.3	—.5
5.5	4.5	2.2	.2	1.8	1.4	.2	3.9	3.4	.5
3.0	1.9	—0.1	.0	1.8	—0.4	—.1	2.2	—2.8	—.3
2.0	0.4	—0.9	—.2	0.5	—0.9	—.2	2.0	—5.7	—.9
3.5	1.7	0.4	—.1	1.7	0.6	—.1	1.0	3.4	—.1
3.0	4.2	—0.6	—.4	4.4	—0.6	—.4	2.4	0.3	—.5
1.5	2.0	—0.3	—.3	2.1	—0.4	—.3	1.4	—1.9	—.4
4.0	2.5	0.7	—.1	2.3	—0.4	—.3	2.2	—1.9	—.4
5.0	2.0	2.0	.1	0.9	1.3	.0	1.6	—2.0	—.4
0.5	4.2	—2.7	—.5	4.6	—2.7	—.5	4.8	—1.3	—.4
2.0	1.1	0.8	.0	0.9	0.7	.0	1.1	—1.7	—.4
2.5	4.7	—0.7	—1.0	4.7	—0.7	—1.0	1.7	—2.1	—.5
0.5	0.8	—1.3	—.2	0.8	—1.3	—.2	0.9	—4.2	—.7
2.0	2.3	—1.0	—.2	3.5	—0.8	—.2	1.5	—0.4	—.5
4.5	9.1	1.4	.2	3.8	1.5	—.5	1.1	1.5	—.5

z_o is depth of the initial earthquakes

$\delta\Delta_1$, δz and δt_{01} are the epicentre, depth and origin time errors for the first location.

$\delta\Delta_2$, δz_2 and δt_{02} are the epicentre, depth and origin time errors for the second location.

$\delta\Delta_3$, δz_3 and δt_{03} are the epicentre, depth and origin time errors for the third location.

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