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On the possibilities of coordinated ionospheric soundings and GPS measurements

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Abstract
Ionospheric soundings carried out at the same site and simultaneously with GPS measurements could contribute to the better understanding of the variations of the total electron content. However, for this purpose the total electron content deduced from GPS measurements must be converted to the place of observation. A method based on an interpolation considered as boundary value problem is recommended for this conversion. Furthermore, the possibilities of the application of data obtained by ionospheric soundings in GPS measurements are discussed and also preliminary results of model calculations are presented.

Key words *ionospheric soundings – GPS measurements – conversion of slant electron content – subpeak electron content*

1. Introduction

Before the introduction of the Global Positioning System (GPS) the technique used for the determination of geographic coordinates of a point by satellites and for the establishment of the total electron content (TEC) in the ionosphere was the TRANSIT or Navy Navigation Satellite System (NNSS). This system enabled the determination of the latitudinal variation of the TEC by the short transit time of its polar orbiting satellites (due to their altitude of 1000 km), since it could be assumed that the ionosphere does practically not change during this time. The disadvantages of this system were on the one hand that in case of a single station the initial phase of the satellite signal could not be determined precisely enough (Leitinger and Putz, 1978; Bencze *et al.*, 1986). On the other hand, the relatively low frequency did not assure sufficient accuracy in the determination of geographic coordinates and the TEC (Hartmann and Leitinger, 1984; Bányai and Kovács, 1992).

The error budget of the GPS system has been studied by Wanninger (1992). Concerning the establishment of the TEC the synchronization error of the code generation on board of the satellites and in the receiver can amount to a value of $12 \cdot 10^{16}$ (el/m^2), which must be taken into account in the data processing (Lányi and Roth, 1988). Besides the multipath propagation is the only significant error source, which can be avoided by the careful selection of the site. However, the transit time of GPS satellites is large because of their orbital characteristics (large height). Thus, though the whole orbit above the horizon can be used, it must be divided in segments, during the traverse of which quasi stationary ionospheric conditions could be assumed. The TEC is defined as the electron content of a vertical column of unit area between the satellite and the surface. However, in the course of the measurements mostly the electron content of a slant column (TEC_s) is obtained, which has to be converted to the electron content of a vertical column (TEC). For the establishment of the mean TEC in case of more satellites observed during time intervals in which quasi stationary ionospheric conditions can be assumed (30 min), a method is suggested.

The accuracy of the determination of the TEC depends not only on the establishment of the slant electron content, but also on its conversion to vertical content. For the investigation of this problem, the knowledge of the electron density distribution in the ionosphere is needed. Ionospheric soundings carried out at the same site and simultaneously with GPS measurements could contribute to the solution of this problem by giving the value of the maximum electron density and its height (see details in section 3).

2. Method for the determination of the mean TEC in the multisatellite case

As it has been mentioned above, the slant electron content TEC_s has to be converted to the vertical content TEC. This is carried out by means of the relation

$$TEC = TEC_s \sin E'$$

where

$$E' = \arccos\left(\frac{R_E}{R_E + h_i} \cos E\right)$$

gives the elevation angle E' at the mean ionospheric height h_i. In this equation R_E is the Earth's radius and E is the elevation angle of the satellite at the point of observation.

The observation equations for pseudoranges (R) and phases (ϕ) denoting the carriers by 1, 2 are given as

$$R_1 = \varrho + c\Delta\tau + \frac{1}{\sin E'}\frac{1}{2}\frac{A}{f_1^2}\,TEC + \Delta R_{trop}$$

$$R_2 = \varrho + c\Delta\tau + \frac{1}{\sin E'}\frac{1}{2}\frac{A}{f_2^2}\,TEC + \Delta R_{trop}$$

$$\lambda_1\phi_1 = \varrho + c\Delta\tau + \lambda_1 N_1 + \\ - \frac{1}{\sin E'}\frac{1}{2}\frac{A}{f_1^2}\,TEC + \Delta R_{trop}$$

$$\lambda_2\phi_2 = \varrho + c\Delta\tau + \lambda_2 N_2 + \\ - \frac{1}{\sin E'}\frac{1}{2}\frac{A}{f_2^2}\,TEC + \Delta R_{trop}$$

where the following notations were used:

ϱ	distance between the receiver and satellite;
λ	wavelength of the carrier;
f	frequency of the carrier;
c	speed of light;
A	constant;
$\Delta\tau$	clock error;
N	phase ambiguity (initial phase unknown);
ΔR_{trop}	tropospheric range error.

The geometry-free linear combination is defined as

$$R_1 - R_2 = -\frac{1}{\sin E'}\frac{A}{2}\frac{f_1^2 - f_2^2}{f_1^2 f_2^2}\,TEC + c\delta\tau$$

$$\lambda_1\phi_1 - \lambda_2\phi_2 = \\ (\lambda_1 N_1 - \lambda_2 N_2) + \frac{1}{\sin E'}\frac{A}{2}\frac{f_1^2 - f_2^2}{f_1^2 f_2^2}\,TEC + c\delta\tau$$

where the signal path bending is neglected and the synchronization error is denoted by $\delta\tau$. From the sum of the two equations the TEC is cancelled, therefore, if precise *P*-code pseudoranges (*e.g.* Turbo Rogue observations) are available the unknown combination of the phase ambiguities can be estimated averaging over a suitable time span. Substituting the average value into the phase combination, an optimal observable for the TEC determination can be given where the absolute precision is characterized by the code measurements and the relative precision is characterized by the phase measurements.

The TEC refers to the place under the

ionospheric point. As it has been mentioned, in case of GPS measurements – though the whole orbit above the horizon can be used – the computation of the TEC must be limited to small segments of this part of the orbit that quasi stationary ionospheric conditions could be assumed. The direction and position of these segments changes according to the configuration of the satellites. Taking an elevation angle of 30° for the satellites, data are collected from a spherical surface of 600 km radius at the mean ionospheric height (350 km) with the point of observation below its centre. This means, that at least four GPS satellites can simultaneously be observed. As an example, the situation above the Sopron station during the GPS measurements in November 1992 is shown (fig. 1) (Bányai and Kalmár, 1993). Thus, considering a period, in the course of which, quasi stationary ionospheric conditions can be assumed, the segments define a complicated surface. Thereby, for the determination of a TEC referring to the point of observation a suitable method is needed.

J. Kalmár proposed a method which is based on interpolation considering it as a boundary value problem. In case of the Dirichlet problem it is assumed that the unknown surface can be described by a harmonic function and the known values are located on a closed curve surrounding the place, for which the interpolation should be carried out. The problem can be solved analytically only in simple cases. Generally, the Dirichlet problem can be solved by the Monte-Carlo method. Referring to the left part of fig. 2, let us suppose that a large number of points start continuously from the source *P* showing Brownian motion and they cross the closed curve *S*. The interpolated value of the parameter in question in *P* (potential of *P*) can be described by the expression

$$f(P) = \sum_{s} \varrho(s) f(s)$$

if a probability density function $\varrho(s)$ is defined giving the probability that the points cross differential parts $\underline{s}$ of *S*, where $s \subset S$ and $S = \cup s$. Dividing the closed curve around *P* into segments by radii starting from *P* and generating equal angles *da* at *P* (fig. 2), it can be assumed that the same

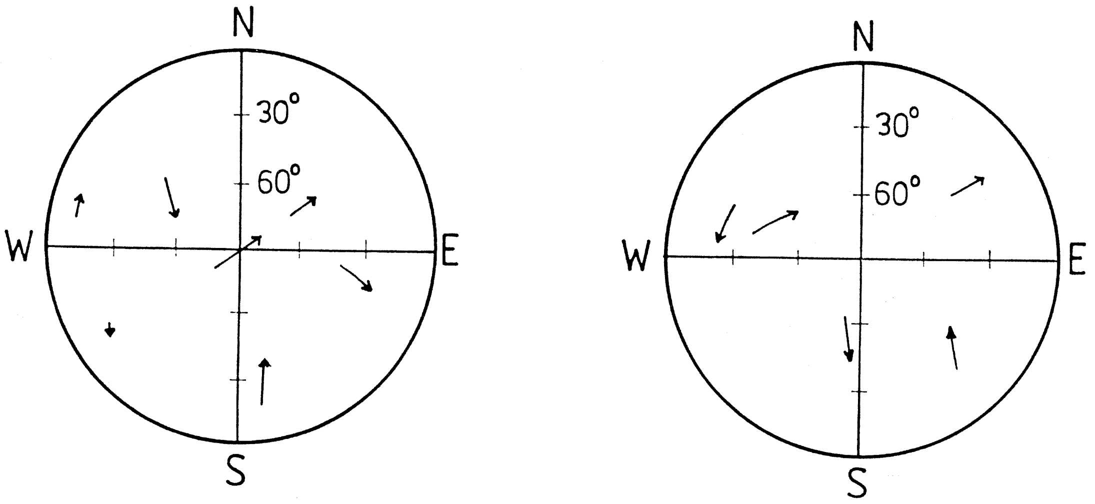

Fig. 1. The best and least favourable satellite configuration for the Sopron station (19.11.1992).

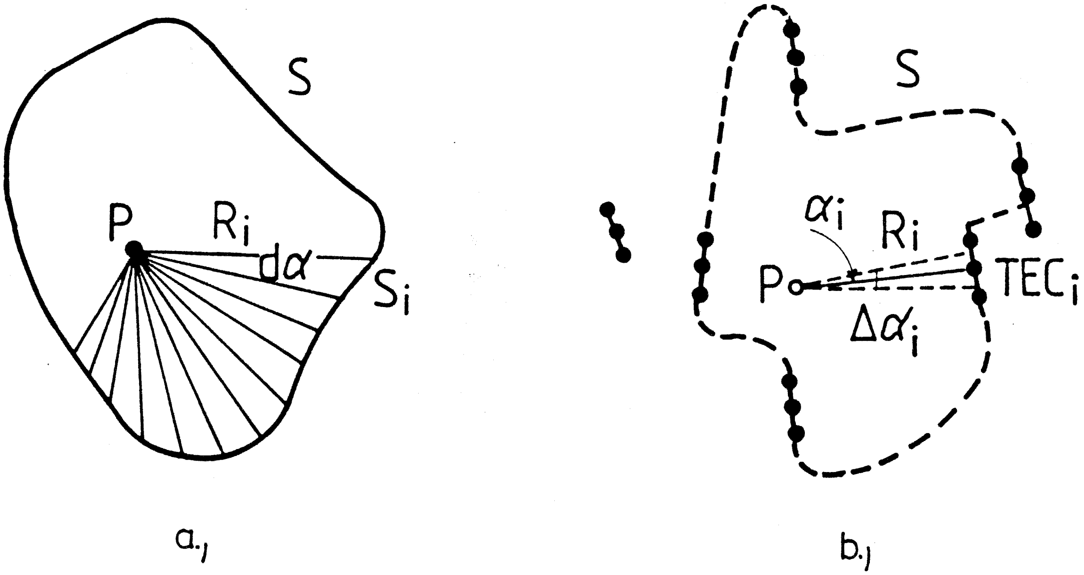

Fig. 2. Geometry of the boundary value interpolation.

number of points from the source *P* cross the curve in each segment. The different segments have different radii R_i and consequently different lengths ($s_i = R_i \cdot d\alpha$). Thus, the unknown density function can be expressed in the form

$$\varrho(s_i) = \frac{1}{R_i}$$

where $\varrho(s)$ is inversely proportional to the radii R_i. But the result for $\varrho(s)$ obtained by the simplification of the Brownian motion should be corrected. Namely the Monte-Carlo method takes only into account the first crossing of the moving points.

Nevertheless, the longer is the radius, the greater is the probability that the moving point has crossed earlier the boundary at another segment. Thus we must decrease the number of crossing points as a function of the radii. Experiences indicate that it is usually better to use R_i^2 instead of R_i. In this case the interpolated value can be given by

$$f(P) = \frac{\oint_0^{2\pi} f(\alpha)/R^2(\alpha)\, d\alpha}{\oint_0^{2\pi} 1/R^2(\alpha)\, d(\alpha)}$$

Replacing $f(\alpha)$ by TEC and taking the distances between the point of observation and the ionospheric points defined by the orbits of the different GPS satellites as radii

$$\mathrm{TEC} = \frac{\sum_0^{2\pi} \mathrm{TEC}(\alpha_i)/R^2(\alpha_i)\, \Delta\alpha_i}{\sum_0^{2\pi} 1/R^2(\alpha_i)\, \Delta\alpha_i)}$$

In reality the TEC values are determined only for small segments of the previously mentioned closed curve. Therefore, it is assumed that the dashed curve shown in the right side of fig. 2 is in the infinite. Thus, since

$$\lim_{R\to\infty} \frac{1}{R^2} = 0 \text{ and consequently } \lim_{R\to\infty} \frac{\mathrm{TEC}}{R^2} = 0$$

this simplification can also be accepted.

The denominator indicates how the closed curve assumption is fulfilled. The greater is the denominator, the better is the satellite configuration. Thereby, this method can also be used for the characterization of the quality of the estimation.

3. Ionospheric soundings and GPS measurements

The common task of ionospheric soundings and GPS measurements is also the investigation of the ionosphere. There are more quantities, which are determined by ionospheric soundings and can be used to improve ionospheric informations deduced from GPS measurements.

A possibility of coordinated ionospheric soundings and GPS measurements is the control of the procedure by which the slant electron content is converted to the vertical. As it is known, this is done by multiplying the slant content with the sine of the elevation angle of the satellite at the mean ionospheric height (Davies *et al.*, 1977). However, the surface at the mean ionospheric height in case of an elevation angle of 30° is already equal to that of a circle of 600 km radius with its centre at the observing site. Thus, it should also be taken into account that the path of the satellite signal traverses ionospheric regions at different local times, among them regions of horizontal electron density gradients (not necessarily around sunrise or sunset) and at different latitudes. Therefore considering the new measuring possibilities it would be useful to control this procedure. The influence of the local time on the vertical electron content might be studied by observing GPS satellites passing the zenith of the observing site in different local times under quiet geomagnetic conditions. Thus, the observed vertical electron content could be compared with the computed vertical electron content, obtained from the slant content for the same local time and for ionospheric points of the same latitude. The effect of the latitudinal variation might be studied by taking GPS satellites, which pass the zenith of observing sites of different latitudes at the same local time under quiet geomagnetic conditions. Then to compare the observed vertical electron content with the computed vertical electron content obtained from the slant content for ionospheric points northward and southward of the sites, the latitude of which is equal to the latitude of the observing sites mentioned above. Thus, the effect of the latitudinal variation could be determined. The conversion function consists of the combination of the local time and latitudinal variations and also depends on the selected mean ionospheric height. In this way the technique of the conversion of slant to vertical electron content might be controlled. For such investigations the great number of GPS satellites could assure favourable conditions.

In this respect it seemed reasonable to study the effect of the ionosphere and the plasmasphere on the propagation of radio waves transmitted by GPS satellites by means of model calculations (Bencze, 1993). As it is known, the main error sources in the evaluation of GPS measurements are the application of a simple formula for the computation of the group refractive index and the use of the geometrical path instead of the true path of radio waves. Thus, for the determination of the group refractive index a more accurate expression including also the electron gyrofrequency has been used in the modelling (the gyrofrequency represents the effect of the geomagnetic field on the propagation of radio waves). The group refractive index and hereby the true path of radio waves has been determined by means of ionospheric and plasmaspheric models. The refraction angle has been computed for points, which are at the same distance from each other (ray tracing). These are breaking points of the true path. The electron density was considered as constant within these sections. The preliminary computations have been carried out for the carrier frequency of 1.57542 GHz in case of south-north and west-east propagation, as well as for different zenith angles. In this study the coordinates of the observing site were given and that of the satellite were computed.

The preliminary results show that also in case of a division of the propagation path into longer sections (50 km in the ionosphere and 500 km above 1000 km in the plasmasphere), the difference between the latitude of the satellite determined by neglecting the effect of the ionosphere, as well as that of the plasmasphere and the latitude obtained by taking into account their influence is less than 0.36 s (south-north propagation). However, using the same division of the propagation path, the difference between the longitude of the satellite determined by neglecting the effect of the ionosphere, as well as that of the plasmasphere and the longitude obtained by taking into account their influence has been at least by one order of magnitude greater than in the former case (west-east propagation). Moreover, the difference increased with increasing zenith angle. This seems to indicate that the effect of the geomagnetic field cannot be neglected and that the influence of the local time being different in different points of the path can be appreciable.

For the illustration of the influence of the ionosphere and the plasmasphere, the difference between the refraction angle and the angle determined by the radius drawn from the Earth's center to the breaking point (of the true path) and the geometrical path are plotted as a function of height in fig. 3. This difference can be considered as an indicator of the effect of the ionosphere and the plasmasphere, respectively. The curve refers to a zenith angle of 20° of the satellite and to west-east propagation. It can be seen that the predominant part of the effect on the propagation of these radio waves arises in the ionosphere below about 300 km.

The operation of an ionosonde at the observing site of the GPS equipment would produce useful information on the distribution of electron density in the subpeak ionosphere. f_0F2 and the real height of the maximum electron density would be the most useful parameters. Repeating the procedure described in the second section several times, the influence of the maximum electron density and its real height on the TEC could be studied.

The electron density in the *F* region shows also anomalies, which are connected with an anomalous diurnal and seasonal variation indicating maximum electron density in the early afternoon and electron densities greater in winter, than in summer, respectively.

Electron density variations are also related to geomagnetically disturbed periods. Geomagnetic storms are accompanied by ionospheric storms (positive or negative depending on latitude) in the *F* region ($>$ 200 km) and in the topside ionosphere. At the same time increased electron densities are observed in the lower ionosphere ($<$ 100 km) at middle and high latitudes. The electron density in the lower ionosphere is increased also after the decline of the geomagnetic activity, whose phenomenon is called storm after effect.

Further common use of ionospheric soundings and GPS measurements could be the investigation of the topside ionosphere and plasmasphere and the plasmasphere. As it is known, the ionograms obtained by ionosondes enable the determination of the electron density profile below the height of the maximum electron density of the ionosphere. Knowing the electron density profile, the electron content below the maximum electron density, the so called (vertical) subpeak electron content can be computed. According to our calculations, the subpeak electron content reaches values 10%, or more of the total electron content (Bencze and Flórián, 1972).

Subtracting from the TEC – obtained by observing GPS satellites passing the zenith of the common observing site of the GPS equipment and the ionosonde – the subpeak electron content, the electron content of the topside ionosphere and plasmasphere might be determined. By means of these data the formulas used in ionospheric and plasmaspheric models for the description of the

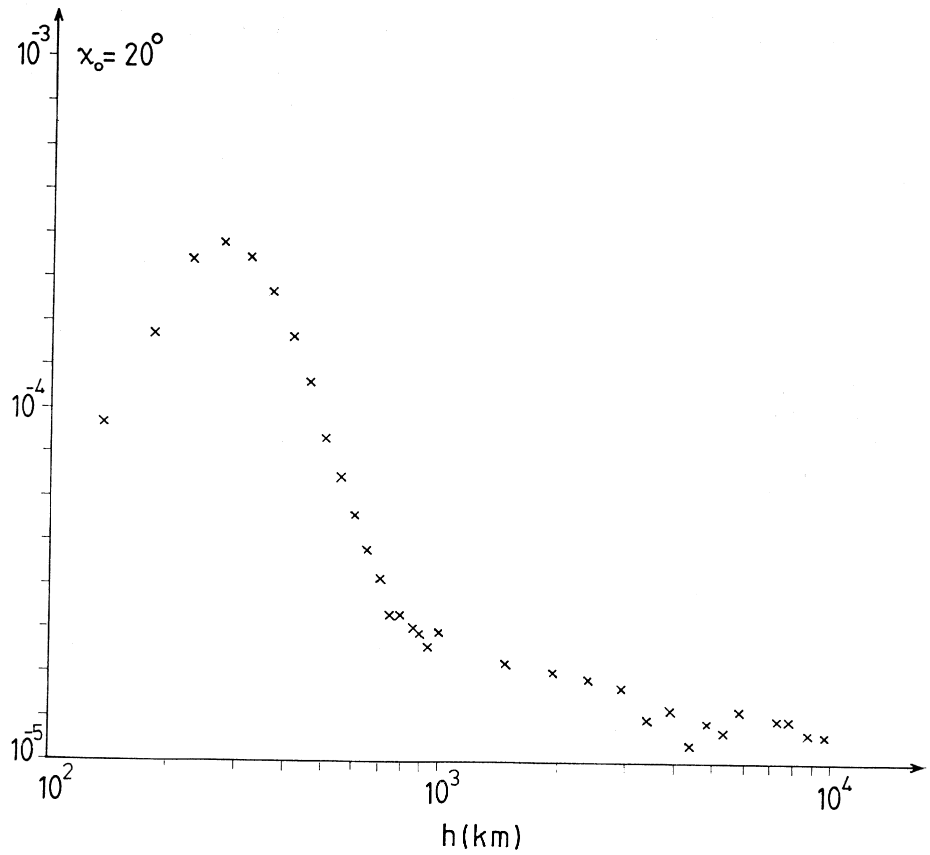

Fig. 3. The difference between the refraction angle and the angle determined by the radius drawn from the Earth's center to the breaking point (of the true path) and the geometrical path as a function of height for zenith angle 20° of the satellite (computed for the Geophysical Observatory Nagycenk, Hungary and for July, 12.00 UT).

electron density distribution in the topside ionosphere and plasmasphere could be controlled. As it is known, the electron density distribution can be given to a height of about 2000 km in the form

$$N = N_0 \exp\left(-\frac{z}{H}\right)^a$$

where N_0 is the maximum electron density computed from f_0F2, z and H are the height and the scale height of ions, respectively. $H = kT_i/m_ig$, in which k is Boltzmann's constant, T_i is the ion temperature, m_i represents the ion mass and g is the acceleration due to gravity, a is a constant. However, the application of an exponential decrease of the electron density above a height of 2000 km can be questioned. For lack of more detailed models, an electron density decrease proportional to B^n along the geomagnetic field lines can be used above this height, with $n = 3$ giving an approximately realistic electron density distribution above the geomagnetic equator (*e.g.* Bauer, 1969).

It should be noted that there are also more useful common applications of ionospheric soundings and GPS measurements. For example it is easier to test the topside electron density profile by means of TEC data, if the slab thickness parameter

$$\tau = \frac{\mathrm{TEC}}{N_{\max}}$$

determined from TEC measurements and ionospheric soundings, is compared with the slab thickness deduced from models (Klobuchar and Leitinger, 1993). Investigations indicate that the slab thickness varies less with latitude than TEC and $N_{\max}$, thus it could be used for the description of the shape of the $F2$ region.

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