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Magnetotelluric sounding of the crust and hydromagnetic monitoring of the magnetosphere with the use of ULF waves

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Abstract
The problem of the unification of the magnetospheric diagnostics and the magnetotelluric sounding with the use of ULF waves is reviewed. Some fundamental problems of magnetotellurics which cannot be resolved without a detailed knowledge of the MHD wave transformation in the magnetosphere and the ionosphere are discussed. Summary of various methods to determine experimentally the ULF spatial structure parameters – resonant frequency, meridional gradient, width of the resonance is given. Some possible directions of further modification of magnetotelluric sounding technique and of the magnetospheric diagnostics are indicated.

Key words *ULF waves – magnetotellurics*

1. Introduction

Nowadays the practical applications of the ground-based observations of ULF waves are mainly related with the hydromagnetic diagnostics (HD) of magnetospheric plasma and with the magnetotelluric sounding (MTS) of the Earth crust. These two approaches were developed practically independently, though the problem of their unification has already been put forward (Gugliel'mi, 1989a,b). In this review we would like to draw attention to the problem of «what MTS specialists should know about magnetospheric field line resonances». As will be demonstrated, the specifics of the MHD wave propagation in the magnetosphere and the ionosphere can influence the fundamentals of the MTS theory. Also, it will be shown that hydromagnetic diagnostics of the magnetosphere could be enriched by the knowledge of all components of a telluric field.

2. Magnetotelluric and magnetospheric field line resonance

Methods of MTS constitute a useful part of geophysical prospecting. It is widely accepted that for the MTS with the use of ULF pulsations the treatment of these waves as magnetic disturbances excited by some ionospheric currents is quite sufficient. Details of ULF generation mechanisms and of MHD wave propagation and transformation in the magnetosphere and ionosphere are not considered. Below we will try to analyze the problem if the existing peculiarities of disregarded processes, in particular – resonant properties of the magnetosphere and mode conversion in the ionosphere, could put in question some principles of interpretation of magnetotelluric data.

2.1. *Tikhonov-Cagniard model*

The basic physical principles of MTS are rather clear and are well explained in many reviews and books (see, for example, Dmitriev and Berdichevsky, 1979; Van'jan and Butkovsakaya, 1980). The horizontal components of the magnetotelluric field $\boldsymbol{E}$ and $\boldsymbol{H}$, recorded at the same point of the Earth's surface, are related through impedance condition. In its turn a surface impedance is determined by the distribution of the crust conductivity σ along depth h. The magnetotelluric problem is reduced to the measurement of a surface impedance and the restoration of a dependence $\sigma(h)$ by the parametric dependence of impedance on frequency. The characteristics of a crust's cross-section are obtained as a result of a fit of model parameters to the MTS curves.

The physical basis of MTS is the Tikhonov-Cagniard (T-C) model based on the following physical considerations. In the atmospheric gap a ULF field has a quasi-static character, because in this frequency range a wavelength in the atmosphere is 10^2-10^3 times longer than the Earth's radius. While penetrating a highly conductive Earth (the condition of «high conductivity» will be given below), the ULF electromagnetic field at the Earth's surface is characterized by an impedance coinciding with the impedance of a plane vertically propagating wave. Thus for shortness, will speak below about T-C model as the model of a vertically incident plane wave, keeping in mind a certain incorrectness of this definition. In general, the transverse components of a magnetotelluric field near the Earth's surface are related by the T-C operator $\hat{Z}$, which parametrically depends on frequency ω and depends on a distribution of $\sigma(h)$:

$$\boldsymbol{E}_\perp = \hat{Z}\,(\boldsymbol{H}_\perp) \qquad (2.1)$$

For the case of stratified media, which constitute a basis of the magnetotelluric theory, the operator $\hat{Z}$ is an integro-differential transformation, *i.e.*

$$\boldsymbol{E}_\perp = \int G(\boldsymbol{r}-\boldsymbol{r}')[\boldsymbol{n}\boldsymbol{H}_\perp(\boldsymbol{r}')]\mathrm{d}s \qquad (2.2)$$

Here $\boldsymbol{n}$ is a versor normal to the Earth's surface. the kernel G of an integral operator can be determined by the spectral impedance $Z^{(h)}(\omega, k_x, k_y)$ (Dmitriev and Berdichevsky, 1979). Herewith axes x, y correspond to north-south and east-west directions, k_x and k_y are spatial spectral parameters.

In the case of a vertically incident plane wave the relation (2.2) results in an extremely simple form

$$\boldsymbol{E}_\perp = -Z_0[\boldsymbol{n}\boldsymbol{H}_\perp] \qquad (2.3)$$

The equation (2.3) is known in the theory of radiowave propagation over high-conductive surfaces as the Leontovich's boundary condition. In the magnetotelluric, Z_0 is the T-C impedance, which is a functional of a conductivity.

To derive the criteria of practical applicability of a T-C model we will remind the asymptotic behaviour of $Z^{(h)}$ for $k_\perp = (k_x^2 + k_y^2)^{1/2} \Rightarrow 0$. For homogeneous Earth ($\sigma = \text{const.}$), it has the following form

$$Z_0^{(h)}(k_x, k_y) = Z_0\left(1 + i\frac{k_\perp^2 d^2}{2}\right)^{-1/2} \qquad (2.4)$$

where $Z_0 = \sqrt{-\dfrac{i\omega}{4\pi\sigma}}$, $d = (2\pi\omega\sigma)^{-1/2}$ is a skin-dept. From (2.4) one can see that if a field is homogeneous over horizontal scales which are much greater than the skin-depth (*i.e.* $k_\perp^2 d^2 \ll 1$), then spectral impedance $Z^{(h)}(k_x, k_y)$ coincides with T-C impedance Z_0 (Wait, 1954).

But T-C model actually has a much wider range of applicability than it follows from Wait's criteria (Price, 1962). It turns out that when spatial variations of horizontal field components are linear, then integral operator (2.2) degenerates to a matrix operator, *i.e.*

$$\hat{Z} = \begin{Bmatrix} O & Z_0 \\ -Z_0 & 0 \end{Bmatrix} \qquad (2.5)$$

The action of this operator is reduced to a simple multiplication by Z_0. Hence, for any steep linear variation of $H(r)$ at the spatial scales not less than 3d a formal definition of T-C impedance is possible.

On the basis of this important conclusion, many leading specialists in magnetotellurics made a conclusion about «nearly universal applicability of T-C model for interpretation of the MTS data» (Dmitriev and Berdichevsky, 1979).

2.2. *Directional analysis*

However for a low conductive layer the condition of T-C model validity may be violated. In this case a layer impedance would be determined not only by a conductivity, but by a spatial structure of an incident field as well. As a model of local ULF horizontal structure it is natural to adopt an inhomogeneous plane wave, *i.e.*

$$(E, H) \sim \exp(-i\omega t + i\boldsymbol{k}\,\boldsymbol{r}) \quad \text{where } \boldsymbol{k} = \boldsymbol{k}' + i\boldsymbol{k}' \tag{2.6}$$

because the experimental data for all types of ULF pulsations indicate their horizontal propagation along the Earth's surface.

The theory of MTS, based on the plane inhomogeneous wave notion (Dmitriev, 1970; Tikhonov *et al.*, 1974), comprises some other physical effects. The ionospheric currents excited by a magnetospheric MHD wave can excite electromagnetic disturbances in the Earth's crust via two mechanisms. The first one, actually assumed above, is an electromagnetic induction. Since the atmospheric gap has some electric conductivity, a galvanic mechanism, *i.e.* the penetration of some part of ionospheric current through the atmosphere into the Earth is also present. The existence of these two mechanisms formally reveals in the splitting of Maxwell's equations into two independent sub-systems. One corresponds to H (magnetic) mode (the above consideration assumes the presence of this mode only) and another to E (electric) mode. These modes have different partial spectral impedances $Z^{(h)}$ $(\omega, k^2_\perp)$ and $Z^{(e)}$ $(\omega, k^2_\perp)$. Only when the condition of strong skin-effect $k^2_\perp d^2 \ll 1$ (or Leontovich's boundary condition) is fulfilled the impedances of these partial modes coincide $Z^{(h)} \sim Z^{(e)} \sim Z_0$. In general case, when E-mode is allowed, T-C operator $\hat{Z}$ comprises not only an integral term (as in (2.2)), but also an integro-differential term

$$E_i(\boldsymbol{r}') = \int G_{ij}(\boldsymbol{r} - \boldsymbol{r}')H_j(\boldsymbol{r})\mathrm{d}r + \\ + \int \Delta G(\boldsymbol{r} - \boldsymbol{r}')\, \hat{L}\,(\boldsymbol{H})\mathrm{d}s \quad (i = x, y) \tag{2.7}$$

where $\hat{L}$ is some 2nd order differential operator (Dmitriev and Berdichevsky, 1979).

Chetaev (1985) developed practical algorithms for splitting an original ULF field into partial H and E modes, basing on simultaneous 6-component ULF data in one point or $\boldsymbol{E}_\perp$, $\boldsymbol{H}_\perp$, H_z measurements in two points. As a result, partial surface impedances $Z^{(e)}$ and $Z^{(h)}$ can be determined experimentally. There are geoelectric structures where effective skin-depth in the Pc3-4 range is comparable to a horizontal field scale (about 100-200 km). In these cases the partial impedances become dependent on wave propagation characteristics. Then for wave packets with certain $k^2_\perp$ the condition of total reflection from an underlaying layer can be fulfilled. In that case the layer boundary is stressed by the characteristics of the wave, and partial impedances must have a pronounced peculiarity near critical values. This interpretation of MTS data forms the basis for a new scheme of MTS suggested by Chetaev (1985). The new scheme of MTS (so called directional analysis) was verified experimentally in 1973 and gave a reasonable profile of upper mantle.

The described above MTS schemes were based on phenomenological notions about a ground-based ULF structure, whereas the physical mechanisms governing the ULF propagation characteristics were out of consideration. Let us discuss now why the «magnetospheric» aspect of the physics of ULF waves might be essential for the problems of MTS data interpretation.

2.3 *Resonant structure of ULF field*

MHD disturbances from remote parts of the magnetosphere (for example, solar wind disturbances, magnetopause surface waves, magnetosheath turbulence, etc.) propagate inside an inhomogeneous magnetosphere and somewhere transform into Alfven field line oscillations. In their turn the Alfven waves impinging the ionosphere in most cases are the sources of ULF geomagnetic pulsations (Pc3-5, Pi2) observed on the ground (Yumoto, 1986). The process of the mode transformation is most effective in the vicinity of the resonant geomagnetic shells where the frequency f of the external source coincides with the local frequencies f_R(L) of Alfven field line oscillations, *i.e.* $f \sim f_R$(L) (Chen and Hasegawa, 1974; Southwood, 1974). Formal mathematical description of the spatial field structure in the magnetosphere near resonant shells is given by the qualitative theory of differential equations (Kivelson and Southwood, 1986; Krylov and Fedorov, 1976; Krylov *et al.*, 1981).

$$\begin{aligned} b_y(x, f) &= c_1 w^{-1} + c_2 k_y^2 ln(w) + \dots \\ b_x(x, f) &= d_1 ln(w) + d_2 + \dots \\ w &= x - x_R(f) + i\in \end{aligned} \qquad (2.8)$$

where x is the coordinate of a magnetic shell; x_R is the point where $f_R(x) = f$; $\in$ is the width of a resonance region; coefficients c_i, d_i are determined by system geometry, source and boundary conditions. The expression (2.8) is valid for arbitrary geometries of geomagnetic fields and is the generalization of the original simplified models of the magnetospheric resonator (Southwood, 1974, Chen and Hasegawa, 1974; Radoski, 1974).

The leading term in the asymptotic expression (2.8) which describes the resonant singularity of the b_y component near a resonant shell, *i.e.* when $|x - x_R(f)| \leq \in$, can be presented in the form

$$b_y(x, f) = \frac{i\in}{x - x_R(f) + i\in} h_0(f) \qquad (2.9)$$

On the grounds of (2.9) the meridional structure of ULF field can be qualitatively imagined as the superposition of a «source» spectrum $h_0(f)$ and a magnetospheric resonance response $\sim [x - x_R(f) + i\in]^{-1}$. The «source» part is related with a disturbance transported by a large-scale fast compressional mode and it slightly depends on the coordinate. But resonant magnetospheric response related with Alfven waves excitation is strongly localized and it causes the rapid variations of amplitude and phase during the crossing over a resonant shell. The radial component $b_x(x, f)$ has a weaker logarithmic singularity near resonance, so the resonant behavior of this component should hardly be noticeable.

After transmission through the ionosphere the spatial structure of large scale oscillations remains the same if the wave parameters are replaced by the rule: $b_x \rightarrow D$, $b_y \rightarrow H$, and $\in \rightarrow \in + h$ (h is the height of the ionospheric E-layer) (Hughes and Southwood, 1976a,b; Leonovich and Mazur, 1991a,b; Al'perovich *et al.*, 1991). The leading term (2.9) which describes amplitude and phase characteristics of H component can also be presented in the form (Gugliel'mi, 1989a):

$$H(x, f) = \frac{h_R(f)}{1 - i\zeta} \qquad (2.10)$$

where $\zeta = (x - x_R)/\in$ is the normalized distance from the resonant point $x_R(f)$, $\in$ is the resonant width, and $h_R(f)$ is the amplitude of the pulsation at the resonant point.

The resonant structure (2.9), (2.10) of ULF pulsations cannot be approximated neither by a vertically incident plane wave, nor by a plane inhomogeneous wave. Hence, magnetospheric resonant effects may cause distortions of a standard MTS curve near local resonant frequency which could be misinterpreted as false features of the Earth crust structure. This situation is illustrated in fig. 1 taken from Al'perovich *et al.* (1991). As an example 4-layer geoelectric cross-section with the following parameters was chosen: $\rho_1 = 30$ Ohm · m, $h_1 = 3$ km, $\rho_2 = 3 \cdot 10^3$ Ohm · m, $h_2 = 50$ Km, $\rho_3 = 3 \cdot 10^2$ Ohm · m, $h_3 = 50$ Km, $\rho_4 = 3 \cdot 10^{-2}$ Ohm · m, $h_4 = \infty$. The numerical calculations of the field structure and corresponding impedances were made for a vertically incident plane wave (solid line) and for a resonant structure (2.8) at latitudes $\phi = 60°$ (dashed line) and $\phi = 55°$ (dotted line). The comparison of apparent resistance curves for classical T-C model $\rho_a(T)$ and for a resonance structure of the incident field $\rho_a^*(T)$ shows additional extrema near the local resonance periods (100 sec and 46 sec, correspondingly).

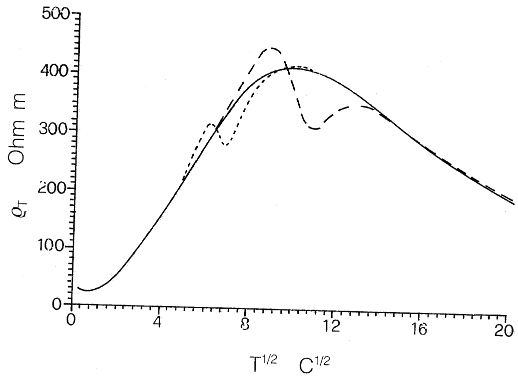

Fig. 1. The apparent resistance $\rho_a(T)$ curves for the classical T-C model (vertically incident plane wave) – solid line and $\rho_a^*(T)$ for the resonance structure of the incident field (latitude $\phi = 60°$ – dashed line; $\phi = 55°$ – dotted line) above 4-layer geoelectric structure.

Deflection of two curves reaches 30%. Distortion of MTS curves by resonant effects would be especially pronounced above low-conductive layers.

Recent experimental studies of the resonance effects role in the formation of a local meridional ULF structure proved the predictions of the resonance theory. Hence the unambiguous application of standard MTS methods requires a preliminary hydromagnetic diagnostics (HD) of magnetospheric resonance frequencies in a region under study. This fact demonstrates once again the necessity of unification of two approaches to the use of geomagnetic pulsations – HD and the magnetotelluric sounding (MTS) (Gugliel'mi, 1989a; Pilipenko, 1990).

Below, some examples which illustrate the fruitfulness of such «combined» approach and indicate new opportunities for HD will be considered.

3. Possible input of electric mode in the ULF pulsation structure

The presence of E-mode modifies the impedance relations for spectral components $\boldsymbol{e}_\perp$, $\boldsymbol{h}_\perp$ of a telluric field

$$e_x = Z^{(h)}h_y + \Delta Z(k_x k_y h_x - k_x^2 h_y)$$
$$e_y = -Z^{(h)}h_x + \Delta Z(k_y^2 h_x - k_x k_y h_y)$$
$$\Delta Z = \frac{Z^{(h)} - Z^{(e)}}{k_x^2 + k_y^2} \qquad (3.1)$$

In a classical magnetotellurics the input of E-mode is considered to be negligible and the terms with a difference of spectral impedances $\sim \Delta Z$ in (3.1) are ignored (Berdichevsky *et al.*, 1971). The theoretical calculations of the magnetospheric MHD wave transmission through the ionosphere (Al'perovich and Fedorov, 1984a,b; Pudovkin *et al.*, 1984) show a very low effectiveness of E-mode excitation. In essence, the rate of the E-mode generation by magnetospheric processes should be proportional to the rate of magnetospheric vertical current penetration into a low-conductive atmosphere. For all theoretical models and for a wide range of ionospheric and pulsation parameters the vertical component of ULF electric field in the air near the Earth's surface does not exceed several V/m. Direct E_z measurements in the air recorded E_z variations in the ULF frequency range with amplitudes less than 10 V/m (Chetaev *et al.*, 1975; Chetaev *et al.*, 1977). As follows from some estimates (Berdichevsky *et al.*, 1971) an E-mode can give an essential input into the pulsation structure and must be taken into account in MTS studies only when the magnitude of vertical electric field component in the air exceeds 10^3 V/m.

On the other hand, some authors (Tikhonov *et al.*, 1974; Chetaev, 1985; Savin *et al.*, 1991) suggested that many experimentally observed paradoxes of magnetotellurics (non-orthogonality of E and H ellipses, large magnitudes of j_z in bore-holes, opposite rotation of E and H polarization ellipses, etc.) cannot be resolved without regard for E-mode existence. They supported their conclusions by the experimental checking up of the directional analysis algorithms for dayside Pc3 pulsations (Chetaev, 1985; Savin *et al.*, 1991). In their experiments with a 6-component telluric Pc3 field registration, the large vertical electric currents in bore-holes reaching 10^{-8} A/m^2 were detected. The authors ascribed these intense currents to an incident ULF field and not to the influence of geoelectric inhomogeneities. Consequently, the conclusion about an essential input of E-mode in the ULF structure was made. In many events partial modes of E-type turned out to be even prevailing. Finally, the directional method of MTS gave a reasonable picture of an upper mantle vertical structure (Chetaev, 1985).

So far there is some uncertainty concerning the possible input of E-mode in the structure of geomagnetic pulsation field and its role in magnetotellurics, in particular, interpretation of bore-holes observation data. This problem is still open and its solution requires specialized 6-component observations of ULF pulsations, including the atmospheric vertical electric field registration. Reliable techniques which will allow to separate a vertical electric field and currents, induced correspondingly by lateral inhomogeneities and by an incident ULF wave, should be developed in greater detail.

4. Hydromagnetic diagnostics of the magnetospheric plasma parameters by the data of ground based observations

The above discussion shows that, for correct MTS or HD, effective and simple methods of operative determination of the resonant frequency distribution in a given region should be elaborated. With this aim in mind, below we will analyze various ground-based methods of determination of f_R and its meridional gradient. Frequencies of a fundamental mode of field line Alfven oscillation and its harmonics may vary considerably in space (from 10 sec at low latitudes till 10 min at high latitudes) and in time. In fact, f_R depends on geomagnetic field B_0, magnetospheric plasma density distribution n, ion plasma composition, and all these parameters vary considerably (by order of magnitudes and more) with local time and with the magnetospheric activity. Therefore, near real-time monitoring of Alfven resonant frequencies is desirable for a productive MTS. Below we will describe some methods of f_R determination with the use of the same magnetotelluric data. Both well-known methods, as well as relatively new ones are considered and compared.

4.1. *Gradient method*

Investigations of the ULF spatial structure have been conducted for tens of years, but only in the last decade the fundamental role of an Alfven field line resonance in the formation of a meridional structure of ULF pulsations at all latitudes was demonstrated. Despite the extreme simplicity of the resonance model (Southwood, 1974; Chen and Hasegawa, 1974), a theoretically predicted amplitude and phase meridional structure well corresponds to the experimental local structure of various types of ULF waves at sub-auroral and middle latitudes: Pc5 (Walker *et al.*, 1979), Pc4 (Green, 1978; Baransky *et al.*, 1985, 1989), Pc3 (Fukunishi and Lanzerotti, 1974; Best *et al.*, 1986) and even Pi2 (Pilipenko *et al.*, 1988). It was found that the existence region of Alfven resonance spreads till rather low latitudes ($L < 2$). The resonant amplification of high-frequency Pc3 band at $L = 1.5$-1.7 was noticed by Hattingh and Sutcliffe (1987), Waters *et al.* (1991), Vellante *et al.* (1989), Green *et al.* (1992).

The principal problem of the experimental $f_R(L)$ determination consists in the fact that in most events the input in a spectral content of ULF pulsations from resonant magnetospheric response and the one from «source» are comparable. So in most cases a spectral peak does not correspond to a local resonant frequency, and the width of a spectral peak cannot be directly used to determine the Q-factor of the magnetospheric resonator. Usually, reasonable results are obtained with the use of statistics, but for individual events the results are contradictory. This ambiguity can be resolved with the help of the listed below experimental methods, in particular – the gradient method proposed by Baransky *et al.* (1985). The precision measurements of gradients of spectral amplitude (desirably – phase also) at a small base allow to exclude an influence of a source spectrum form and to reveal even relatively weak resonant effects.

Here we will summarize the simple relationships stemming from function (2.10) properties which describe the changes in the ULF spectral characteristics between two near-by stations along a meridian.

The ratio between amplitude spectra and the difference of phase spectra of ULF H-components, recorded in points x_1 and x_2 ($x_1 > x_2$) have the following form:

$$G(f) = \frac{|H(x_1, f)|}{|H(x_2, f)|} = \left(\frac{1 + \zeta_2^2}{1 + \zeta_1^2}\right)^{1/2} \tag{4.1}$$

$$\Delta\varphi(f) = \text{arctg}\left(\frac{\zeta_2 - \zeta_1}{1 + \zeta_1\zeta_2}\right) \tag{4.2}$$

In a schematic way these model functions $G(f)$ and $\Delta\varphi(f)$ are shown in fig. 2. We will stress the typical features of these functions:

a) $G(f_A) = 1$ when $\zeta_1 = -\zeta_2$, *i.e.* at point $x_R(f_A) = (x_1 + x_2)/2 = x_c$ laying between two stations;

b) $G(f)$ reaches extremal values G_+ and G_- at certain points $x_R(f_\pm) = x_c \pm [\epsilon^2 + (\Delta x/2)^2]^{1/2}$, where $\Delta x = x_1 - x_2$;

c) $G_+G_- = 1$ and $G_+ - G_- = \Delta x/\epsilon$;

d) $\Delta\varphi(f)$ reaches extremal value $\Delta\varphi^* = 2\text{arctg}(\Delta x/2\epsilon)$ at resonant frequency $f_A = f_R(x_c)$.

The given above properties of functions $G(f)$ and $\Delta\varphi(f)$ allow to estimate the resonant frequency of the field line between the stations (a,d); the width of a resonant peak ϵ (c,d); magnitude and sign of an Alfven frequency gradient in the magnetosphere (b,a,d). All these relationships constitute, in essence, the practical background of the gradient method and they were used in one or another form elsewhere (Baransky *et al.*, 1985, 1989, 1990; Kurchashov *et al.*, 1987; Waters *et al.*, 1991).

From the point of view of the theory of signals (Bendat and Piersol, 1986) the analysis of the gradient measurement data represents the problem of estimation of linear system parameters with one input and two outputs. An input signal $X_0(f)$ is produced by a joint unobserved source (magnetospheric disturbance), and output signals $X_i(f)$ ($i = 1,2$) (at ground-based stations) are a superposition of an input signal passed through the magnetospheric resonator with transfer function $H_i(f) = |H_i|e^{i\psi_i}$ and additive non-correlated noise $Z_i(f)$. Cross-spectral analysis gives the estimates of auto and cross power spectra Γ_{11}, Γ_{22}, Γ_{12} which are related with unknown spectral power densities of input signal Γ_{00} and of noises N_{11}, N_{22} by relationships

$$\begin{aligned} \Gamma_{11} &= |H_1|^2\Gamma_{00} + N_{11} \\ \Gamma_{22} &= |H_2|^2\Gamma_{00} + N_{22} \\ \Gamma_{12} &= H_1{\star}H_2\Gamma_{00} \end{aligned} \tag{4.3}$$

Formally, the existing information is not sufficient for the determination of the ratio $H_2(f)/H_1(f)$ which depends on the needed parameters of the magnetospheric resonator. But if one supposes that ratio signal/noise N_{ii}/Γ_{ii}, is the same at two stations, then from (4.3) one can determine the amplitude and phase parameters of the system:

$$\begin{aligned} |H_2/H_1| &= (\Gamma_{22}/\Gamma_{11})^{1/2} \\ \psi_2 - \psi_1 &= \text{arctg}\left(\frac{Im\Gamma_{12}}{Re\Gamma_{12}}\right) \end{aligned} \tag{4.4}$$

Then the calculated according to (4.4) amplitude and phase characteristics of ULF signals at two stations of a meridional profile can be compared with functions (4.1), (4.2) resulting from the theory of magnetospheric resonator.

However, all the developed above ideology of the gradient method can be applied only when both stations are situated in the same geoelectrical conditions. The lateral geoelectrical inhomogeneity, especially when the condition of a strong skin-effect is violated, may substantially distort a resonant structure of pulsations. The disturbing influence of geoelectrical structure would be mainly seen in the behavior of the component oriented across the structure's spreading. Above a rock with higher conductivity an abnormal magnetic field DH increases and an additional phase shift $\Delta\varphi^0 > 0$ as compared with the incident field

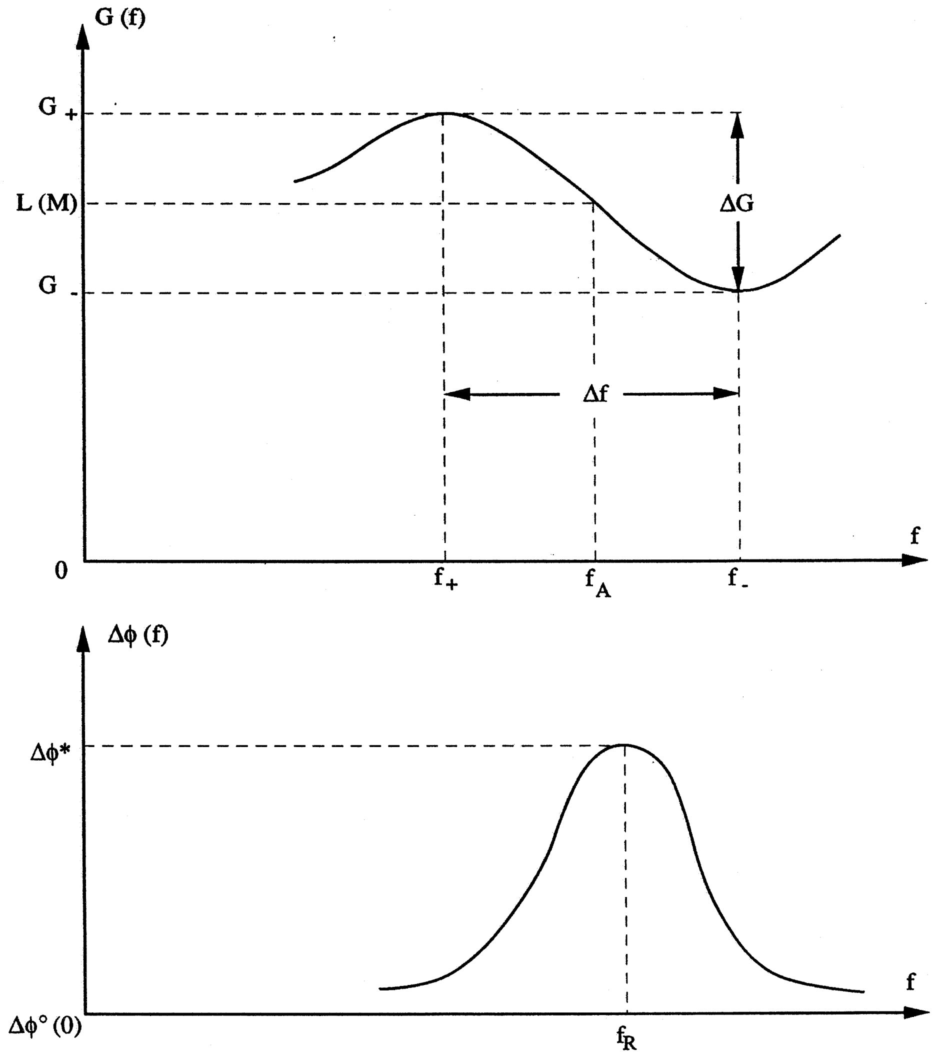

Fig. 2. Schematic plots of the amplitude spectra ratio $G\ (f)$ and phase spectra difference $\Delta\varphi(f)$ between two near-by stations, as predicted by the resonance theory.

appears. This phase shift can reach values up to $\Delta\varphi^0 \sim (180/\pi)\Delta H/H$.

The theoretical modification of the gradient method of HD for the case of a crust geoelectrical inhomogeneity requires numerical calculations of a complicated self-consistent problem. For the elimination of geological influence one may attempt to use the following simple phenomenological method. Let us suppose that the influence of a geoelectrical inhomogeneity can be expressed as some coefficient M, which the ratio $G = H^{(N)}/H^{(S)}$ is multiplied by; and an additional phase shift $\Delta\varphi^0$, which is added to the phase difference $\Delta\varphi = \varphi^{(1)} - \varphi^{(2)}$. Then the experimentally measured functions G' and $\Delta\varphi'$ will be presented in the form: $G' = MG$ and $\Delta\varphi' = \Delta\varphi + \Delta\varphi^0$. Further, we will neglect a weak dependence of the unknown coefficients $M(f)$ and $\Delta\varphi^0(f)$ on frequency for a limited frequency band near resonant frequency, as compared with $G(f)$ and $\Delta\varphi(f)$ (3.1), (4.1). The coefficient M can be found experimentally from the following set of simple relationships resulting from the properties (c) of function $G(f)$:

$$G'_+ = MG_+, G'_- = MG_-, G_-G_+ = 1 \quad (4.5)$$

Finally, the correction coefficient for amplitude ratio can be estimated as

$$M = \sqrt{G'_-G'_+} \quad (4.6)$$

As a rough estimate of $\Delta\varphi^0$, the constant level of phase shift away from resonant phase excursion can be taken. So, the amplitude ratio (3.1) and phase difference (4.1) will be shifted to the new levels M and $\Delta\varphi^0$ (see fig. 2). The modified gradient method was successfully applied to the data processing of Soviet-American experiment in Kirgizia (Green *et al.*, 1992).

The meridional gradient of Alfven frequency can be roughly estimated from the data of gradient measurements as

$$\frac{\partial f}{\partial x} \sim \frac{f_+ - f_-}{x_+ - x_-} = 2\Delta f\,[\epsilon^2 + (\Delta x/2)^2]^{-1/2} \quad (4.7)$$

The gradient method can be used even for the restoration of smooth $f_R(L)$ profile in a limited interval of latitudes. The properties (a,b,c) of $G(f)$ function give possibility to estimate the width of resonance ϵ, and 3 resonant frequency f_A, f_+, f_-, of field lines crossing the meridian between two stations ($x = x_c$) and at some points to the north and to the south ($x = x_-$, $x = x_+$). Then the dependence of resonant frequency on latitude $f_R(\Phi)$ can be approximated by some function, for example $f_R = k \cos^r \Phi/\sin \Phi$. The unknown parameters of this distribution can be determined by a least square fit to 3 experimentally determined values. This method of $f_R(L)$ restoration was suggested and applied in Baransky *et al.*, (1989).

4.2. *Amplitude-phase gradient method*

This modification of a gradient method enables one to determine the resonant frequency as a function of distance between two observation points of the Earth (Gugliel'mi *et al.*, 1989). Assuming that the dependance of the complex amplitude $H(f, x)$ has the form of (2.10) and using the measurements of H at two points x_1 and x_2, a system of two linear equations with complex coefficients and three unknown parameters x_R, ϵ, h_R can be obtained. As a result, according to the measurements of phase difference $\Phi(f) = \Phi_1 - \Phi_2$ and amplitude ratio $h(f) = |H_1|/|H_2|$, one can obtain the dependance of the resonance position on frequency $x_R(f)$ and resonance width ϵ:

$$\frac{x_R(f) - x_1}{\Delta x} = \frac{1 - h\cos(\Phi)}{h^2 - 2h\cos(\Phi) + 1} \quad (4.8)$$

$$\frac{\epsilon}{x_2 - x_1} = \frac{h\sin(\Phi)}{h^2 - 2h\cos(\Phi) + 1} \quad (4.9)$$

As usual the value $h = 1$ corresponds to the local resonant frequency f_A of field line between the stations, *i.e.* $x_R(f_A) = (x_1 + x_2)/2$. The gradient of f_R at this point can be estimated by formula

$$\frac{\partial f}{\partial x} \sim \left(\frac{\partial h}{\partial f}\right)^{-1} (1 - \cos(\Phi))/\Delta x \quad (4.10)$$

As before, the distribution of resonant frequency on L-value can be restored, after reversing of dependance (4.8).

4.3. *Multi-point gradient method*

For the restoration of a continuous $f_R(L)$ profile another method proposed by Gugliel'mi *et al.* (1989) can be used. This technique only uses the amplitude measurements of H-component at a meridional net of stations (not less than 3). The following algorithm should be applied:

– a distribution $H_f(x_i)$ (x_i – station's coordinate) for each frequency f is plotted;

– a local maximum x_R at every frequency f is determined;

– a dependence $x^i_R(f)$ is interpolated to obtain a smooth curve $x_R(f)$;

– a curve $x_R(f)$ is reversed to obtain $f = f(x_R)$ plot.

This scheme was tested for the restoration of $f_R(L)$ profile by Pc5 pulsations at meridional network near Norilsk (Gugliel'mi *et al.*, 1989).

5. Polarization methods of the ULF resonant structure study

The mentioned above remarkable asymmetry of resonant properties between various components of the ULF field prompts that resonance effects would manifest themselves not only in the spatial structure, but in their polarization properties too. On the basis of this idea a number of polarization methods of f_R determination were suggested. These methods could supplement the gradient method and could even be used for diagnostics of $f_R(x)$ distribution by the data of one station only.

5.1. *Polarization of horizontal magnetic components*

Polarization of horizontal magnetic components is the sensitive indicator of spatial ULF structure. As (2.8) shows, the resonant response of the magnetosphere is characterized by a pronounced asymmetry of H and D components. At the same time a source spectrum is imposed on both components in the same way. Hence, even when a resonance response is masked by a source spectrum nevertheless a ratio $H(f)/D(f)$ demonstrates the maximum at a resonant frequency. The ratio of the spectra of horizontal components H/D for the determination of a local resonant frequency was successfully used in Baransky (1990) and Green *et al.* (1992).

The useful information about pulsations structure is hidden in polarization parameters – ellipticity k and orientation of main axis ψ. Keeping the leading asymptotic terms in the solution of MHD equations which describes a transverse wave structure in the magnetosphere near resonant field line, *i.e.* when $s = (x - x_R)/x_R < \eta$ ($\eta = \in/x_R$ is a normalized resonance width), one can obtain (Chen and Hasegawa, 1974; Hasegawa, 1978)

$$b_x/b_y = g + ih$$
$$g = m\eta ln(\eta) \qquad (5.1)$$
$$h = m\eta(\pi/2 - s/\eta - ln(\eta))$$

The parameters g and h are related with ellipticity k and angle ψ by known formulae (Hasegawa, 1978). As follows from (5.1) at some critical frequency, close to resonant one, the polarization of pulsation becomes linear and the sense of rotation changes. But because of rather low Q-factor of a real magnetospheric resonator $\eta \leq 10^{-1}$, the effect of a polarization switch would be hardly noticeable during ground-based observations in contrast with the expectations of the resonance theory (Chen and Hasegawa, 1974; Southwood, 1974). Moreover, the polarization features of horizontal magnetic components are difficult to use for a practical determination of a resonant frequency, because the shift of a polarization switch point from a resonant one depends on a number of hardly known parameters – azimuthal wave number m, scale of magnetic field inhomogeneity and plasma anisotropy. Nevertheless, the polarization parameters can give additional information about the direction of an azimuthal phase velocity and sign of radial gradient of Alfven velocity in the magnetosphere according to the data of one station only. For that purpose only the signs of ellipticity k and angle ψ can be used.

5.2. *Resonant properties of the vertical magnetic component*

The vertical component of ULF magnetic field is a sensitive indicator of inhomogeneity of both the ULF field and crust conductivity. So, the use of H_z component data might be very promising for the elucidation of resonant features of the geomagnetic pulsation spatial structure.

In a case of strong skin-effect (this condition is well fulfilled for typical Pc3-4 pulsations over low-resistive crust) the relationship between vertical and horizontal magnetic components of the ULF field is given by the following formula (Wait, 1982; Gugliel'mi, 1989b):

$$H_z(x,f) = \frac{icZ_0}{2\pi f}(\text{div}\boldsymbol{H}_\perp + \boldsymbol{H}_\perp \nabla Z_0) \quad (5.2)$$

For regions with homogeneous conductivity of the Earth's crust, estimates show that the second term of the above equation can be neglected for typical Pc3-5 and Pi2 pulsations (with the exception of specific classes of pulsations, *e.g.* Pg). Then

$$H_z(x,f) = \frac{icZ_0}{2\pi f}\frac{\partial H}{\partial x} + \ldots\ldots \quad (5.3)$$

Under these conditions, as follows from (2.8), (5.3), the resonant effects in the behavior of H_z component might be even more pronounced than in H-component. This conclusion was supported by the experiment at meridional profile near Boulder (Green *et al.*, 1991).

The complex value of the surface impedance Z_0 in (5.3) can be determined with a preliminary study of the Earth's crust or it can be excluded from consideration with the help of impedance relations (2.3). In latter case the relationship between components H_z and E_y will be given as

$$b_z/E_y = -\frac{c}{\omega \in}(1 + i\zeta)^{-1} \quad (5.4)$$

The qualitative plot of amplitude curve $|b_z/E_y| \sim (1+\zeta^2)^{-1}$ and a phase one $\psi = \arg b_z - \arg E_y = \pi - \text{arctg}(\zeta)$ are shown in fig. 3. Just at the frequency of an Alfven resonance $f = f_R$ the ratio $|H_z/E_y|$ has a local maximum, and the components H_z and E_y must be in anti-phase. The slope of the curve $\psi(\omega)$ at the frequency $f = f_R$ is determined by the resonance width $\in$ and by the Alfven frequency gradient:

$$\frac{\partial f_R}{\partial x} \sim \frac{1}{\in}\left(\frac{\partial \varphi}{\partial f}\right)^{-1}_{f=f_R} \quad (5.5)$$

This formula can be used to estimate of a local Alfven frequency gradient in the magnetosphere.

Through the use of H_z and other components of the ULF field it is possible even to reconstruct the functional dependance $f_R(L)$ in the vicinity of the observation point, with the data from a single station. From (5.4), the distance between the observation point ($x = 0$) and the resonant shell $x_R(f)$ as well as the

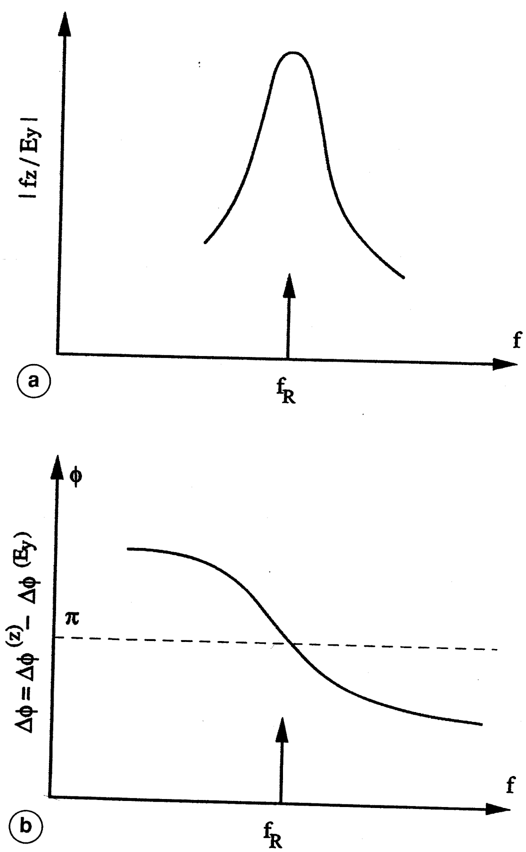

Fig. 3. Schematic plots of the relationships between spectra of H_z and E_y components: a) ratio of amplitudes; b) phase difference.

width of the resonance $\in$, can be determined by

$$x_R = \frac{c}{\omega} |E_y/H_z| \sin(\psi)$$
$$\in = -\frac{c}{\omega} |E_y/H_z| \cos(\psi) \qquad (5.6)$$

The relationships obtained allow the use of multicomponent data as «distance meter» for the determination of the distance to the resonant field line at a given frequency f (Gugliel'mi, 1989a). Reversing the dependence (5.6) $x_R(f) \to f_R(x)$, the distribution of resonant frequency $f_R(x)$ can be restored in the vicinity of the observation point.

6. Concerning group velocity of ULF signals along the Earth surface

While investigating the wave properties of ULF pulsations by the data of ground-based observatories, it is possible to measure not only a phase delay between stations but a group velocity as well. For that purpose, for example, a spectral-temporal analysis can be used. Then a group delay can be determined by the shift between the wave packet envelopes at two stations. One may expect that a group velocity calculated in this way indicates the energy flux propagation and helps to reveal a physical source of a wave signal.

But the resonant wave transformation in the magnetosphere may raise doubts about this obvious, as it seems, approach. In fact, phase V_{ph} and group V_{gr} velocities of a pulsation packet between stations separated by Δx, could be formally defined according to standard formulas:

$$V_{ph} = \frac{\omega \Delta x}{\Delta\varphi(\omega)}$$
$$V_{gr}^{-1} = \frac{d}{d\omega}\left(\frac{\omega}{V_{ph}(\omega)}\right) = \frac{1}{\Delta x}\frac{d\varphi(\omega)}{d\omega} \qquad (6.1)$$

From the latter formula it follows, that at specific frequency, where $d\varphi(\omega)/d\omega \to 0$, $V_{gr} \to \infty$, *i.e.* wave packet propagates with super-light velocity. But the resonance theory just predicts that a phase delay has an extremum at a resonant frequency (see fig. 2). Hence, the formal application of (6.1) to meridional propagation of an ULF packet with central frequency near local resonant one would give a non-physical result.

The reason for the above paradoxis consists in the fact that in the region of an Alfven field line resonance the standard expression (6.1) turns out to be invalid. Let us remind, following (Ginzburg, 1970), that the «classical» formula $V_{gr} = d\omega/dk$ is derived only when $\Delta\varphi(\omega)$ could be presented as a linear expansion near central frequency ω_0

$$\Delta\varphi(\omega) = \Delta\varphi(\omega_0) + \varphi'(\omega_0)(\omega - \omega_0) + \frac{1}{2}\varphi''(\omega_0)(\omega - \omega_0)^2 + \ldots \qquad (6.2)$$

Only when non-linear terms in (6.2) are small, a wave impulse slightly changes its form at a wave length and propagates as a whole with group velocity (6.1). In the resonance region, $\varphi'(\omega_0) \to 0$ and higher terms $\sim (\omega - \omega_0)^2$ in expansion (6.2) become essential. Then the form of an impulse is severely distorted and the notion of a group velocity becomes ambiguous. The correct treatment of a problem requires special calculations for a particular packet form (Ginzburg, 1970). So, in fact, an Alfven field line resonance region manifests itself for a pulsation packet as a region with anomalous dispersion for electromagnetic waves. Hence the determination of the ULF group velocities along a meridian should be performed with great care.

7. Conclusions

By this review we have attempted to draw the attention of geophysicists to the necessity to merge the theory of the magnetospheric MHD waves and the MTS methods. Disregard of one of these aspects of ULF physics may lead to principal errors. For example, the influence of lateral geoelectrical inhomogeneity may obscure the magnetospheric resonance effects in the ground-based structure of ULF pulsations, which makes the HD impossible. In their turn, the resonant effects cause the distortions of MTS curves that may be misinterpreted as some features of a geoelectric structure.

Simple modification of the gradient method described above allows to exclude in zero-approximation the influence of geology and to reveal the resonance effects. But this method requires a theoretical justification. For that purpose numerical calculations of a complex problem of the resonant ULF field reflection from laterally inhomogeneous geoelectric structure should be done.

The new methods of HD which use multi-component structure of a magnetotelluric field seem rather promising. These methods should be experimentally verified and the results must be compared with other HD methods (f.e. gradient one). It would be especially interesting to apply these methods for the study of the role of resonant effects near plasmapause projection and at very low latitudes. That would enable to study the peculiarities of an Alfven resonance at steep gradient and to determine experimentally an equatorial border of the ULF field line resonance.

As was demonstrated above, the bandwidth near resonance frequencies should be excluded from the conventional «local» MTS scheme. But the resonance spatial structure is well described by the model function (2.9), (2.10) with parameters $x_R(f)$, $\in$, which can be determined from experimental data. Hence a «spectral» scheme of MTS, which uses the spectral transformations given by (3.1), could be applied (Dmitriev and Berdichevsky, 1979). This scheme allows to determine the distribution of $\sigma(z)$ averaged over a region under study.

A very promising approach is the study of statistical properties of ULF time series recorded at magnetic network. Polyakov and Potapov (1989) demonstrated the possibility to separate a «source signal» and a «resonance response» in (2.9) with the use of the statistical theory of oscillations. New information about irregular properties of a crust conductivity («turbidity of a crust medium») could be obtained with the «statistical theory of MTS». The attempts to develop such theory were reported in Treumann and Shafer (1977).

The regular monitoring of the dynamics of geoelectric conductivity in seismoactive regions with the use of MTS methods might play an important role in the solution of an earthquake prediction problem. For successful application of MTS methods it will be essential to choose the parameters of magnetotelluric ULF variations for which a magnitude of the expected anomalous effect and the sensitivity to the conductivity dynamics are highest (Sholpo, 1990). Probably, the optimal spectral parameters should correspond to the strong skin-effect limit near the boundary of a seismic source. On the basis of the existing knowledge of natural ULF field structure new algorithms for the detection of seismo-magnetotelluric anomalies could be developed.

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