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A single-station prediction model as a contribution to instantaneous mapping

Iwona Stanislawska
Space Research Centre, Polish Academy of Sciences, Warszawa, Poland

Abstract
The paper presents two opposite approaches for single-station prediction and forecast. Both methods are based on different assumptions of physical processes in the ionosphere and need the different set of incoming data. Different heliogeophysical data, mainly f_0F2 parameters from the past were analyzed for f_0F2 obtaining for the requested period ahead. In the first method – the autocovariance prediction method – the time series of f_0F2 from one station are used for daily forecast at that point. The second method may be used for obtaining f_0F2 not only at the particular ionospheric station, but also at any point within the considered area.

Key words *ionospheric prediction – single-station model – instantaneous mapping*

1. Introduction

The paper presents two opposite approaches for single-station predictions and forecasts. Both presented methods are based on different assumptions of physical processes in the ionosphere and need the different set of incoming data. In the first model – the autocovariance prediction – the prediction estimation is computed as a function of all observed variables, which are only f_0F2 parameters, in such a way, that the autocovariance estimation remains unchanged in the minimum least square sense. The nature of the described phenomenon can be unknown. The only information needed is the period of observations long enough to be used for prediction of ionospheric parameters, and is in all cases independent of the future solar-geophysical conditions. On the contrary, another method SRC (ionospheric disturbance prediction) is based upon the knowledge of the ionospheric disturbance behaviour. The wave-like disturbance at every station is approximated by the set of function depending on the local time, universal time, and geographical coordinates. The obtained functions at every station are then averaged. This method demands more information as an input: physical parameters of disturbance, geophysical coordinates, local time and also a kind of indicator of heliogeophysical situation. The requirements of this method are more complicated, but become more convenient in the understanding of the physical processes in the ionosphere.

2. The autocovariance prediction method

The autocovariance prediction is the original new approach (Kosek, 1992) based on the characteristic property of the autocovariance function: for stationary stochastic process the autocovariance is a function of a lag time k only and it contains all the information about the process. Let $X = X_1, X_2, \ldots, X_n$ be an equidistant stationary stochastic process of N observations and X'_{n+1} the prediction in a time of $N+1$. The stochastic process X_t is completely defined by its mean $E(X)$ and its autocovariance $c(t,k)$ for all t and k (De Larminat and Thomas, 1975).

In the autocovariance prediction method the first prediction estimation point satisfies the following condition:

$$P = \sum_{k=0}^{N-1} (c'_k - c_k)^2 = \min \qquad (2.1)$$

where

$$c_k = \frac{h_k}{N-k} \sum_{t=1}^{N-k} X_t X_{t+k}$$

$$\text{for } k = 0, 1, \ldots, N-1 \qquad (2.2)$$

and

$$c'_k = \frac{h_k}{N-k+1} \sum_{t=1}^{N-k+1} X_t X_{t+k}$$

$$\text{for } k = 0, 1, \ldots, N \qquad (2.3)$$

are the biased autocovariance estimations, first of the data and second of the data with added unknown prediction estimation point X', h and h' are lag windows with the length of N and $N+1$ respectively.

The unknown autocovariance estimation (2.3) can be expressed by the known one:

$$c'_k = \frac{c_k(N-k)h'_k/h_k + X_{N+1}X_{N-k+1}h'_k}{N-k+1}$$

$$\text{for } k = 0, 1, \ldots, N-1 \qquad (2.4)$$

so the function P has the following form:

$$P = \sum_{k=0}^{N-1} \left(\frac{X'_{N+1}X_{N-k+1}h'_k + ((N-k)(h'_k/h_k) - 1) - 1)}{N-k+1} \right) \qquad (2.5)$$

It is a minimum when

$$\frac{\partial P}{\partial X'_{N+1}} = 0 \qquad (2.6)$$

The result of this differentiation is a 3rd degree polynomial:

$$X^3_{N+1} + 3pX_{N+1} + 2q = 0 \qquad (2.7)$$

where

$$p = \sum_{k=1}^{N-1} X^2_{k+1} \left(\frac{h'^2_{N-k}\,(N+1)^2}{6\,h'^2_0\,(k+1)^2} + \right.$$

$$\left. - \frac{h_0}{3h'_0 N} \right) - \frac{h^2_0 X_1}{3h'_0 N} \qquad (2.8)$$

and

$$q = - \frac{(N+1)^2}{4h^2_0}$$

$$\sum_{k=1}^{N-1} \frac{h'_k c_k X_{N-k+1}\,(1 - (N-k)\,(h'_k/h_k - 1))}{(N-k+1)^2} \qquad (2.9)$$

The first prediction estimation X' added to the end of data enables computation of the next prediction estimation X' etc. In time series analysis all lag windows h_k (Priestley, 1981) are made so that there is never more than 1 real root. The time series of the hourly data of f_0F2 from the previous week were used for daily forecast. The results of calculations are presented in fig. 1. The data from Poitiers station on 3-6 April

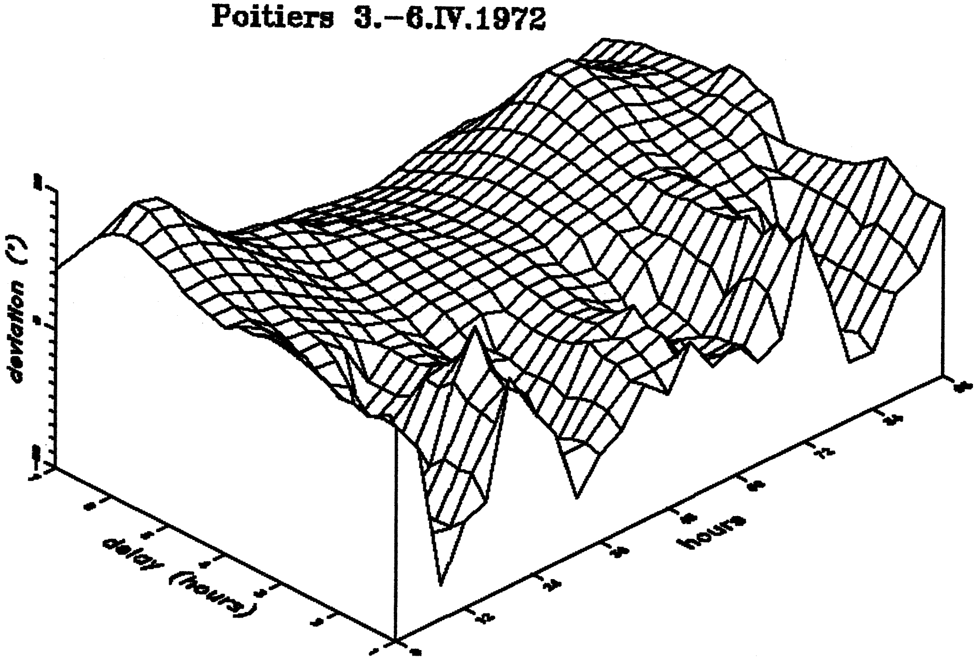

Fig. 1. The percentage deviation of forecast f_0F2 parameter from measurements with time delay between last measurements and the forecast varied from 1 to 7 h at Poitiers station on 3-6 April 1972.

1972, obtained from PRIME data bank in Lannion (Hanbaba, 1992) are compared to the forecast. The accuracy of the forecast highly depends on the time lag between the last used measurements and the first forecasted values. A more detailed analysis is presented by Stanislawska (1993a).

3. The SRC ionospheric disturbance prediction

The SRC ionospheric disturbance prediction method only describes the regular part of the ionospheric $F2$ layer, during quiet and disturbed conditions. The method is based upon the analysis of the wave-like form of the ionospheric disturbance and is similar to the method proposed by Kuleshova *et al.* (1978), Reinish *et al.* (1993). This method, combined with the models of the variations of ionospheric parameters during disturbances, enables to forecast the ionospheric dynamics during storms.

The analytical form of critical frequency for the day time period f_0F2 consists of the periodic component with the period of 24 h, measured in local time (SD), and the part which changes with storm time, that is represented in universal time (DD).

The f_0F2 parameter can be written as

$$f_0F2\,(t) = SD(t') + DD(t) \qquad (3.1)$$

where t is universal time and t' is local time.

$$SD(t') = c_1 \sin(15t') + c_2 \cos(15t') + \\ + d_1 \sin(30t') + d_2 \cos(30t') \qquad (3.2)$$

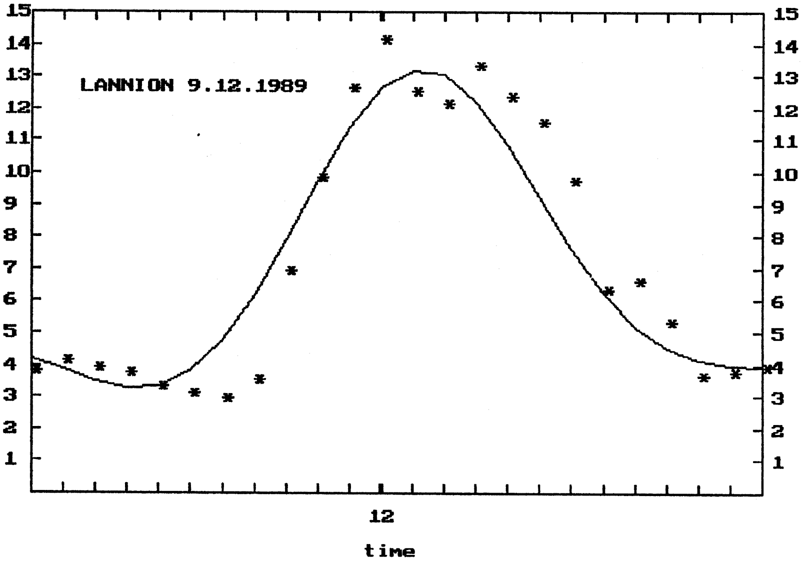

Fig. 2. f_0F2 parameter at Lannion station in December 9, 1989. $*$: Measurements; full line: SRC model.

$$DD(t) = a + b\,t\,\exp(-t) \qquad (3.3)$$

The daily curve of f_0F2 at single station can be described with 6 parameters: a, b, c_1, c_2, d_1, d_2 using a least squares fitting method. The simple prolongation of the obtained curves gives the forecast for a required period ahead. The average within the limited area of longitude and latitude parameters, can be used for calculation f_0F2 at the whole required area, composed the f_0F2 map for the requested period. In such case the f_0F2 is determined by the set of coefficients: a_i, b_i, c_{i1}, c_{i2}, d_{i1} and d_{i2}, where i is the number of specified regions within the required area. This can describe the f_0F2 parameter in some additional screen-points and so it can be a useful tool for instantaneous mapping. Figure 2 presents f_0F2 measurements (stars) and modelled values (full line) obtained only at one station. A more detailed analysis of the method and its application to the prediction is described by Stanislawska (1993b). The comparison within PRIME area is presented by Stanislawska and Zbyszyński (1993).

4. An application to instantaneous mapping

Both methods represent two different approaches of combining the modelled data and the measurements for instantaneous mapping:

1) synthesis of data in regions without measurements by models, like the autocovariance prediction model. Then the chosen mapping technique is used to cover the whole area by the considered ionospheric parameter;

2) the mapping technique is applied not to individual measurements, but to the parameters of model: the series of data at the single station are modelled, like in SRC ionospheric disturbance prediction model. In every station the same modelling procedure is used and then the obtained parame-

ters of model are averaged, interpolated, or fitted by the chosen function.

The short recapitulation of proposed options is presented by Cander *et al.* (1993).

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