Volcanomagnetic anomalies: a review and the computation of the piezomagnetic field expected at Vulcano (Aeolian Islands, Italy)

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Abstract
The volcanic area of Vulcano experienced major unrest, which brought the fumarolic field temperatures from slightly less than 300 °C to ca. 700 °C between 1988-1993. The structure underlying the crater, investigated by drillings and by different geophysical techniques, is relatively well-known. This led us to attempt modelling the magnetic anomaly which could be generated by sudden pressure variations in the magma chamber at shallow depth. The rocks embedding the intrusive rock penetrated by drill-holes to a depth of ca. 2000 m are characterized by high susceptibility, which points to the possibility of obtaining significant magnetic anomalies with acceptably weak pressure pulses. The model for straightforward computing of the anomalous field was drawn accounting for (1) the inferred geometry of the Curie isotherm, (2) presence of a spherical magma reservoir, 2 km wide and centred at a depth of 3.5 km, overlain by (3) a 0.5 km wide and 1.5 km high cylinder simulating the intrusion first revealed by drillings. The model elements (2) and (3) behave as a single source zone and are assumed to lie beyond the Curie point, the contribution to the piezomagnetic effect being provided by the surrounding medium. Under such conditions, a 10 MPa pressure pulse applied within the source-zone provides a 4 nT piezomagnetic anomaly, compatible with the amplitude of the anomalies observed at those volcanoes of the world where magnetic surveillance is routinely carried out. The analytical method used for computation of the magnetic field generated by mechanical stress is extensively discussed, and the contribution of piezomagnetism to rapid variations of the magnetic field is compared to other types of magnetic anomalies likely to occur at active volcanoes.

Key words *Vulcano – piezomagnetism – modelling*

1. Introduction

Since the early part of the century (*e.g.* Wilson, 1922), local variations in the Earth's magnetic field at volcanic areas were inferred to be caused by endogenous activity. Quantitative evidence for this was however collected only decades later, when the availability of proton precession magnetometres allowed continuous measurements of the magnetic field to be made at several active volcanoes of the world (Johnston and Stacey, 1969; Davis *et al.*, 1973; Zlotnicki and Le Mouel, 1988; Yukutake *et al.*, 1990).

Two types of volcanomagnetic effects are known, *i.e.*: a) slow variations of the magnetic field intensity, due to magnetization-demagnetization cycles of thermal origin; b) swift changes of the total field. The latter are generally attributed to rock piezomagnetism (that is, to the magnetic response of the medium to changes in the local stress field) and/or to the electrokinetic effect (that is, to changes in the magnetic field associated to rapid re-distribution of electric charges in fluids).

Both such effects are likely to occur at Vulcano (Aeolian Islands, Southern Tyrrhenian Sea), which last erupted at the end of the last century (1888-1890) and, since then, has remained in quiescent stage characterized by minor fumarolic and microseismic activity. Timing 1987, however, sharp changes affected both the chemical parameters of the fumarolic gases and the gas temperatures. The latter rose from ca. 270 °C in 1988 to ca. 400 °C in 1989, then reached 650 °C in early 1991 and slightly less than 700 °C in 1993.

Such changes were accompanied by an increase in both the flow of magmatic gases and the CO_2 concentration in soils (Chiodini *et al.*, 1991), but not by comparable changes in the geophysical parameters. Indeed, the shallow and weak seismic activity typical of the crater had dropped since late 1988 (Vilardo *et al.*, 1991), while no significant ground deformation has been observed from 1988 onwards. The most relevant episode to occur during these years of unrest was in April 1988, when a large landslide removed several thousand hundreds m^3 of pyroclastic cover from the upper eastern flank of the crater, which is still showing minor flank instability episodes.

In spite of the steadiness of geophysical indicators, the eruptive history of Vulcano (Frazzetta *et al.*, 1984) and the character of its present unrest suggest that magnetic surveillance is a potentially profitable tool for detecting changes in the shallow plumbing system. Indeed, the La Fossa crater area shows:

1) strong thermal anomalies at the surface, testified by the major increase in gas temperatures observed from late 1987 onwards (Barberi *et al.*, 1991);

2) an intrusive body at shallow depth, first revealed by geothermal drillings (Faraone *et al.*, 1986) and later confirmed by seismic tomography (Ferrucci *et al.*, 1991);

3) high susceptibility volcanic rocks, characterized by presence of magnetite among the primary components of the intrusive body (Gioncada and Sbrana, 1991).

This study attempts the quantitative assessment of short-to-mid term magnetic precursors to eruptions at Vulcano on a theoretical basis.

2. Magnetic transients

Magnetic transients at Vulcano could be engendered by:

– demagnetization by heat transfer from a hot intrusive body to embedment;
– dislocation of magnetic volumes within the Earth;
– electrokinetic effects (electro-filtration);
– effects involving dilatancy;
– piezomagnetic effects.

The first of the transients listed above relates to heating of the medium by uprising magma bodies, which bring increasingly large volumes of rocks above the Curie Point. This process may take years before being recognized on a local scale, and cannot be considered either a short or a mid-term precursor to eruptions, even though it could be speeded-up by convection occurring in highly fracturated media as ours. As for the dislocations of magnetic bodies underneath, the lack of significant ground deformations (Falsaperla *et al.*, 1989; Barberi *et al.*, 1991) or gravity anomalies with time at Vulcano (only a few tens μGal; Berrino *et al.*, 1988) excludes the fact that such events might have occurred there in recent times.

In our case, the only phenomena able to generate relatively fast magnetic transients are: electro-filtration, dilatancy and piezomagnetism, which display strict links. Indeed, changes in the stress field acting on the plumbing system may modify not only the natural and the induced magnetization of rocks (Martin *et al.*, 1978; Zlotnicki *et al.*, 1981), but also the interconnected microcrack networks and the circulation of underground waters. In turn, resistivity changes in crustal volumes affected by dilatancy include the effects due to change in pore networks (Nur, 1972), while mechanical distortion of dilated bodies may breed piezomagnetic effects, as well as interstitial pressure gradients acting on dilated regions where waters circulate may breed electrokinetic effects (Mitzutani *et al.*, 1976; Fitterman, 1979). In the case of Vulcano, three such reasons for magnetic anomalies conceivably coexist.

Streaming potentials generated by underground water flow are considered the most probable reason for changes in self-potentials linked to volcanic activity (Fitterman. 1978; Ishido and Mitzutani, 1981). Self-potentials induce telluric currents which, in turn, may generate a magnetic field (Fitterman, 1979). Recent theoretical studies show that, with ordinary values for Earth resistivity and water flow velocity, the electrokinetic effects are too weak to give rise to observable magnetic fields (Fitterman, 1981; Murakami, 1989), even though they might become detectable when hydrothermalism is present (Zlotnicki and Le Mouel, 1988).

One relevant aspect of dilatancy is the mechanical distortion of rocks, which implies stress and mass readjustments to give rise to magnetic and gravity anomalies. Constant-stress, dilatancy-driven changes in the remanent magnetization of rocks were observed by Martin *et al.* (1978). Sasai (1985) demonstrated that Hagiwara's (1977b) expanding spherical-source method, based on Mogi's (1958) point-source deformation model and allowing for the computation of dilatancy-induced gravity changes, is applicable to magnetism.

Comparison of computed changes in the magnetic field (Appendix I) emphasizes clear predominance of the piezomagnetic over the dilatancy effect. This leads to the assessment that, at least from a theoretical standpoint, only the piezomagnetic effect is significant and detectable.

3. The piezomagnetic effect

Reversible or non-reversible changes in rock magnetization can occur as a function of changes in the stress field acting on an assigned crust volume. This effect is known as the piezomagnetic effect. Non-reversible piezomagnetic changes are generally small (some tenths of A m^{-1}; Nagata, 1970). Reversible changes are roughly proportional to the applied stress, which allows for relatively plain computation of the piezomagnetic field.

The theoretical aspects of piezomagnetism were developed by Nagata (1970, 1971) and Stacey and Johnston (1972). In ferromagnetic minerals with single-domain structure, reversible magnetization can be explained in terms of the rotation of the spontaneous magnetization, while application of large stresses to multi-domain ferromagnetic minerals gives rise to non-reversible magnetization because of the displacement of the domain walls. It is worth noting that the remanent natural magnetization of some rocks often displays non-linear behaviour as a function of the applied stress (Henyey *et al.*, 1978; Revol *et al.*, 1978). Conversely, the stress-sensitivity (that is, induced magnetization vs. stress) increases with increasing hydrostatic pressure (Nulman *et al.*, 1978), and abruptly decreases when temperatures exceeds ca. 300 °C (Pozzi, 1977).

The fundamental concept of the stress-induced volcanomagnetic effect was first proposed by Stacey *et al.* (1965) who, having assigned a stress distribution around a magma chamber, computed a theoretical piezomagnetic anomaly. Several examples of magnetic anomalies due to volcanic activity are reported in literature. Among them, magnetic variations were observed before and during the 1968 eruption at Mt. Ruapehu (New Zealand; Johnston and Stacey, 1969), during an eruption of Kilauea (Hawaii; Davis *et al.*, 1979), immediately after the May 18, 1980 eruption of Mt. St. Helens (Johnston *et al.*, 1981), and during the November 1986 fissural eruptions of lzu-Oshima (Japan; Sasai *et al.*, 1990). It can be inferred that magma intrusions, or sudden pressure changes within a magma chamber, may alter the stress field acting on the embedment and give rise to piezomagnetic transients.

Schematically, this stress field can be decomposed into a hydrostatic and a deviatoric component. The piezomagnetic effect would derive from the latter, provided that the temperature of the embedment is below the Curie Point. Prior to an eruption, it can be expected that magmatic activity may lead to an increase in the deviatoric stress, and to the consequent increase in the local magnetic field, whereas stress release occurring during an eruption would be consistent with a decrease in the magnetic field (Parkinson, 1983).

The viscous magmatic products of Vulcano are highly saturated. This petrochemical character points to long residence times in a magma chamber for *in situ* differentiation of the primary magmas of mantle origin. Then, sudden pressure changes in the magma chamber are a very likely stress source to give rise to piezomagnetic effects and forerunning an eruption.

Below, we will compute the piezomagnetic field expected at the La Fossa crater, based on the occurrence of a pressure pulse in the magma chamber.

4. The model

Magnetic field variations relating to the stress generated by a pressure pulse applied within a cylindric source domain were first calculated by Yukutake and Tachinaka (1967). This model can be considered a two-dimensional version of Mogi's model, which was first used in application to modelling of the source of the volcanomagnetic effects observed at Kilauea by Davis (1976). Sasai (1979) obtained an analytical solution for the piezomagnetic variations generated by an expanding sphere, by Fourier-transform solving of the convolution integrals related to the magnetic potential.

We compute the expected piezomagnetic field following the extended Mogi's model (Hagiwara, 1977b; Sasai, 1986) which, from a mechanical standpoint, is equivalent to a set of tensile cracks with arbitrary orientation, centred on a point at depth. In this model, increase in the hydrostatic pressure within all cracks, that is within a spherical magma chamber, generates stress variations in the embedding medium (the volcanic body).

In terms of depth, radius and pressure variation of the magma chamber, the model boundaries are:

Depth – The fumarolic system of Vulcano is reported to be fed by a magmatic source, relating to the hottest fumaroles of the La Fossa crater, and by a shallow source, relating to rain and seawaters heated by hot deep fluids and giving rise to intermediate-temperature fumaroles (Cioni and D'Amore, 1984). The isotopic ratio of the principal gas components points to a magmatic origin of gases (Italiano *et al.*, 1989; Chiodini *et al.*, 1991). In turn, the temperature of ca. 700 °C reached by the hottest fumaroles nowadays is close to the melting point of Vulcanian magmas, and points to a certainly limited depth of the source melts.

In addition, it is worth noting that the typical seismicity of Vulcano, substantially lacking since late 1988, spread in depth between 0-2 km b.s.l. (Vilardo *et al.*, 1991), and that the largest focal depths beneath the island have never exceeded 4 km b.s.l.

These arguments point to the presence of a low-rigidity medium between 2-4 km b.s.l., and suggest that the centre of the hypothesized magma chamber would lie some 3 km beneath the crater.

Radius – A geothermal drilling carried out in 1983-1984 on the southwestern foot of the volcano reached an intrusive body at a depth of 1360 m, and penetrated it down to the depth of 2050 m (Faraone *et al.*, 1986; Barberi *et al.*, 1989) where temperature exceeded 419 °C (the Zinc melting point). A deviated well emphasized higher temperatures at comparable depths, leading to infer lesser depths of the top of the intrusion towards the north. These data are compatible with the indications provided by seismic (Ferrucci *et al.*, 1991), gravity (Faraone *et al.*, 1986) and magnetic surveys (Iacobucci *et al.*, 1977), but not with the results of another geothermal well located at the northeastern foot of the crater. This well, 1000 m deep, remained within the volcano-sedimentary serie and did not exceed the temperature of 250 °C (Gioncada and Sbrana, 1991), leading to the assumption that the intrusive body was not large.

In conclusion, geophysical evidence does not allow the dimension of the magma chamber to be constrained. However, assuming that the chamber has a spherical shape and its inferred centre lies 3 km b.s.l., and considering that the deepest well, offset 2 km with respect to the crater axis, has reached high temperatures 2 km b.s.l., it seems acceptable to adopt a magma chamber radius of 1 km or slightly less.

Pressure – A magmatic melt at depth is generally less dense than the embedding medium, and tends to uprise in a gravitational field. The difference between buoyancy and lithostatic load can be increased by non-hydrostatic processes as: convective motion of rocks at depth (Elder, 1976), volume changes during the melting process (Fedotov, 1977), exolution of gases because of convective mixing (Sparks *et al.*, 1978), or partial crystallization of magmas. In volcanological literature, local pressure estimates spread over one order of magnitude, ranging from 20 MPa for viscous magmas (Popov, 1973) to 2 MPa for fluid magmas (Aki *et al.*, 1977). Considering the high viscosity of vulcanian magmas and the shallow depth of the hypothesized magma chamber at Vulcano, we adopted an average overpressure value of 10 MPa for computation of the piezomagnetic field.

As the initial conditions, we assume the weight of the medium to be balanced by the thermoelastic stresses and the hydrostatic magma pressure P in the reservoir. The medium is assumed to be homogeneous, isotropic and perfectly elastic. The mechanical properties of the rocks are assumed not to depend on temperature and pressure. The perfect elasticity hypothesis is supported by some examples collected on volcanoes (for instance at Mt. Etna: Hirn *et al.*, 1991; Ferrucci *et al.*, 1993) where Lamé's constants λ and μ have been inferred to be generally close in value, independent of their heterogeneity in space. In turn, the hypotheses of homogeneity and isotropy are assumed to be valid at the scale of the volcano, and would be supported by the presence of tight intercalation of lavas and pyroclastic deposits, distributed in near-axial symmetry and constituting the volcanic edifice.

The magma reservoir is modeled by piling two bodies with simple geometry, that is: a sphere simulating the magma chamber, whose radius is 0.75 km and whose centre lies 3.5 km b.s.l., and a cylinder, 1.5 km high and a 0.5 km radius, simulating the intrusion. It is worth noting that, in the real case, the rocks nearest to the magma reservoir are also the site of partial melting which may lead to mechanical and petrochemical changes, as well as to inflitration of gases of magmatic origin. This near-chamber domain must have both temperatures close to the melting temperatures (some 800 °C at Vulcano), and display plastic behaviour. Below, we will refer to the «magma reservoir» to include the ensemble of the magma chamber, the intrusion, and the rocks with plastic behaviour.

5. The stress field in Mogi's model

Let us take, as the reference system, a Cartesian frame (O, x, y, z) with x, y and z oriented to the north, to the east and downwards respectively. A perfectly elastic half-space $(z > 0)$, uniformly magnetized in the range $0 < z < H_0$, includes a small sphere centred in $A\ (0, 0, D)$, with radius a. The sphere is subjected to a change P of its hydrostatic internal pressure (fig. 1).

The displacement field predicted by Mogi's model is equivalent to that given by a deformation source (dilation nucleus), whose components are (Anderson, 1936; Yamakawa, 1955):

$$u_x = \frac{C}{2\mu}\left\{\frac{x}{R_1^3} + \frac{\lambda+3\mu}{\lambda+\mu}\frac{x}{R_2^3} - \frac{6xz(z+D)}{R_2^5}\right\} \quad (5.1a)$$

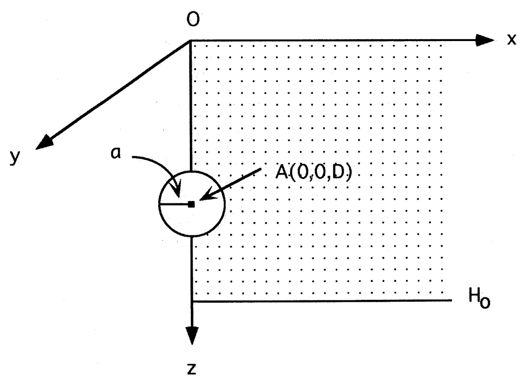

Fig. 1. Reference system for Mogi's model. The hydrostatic pressure increase is applied within a spherical source of radius a, centred in A (0, 0, D). H_0 is the depth of the Curie isotherm.

$$u_y = \frac{C}{2\mu}\left\{\frac{y}{R_1^3} + \frac{\lambda+3\mu}{\lambda+\mu}\frac{y}{R_2^3} - \frac{6yz(z+D)}{R_2^5}\right\} \quad (5.1b)$$

$$u_z = \frac{C}{2\mu}\left\{\frac{z-D}{R_1^3} + \frac{(\lambda-\mu)z-(\lambda+3\mu)D}{(\lambda+\mu)R_2^3} - \frac{6z(z+D)^2}{R_2^5}\right\} \quad (5.1c)$$

where

$$R_1 = [x^2+y^2+(z-D)^2]^{1/2} \quad (5.2a)$$

$$R_2 = [x^2+y^2+(z+D)^2]^{1/2} \quad (5.2b)$$

$$\lambda = \mu \quad (5.2c)$$

λ e μ are Lamé's constants, while C has the dimension of a moment and describes the dilation nucleus intensity. Under the condition that a is small enough ($a \cong D/4$; Davis, 1976), C is

$$C = -\frac{1}{2}a^3 \Delta P \quad (5.3)$$

Note that, in Mogi's model, ΔP (dilation) and a (radius of the domain) are not independent of each other. The first term in each of the eqs. (5.1a-c), indeed, represents the displacement field given by a dilation source in an infinite medium, while the two remaining terms describe the free-surface effect.

In agreement with the elasticity theory (Means, 1976), the strain components are obtained by the displacement field through differentiation of eqs. (5.1a-c), that is

$$e_{xx} = \frac{C}{2\mu}\left(\frac{1}{R_1^3} + \frac{2}{R_2^3} - \frac{6\{z(z+D)+x^2\}}{R_2^5} - \frac{3x^2}{R_1^5} + \frac{30x^2z(z+D)}{R_2^7}\right) \quad (5.4a)$$

$$e_{yy} = \frac{C}{2\mu}\left(\frac{1}{R_1^3} + \frac{2}{R_2^3} - \frac{6\{z(z+D)+y^2\}}{R_2^5} - \frac{3y^2}{R_1^5} + \frac{30y^2z(z+D)}{R_2^7}\right) \quad (5.4b)$$

$$e_{zz} = \frac{C}{2\mu}\left(\frac{x^2+y^2-2(z-D)^2}{R_1^5} + \frac{6D(z+D)}{R_2^5} - \frac{6(z+D)\{3(z+D)R_2^2-5z(z+D)^2\}}{R_2^7}\right) \quad (5.4c)$$

$$e_{xy} = \frac{C}{2\mu}(-3xy)\left(\frac{1}{R_1^5} + \frac{2}{R_2^5} - \frac{10z(z+D)}{R_2^7}\right) \quad (5.4d)$$

$$e_{yz} = \frac{C}{2\mu}(-3y)\left(\frac{z-D}{R_1^5} + \frac{z}{R_2^5} + \right. \tag{5.4e}$$

$$\left. + \frac{\{(2z+D)(x^2+y^2)+(z+D)^2(-8z+D)\}}{R_2^7}\right)$$

$$e_{zx} = \frac{C}{2\mu}(-3x)\left(\frac{z-D}{R_1^5} + \frac{z}{R_2^5} + \right. \tag{5.4f}$$

$$\left. + \frac{\{(2z+D)(x^2+y^2)+(z+D)^2(-8z+D)\}}{R_2^7}\right)$$

According to Hooke's law, in the case of isotropic media the relations between the stress tensor and the strain tensor components reduce to

$$\begin{bmatrix} \tau_{xx} \\ \tau_{yy} \\ \tau_{zz} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix} = \mu \begin{bmatrix} 3e_{xx}+e_{yy}+e_{zz} \\ e_{xx}+3e_{yy}+e_{zz} \\ e_{xx}+e_{yy}+3e_{zz} \\ 2e_{xy} \\ 2e_{yz} \\ 2e_{zx} \end{bmatrix} \tag{5.5}$$

6. Computation of the piezomagnetic anomaly

Volcanic rocks are characterized by two magnetization terms, *i.e.*: thermo-remanent magnetization (acquired when cooling), and the magnetization induced by the present Earth's Magnetic Field (EMF). Susceptibilites of the principal bulk lithotypes sampled at Vulcano range from 1 to 3×10^{-2} SI, while susceptibilities of pyroclastic and *lahar* deposits range from 0.3 to 2×10^{-2} SI (Barberi *et al.*, 1993). In our model, 1×10^{-2} SI is taken as the representative susceptibility value.

As for remanent magnetization, its intensity may range from null values in *surge* and *lahar* deposits, to values even greater than 7 A m^{-1} in the intrusive body penetrated by drillings (Barberi *et al.*, 1993). In our case, 2 A m^{-1} represents a reliable average of this parameter, in spite of its wide variability.

Aimed at forward modelling of the piezomagnetic anomaly, we will assume the magnetization of the volcanic edifice to be uniform and equal to 2 A m^{-1} and 0.5 A m^{-1} for the remanent and the induced magnetization respectively. Overall, the orientation of the natural remanent magnetization is that of the average EMF (the volcanic products of the Aeolian arc are recent). Since the induced magnetization is also parallel to the EMF, we can write the components of the total magnetization vector $\boldsymbol{J}_t$ (the sum of the induced and remanent magnetizations) as:

$$\begin{aligned} J_x &= J_t \cdot \cos I_t \cdot \cos D_t \\ J_y &= J_t \cdot \cos I_t \cdot \sin D_t \\ J_z &= J_t \cdot \sin I_t \end{aligned} \tag{6.1}$$

where I_t and D_t are the EMF's inclination and declination respectively.

Stacey (1964) established the following linear relationships to hold, in rocks rich in ferromagnetic minerals, between applied stress and magnetization changes J:

$$\Delta J^{\parallel} = \beta\sigma J_0^{\parallel} \tag{6.2a}$$

$$\Delta J^{\perp} = -\frac{1}{2}\beta\sigma J_0^{\perp} \tag{6.2b}$$

where β is the stress sensitivity, symbols $\parallel$ and $\perp$ indicate the parallel and the normal components of the stress respectively, and the subscripts indicate the magnetization values at rest. According to Carmichael (1977), who assessed the upper crust to be characterized by uniform magnetization and approximately constant stress sensitivity, β is of the order of 10^{-3} MPa^{-1}.

Equations (6.2a,b) for three dimensions were obtained by Stacey *et al.* (1965), relying on the applicability of the superposition principle to reversible piezomagnetic effects. Decomposition of $\boldsymbol{J}_0$ (J_1, J_2, J_3) along the directions of principal stresses σ_1, σ_2, σ_3 through eqs. (6.2a,b), leads to the following components of the stress-induced magnetization:

$$\left.\begin{array}{c} \Delta J_i \boldsymbol{e}_i = \beta J_i \left(\sigma_i - \dfrac{\sigma_j + \sigma_k}{2} \right) \boldsymbol{e}_i \\ (i,j,k = 1,2,3. \;\; i \neq j \neq k) \end{array}\right\} \tag{6.3}$$

where $\{\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3\}$ are the unit vectors defining the directions of principal stresses.

Equation (6.3) is equivalent to the so-called «generalized equation of linear piezomagnetism» (Sasai, 1980):

$$\Delta \boldsymbol{J} = \frac{3}{2} \beta \boldsymbol{T}' \boldsymbol{J} \tag{6.4}$$

whose main advantage with respect to eq. (6.3) is that it does not require any estimate of directions and moduli of principal stresses. In eq. (6.4), $\boldsymbol{T}'$ is the deviatoric stress tensor, related to the stress tensor $\boldsymbol{T}$ and to the average stress σ_0 through

$$\boldsymbol{T} = \sigma_0 \boldsymbol{E} + \boldsymbol{T}' \tag{6.5a}$$

In eq. (6.5a) $\boldsymbol{E}$ is the unit matrix, while σ_0 and T' are

$$\sigma_0 = \frac{1}{3}(\tau_{xx} + \tau_{yy} + \tau_{zz}) \tag{6.5b}$$

$$T' = \begin{bmatrix} \tau_{xx} - \sigma_0 & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} - \sigma_0 & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} - \sigma_0 \end{bmatrix} \tag{6.5c}$$

Equations (6.4) and (6.5a-c) imply rock piezomagnetism is due to deviatoric stresses only. This statement directly derives from eq. (6.2a,b), where the magnetization in the direction normal to the applied stress varies at a rate half that parallel to the stress direction. Accounting for (6.5a-c), relation (6.4) can be rewritten as

$$\begin{bmatrix} \Delta J_x \\ \Delta J_y \\ \Delta J_z \end{bmatrix} = \beta \begin{bmatrix} \tau_{xx} - \dfrac{\tau_{yy} + \tau_{zz}}{2} & \dfrac{3}{2}\tau_{xy} & \dfrac{3}{2}\tau_{zx} \\ \dfrac{3}{2}\tau_{xy} & \tau_{yy} - \dfrac{\tau_{xx} + \tau_{zz}}{2} & \dfrac{3}{2}\tau_{yz} \\ \dfrac{3}{2}\tau_{xz} & \dfrac{3}{2}\tau_{yz} & \tau_{zz} - \dfrac{\tau_{xx} + \tau_{yy}}{2} \end{bmatrix} \begin{bmatrix} J_x \\ J_y \\ J_z \end{bmatrix} \tag{6.6}$$

with ΔJ_x, ΔJ_y and ΔJ_z the components of the magnetization variation. Such linear relationship holds even in case of dependence of β e $\boldsymbol{J}$ on their position in space: but it does not for deviatoric stresses exceeding some tens MPa (Sasai, 1980).

Formula (6.6) allows us to evaluate the stress-induced magnetization. Indeed, the magnetic potential dW in a point P (x, y, z), due to a magnetic dipole $\Delta \boldsymbol{J} dV'$ located in Q (x', y', z'), is

$$dW = \frac{\Delta \boldsymbol{J} \cdot \boldsymbol{r}}{r^3} dV' \tag{6.7}$$

where

$$\begin{aligned} \Delta \boldsymbol{J} &= (\Delta J_x, \Delta J_y, \Delta J_z) \\ \boldsymbol{r} &= (x - x', y - y', z - z') \\ r^2 &= (x - x')^2 + (y - y')^2 + (z - z')^2 \end{aligned} \tag{6.8}$$

Since $\boldsymbol{F} = -\nabla W$, the ΔX, ΔY, ΔZ components of the magnetic field $\Delta \boldsymbol{F}$ calculated in P (x, y, z) can be written, for a plate of thickness H_0, as

$$\Delta X = \int_0^{H_0} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (\Delta J_x U_1 + \Delta J_y U_4 + \Delta J_z U_5) dx' \, dy' \, dz'$$

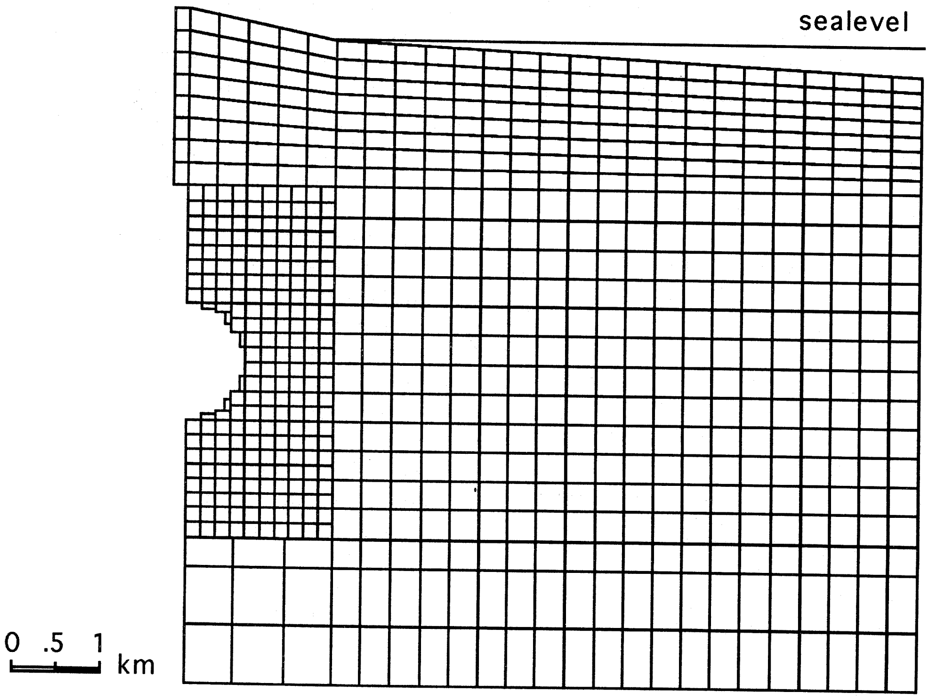

Fig. 2. Cross-section of the discretization grid for finite-elements computation of the magnetic anomaly.

$$\Delta Y = \int_0^{H_0}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (\Delta J_x U_4 + \Delta J_y U_2 + \Delta J_z U_6) dx' \, dy' \, dz'$$

$$\Delta Z = \int_0^{H_0}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (\Delta J_x U_5 + \Delta J_y U_6 + \Delta J_z U_3) dx' \, dy' \, dz' \tag{6.9}$$

with U_i ($i = 1, ..., 6$) depending on the coordinates of P and Q (Appendix II).

The piezomagnetic variations at the Earth's surface can be computed solving eqs. (6.9). Integrals in eqs. (6.9) can be solved by Sasai and Ishikawa's method (Sasai and Ishikawa, 1978; see also Appendix II), that is, by discretizing the volcanic edifice into finite elements of rectangular section and assuming that magnetization is constant within each element. The computation precision is a function of the level of discretization (fig. 2), which implies that it is worth using smaller elements in the central part of the volcano, where forces are applied and large changes of magnetization occur, and near the surface, where magnetic variations are calculated. In order to avoid numerical singularities (Yukutake and Tachinaka, 1967), the field is calculated 10 m above the free-surface.

As for the boundary conditions, we assume that vertical displacements are null beyond the depth of 7 km, and horizontal displacements vanish 8 km off the crater axis. Laboratory measurements on sample rocks of Vulcano indicate that the magnetization intensity is reduced by 50% at 400-500 °C, and total demagnetization is reached at temperatures of 650 °C (Barberi *et al.*, 1993). Extrapolation of the depth of the equal-Curie point surface from bore-hole temperature data allows us to infer that magnetic susceptibility would be null at depths of ca. 3 km along the crater axis and ca.

5 km at the external boundaries of the discretization domain (fig. 3).

The magnetic field variation (ΔX, ΔY, ΔZ) at an assigned point P is the sum of the contributions of each element. Its horizontal component ΔH is

$$\Delta H = \Delta X \cdot \cos D_t + \Delta Y \cdot \sin D_t \qquad (6.10)$$

while the change in total intensity ΔF is

$$\Delta F = \Delta H \cdot \cos I_t + \Delta Z \cdot \sin I_t \qquad (6.11)$$

The results of calculations are shown in fig. 4 and relate to the entry values detailed in table I. The amplitude of the expected piezomagnetic anomaly is 4 nT.

7. Discussion and conclusions

The amplitude of the computed piezomagnetic anomaly (4 nT for a 10 MPa pressure pulse) is in agreement with magnetic anomalies observed or calculated at several volcanoes of the world (Johnston and Stacey, 1969; Davis *et al.*, 1973; Pozzi *et al.*, 1979; Johnston *et al.*, 1981; Zlotnicki and Le Mouel, 1988; Yukutake *et al.*, 1990; Sasai *et al.*, 1990).

It is worth emphasizing that the simple model we used does not account for the expected complexity of the real distributions of stress, strain and magnetization within and beneath the volcanic edifice of La Fossa. Indeed, we assumed that (1) the mechanical properties of rocks are homogeneous and temperature independent, and (2) excluded that changes in the pore pressure distribution might significantly modify the state of stress and, consequently, the piezomagnetic coefficients. We assumed also that (3) the medium lying above the equal-Curie temperature surface is uniformly magnetized.

In real cases of layered media, with layers displaying different magnetic characteristics,

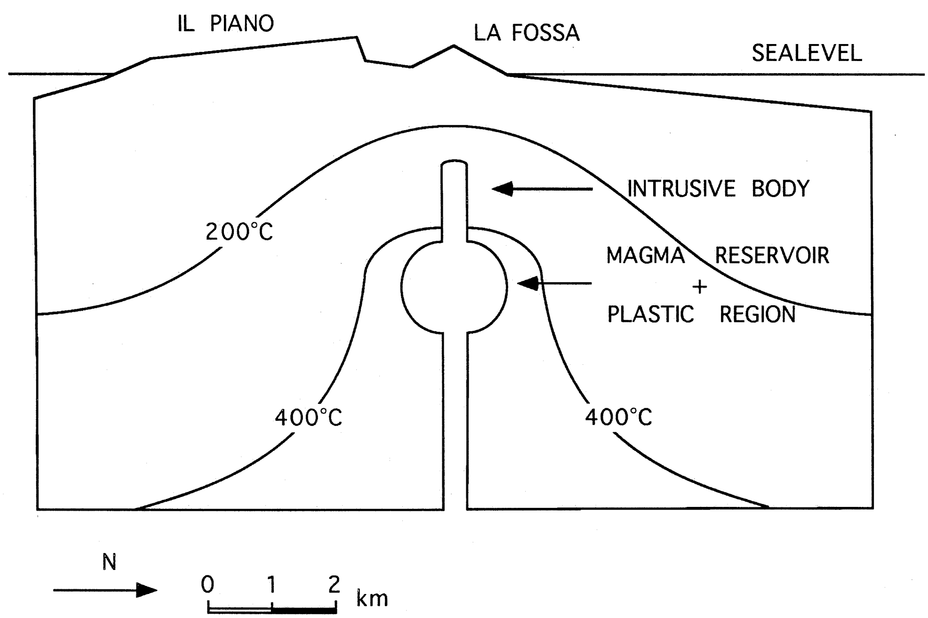

Fig. 3. Extrapolated temperature profile at Vulcano, constructed after the geothermal well parameters reported in Faraone *et al.* (1986).

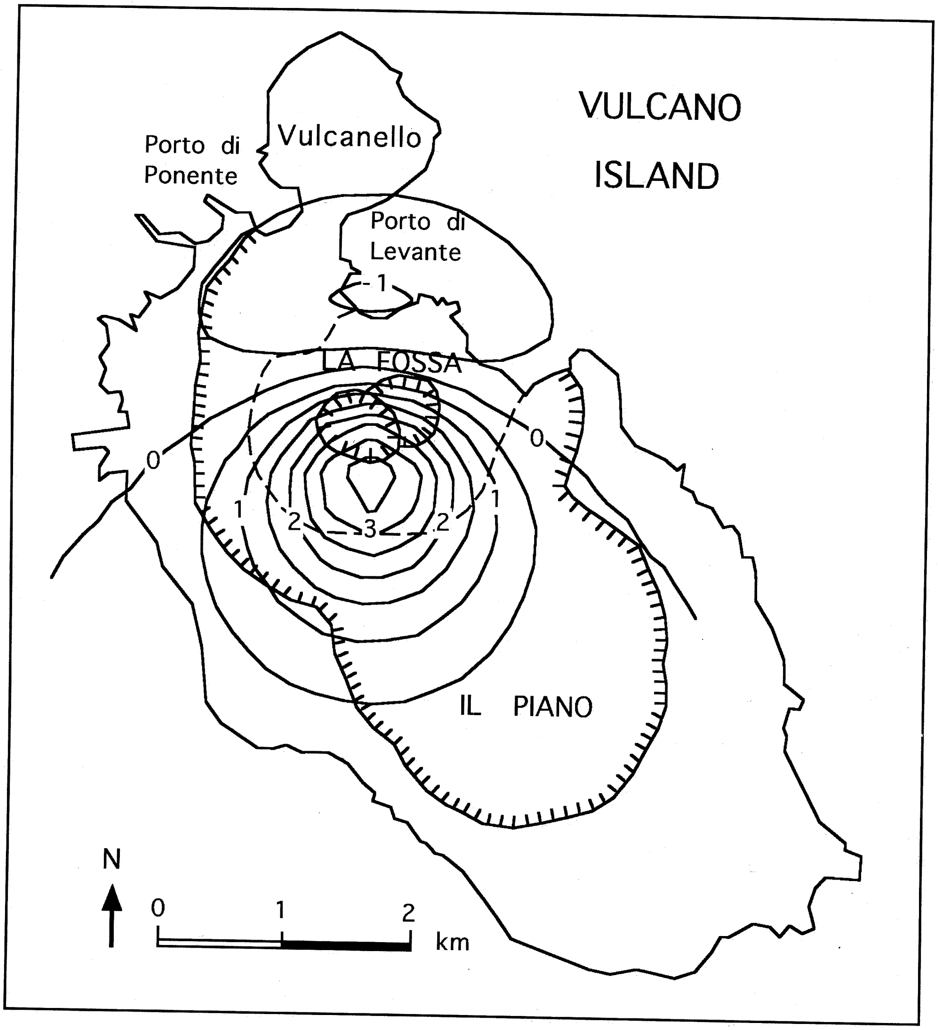

Fig. 4. Map of the computed piezomagnetic anomaly, as given by a 10 MPa pressure pulse applied at the centre of a spherical magma chamber, located at a depth of 3.5 km. Equidistance: 0.5 nT.

significant strengthening of the piezomagnetic effect may occur *(e.g.* Zlotnicki and Cornet, 1986). This can be engendered by strong magnetization and high stress sensitivity of deeper layers, and/or by non-uniformity in rock magnetization, stress sensitivity and elasticity in the Earth's crust (Oshiman, 1990). In other words, the heterogeneity proper to the shallow crust in volcanic environment may be a crucial factor in giving rise to volcanomagnetic effects much larger than those calculated in a homogeneous Earth (Budetta *et al.*, 1991).

We can preliminarily conclude that the limitations of our model (essentially: homogeneity and uniformity of magnetization) lead to locating the computed value of 4 nT at the lower

Table I. Parameters used for computation of the piezomagnetic field.

Depth of the centre of the sphere	D	3.5 km	Stress sensitivity	β	1.0×10^{-3} Mpa^{-1}
Radius of the sphere	a	0.75 km	Lamé's constants	λ, μ	3.0×10^{4} Mpa
Radius of the intrusion		0.25 km	Total magnetization	J_t	2.5 A m^{-1}
Height of the intrusion		1.5 km	Inclination (Earth's M.F.)	I_t	54°
Pressure variation	P	10 MPa	Declination (Earth's M.F.)	D_t	1°

limit of a wide band of stronger effects which would be driven by application of a 10 MPa pressure pulse to the magma reservoir.

In a pure piezomagnetic model as ours, the generation of fluid motions because of the change of the stress conditions at all scales (pores to fractures), and the consequent generation of magnetic effects of electrokinetic origin (Mizutani, 1976), is not accounted for. The magnetic effects of piezomagnetic origin, however, can be distinguished from those of electrokinetic origin since electric fields or current flows are associated only to the latter. From an experimental standpoint, this difference emphasizes the need to carry out joint measurements of magnetic and electric fields, since the good correlation of electric and magnetic anomalies in time and space would provide evidence in support of a streaming potential source (Fitterman, 1979; Davis *et al.*, 1989).

In conclusion, the results of the present study demonstrate the possibility of observing magnetic transients of piezomagnetic origin at Vulcano, since the wide amplitude of the least theoretical anomaly (4 nT) is comparable to the worst environmental magnetic noise observed there (Budetta *et al.*, 1991). In the light of the volcanological history of the island, which mainly experienced explosive eruptions, this possibility may be relevant for revealing sudden stress changes forerunning future eruptive episodes. Accounting for the space character of the expected anomaly highlighted in fig. 4, an array composed of at least three total-field magnetic stations would be adequate for monitoring purposes.

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Appendix I. Predominance of the piezomagnetic effect in Mogi's model.

Dilatancy mechanisms of spherical or point sources are often used for straightforward modelling of ground deformations at volcanic areas. As stressed by Rundle (1978), however, a satisfactory model for dilatancy is still lacking, and the sphere is seen as a kinematic approximation only. Based on Mogi's model of a spherical expanding magma chamber, Hagiwara (1977a) singled out four contributions to the gravity changes associated to dilatancy, that is (fig. A.1):

– G1: free-air gravity change because of the uplift of the observation point;
– G2: Bouguer anomaly change due to the uplifted portion of the ground;
– G3: mass redistribution because of the expansion of the magma chamber walls;
– G4: gravity field associated to density changes in the elastic half-space.

Similarly, Sasai (1985) singled out four types of magnetic changes linked to Mogi's model:

– M1: «free-air» magnetic effect generated by displacement of the observation point;
– M2: magnetic anomaly due to uplift of the topographic surface;
– M3: loss of magnetic mass because of the expansion of the magma chamber walls;
– M4: piezomagnetic effect.

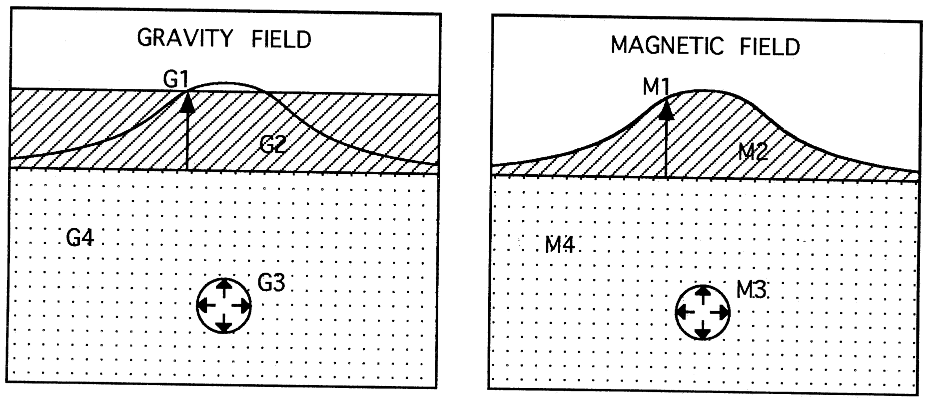

Fig. A.1. Sketch map of Mogi's model contributions to gravity (left) and magnetic (right) variations. After Sasai (1986).

Hagiwara (1977a) demonstrated that, if the spherical source is filled with gas, then:

$$|G1| > |G2| > |G3| > |G4|$$

while, when it is filled with magma:

$$|G1| > |G2| > |G4| > |G3|$$

Even though the contribution of G3 and G4 to the overall gravity variation is less than that of G1 and G2, both these terms must be taken into account since they allow information on the density of materials filling the fracture network to be retrieved. In the magnetic case, the fracture is seen as empty, because gas, water and magma do not contribute in the local magnetic field. In application to Mogi's model (Sasai, 1985), we have:

$$|M4| \gg |M2| \approx |M3| \gg |M1|$$

In conclusion, since the term M4 is clearly predominant with respect to the others, the volcano-magnetic field deduced by application of Mogi's model can be considered of pure piezomagnetic origin.

Appendix II. The Sasai-Ishikawa method.

The integrals in the right-hand terms of eqs. (6.9) can be computed by use of the following method, proposed by Sasai and Ishikawa (1978). A Poissonian half-space is discretized by means of rectangular prisms. The integrals can be easily calculated assuming that magnetization is constant within each prism. Taking the point $Q\,(x', y', z')$ as the centre of a rectangular prism with dimensions $2\Delta x$, $2\Delta y$, $2\Delta z$, we obtain

$$\begin{aligned} \Delta X &= \sum_i (\Delta J_{xi}\,\Delta U_{1i} + \Delta J_{yi}\,\Delta U_{4i} + \Delta J_{zi}\,\Delta U_{5i}) \\ \Delta Y &= \sum_i (\Delta J_{xi}\,\Delta U_{4i} + \Delta J_{yi}\,\Delta U_{2i} + \Delta J_{zi}\,\Delta U_{6i}) \\ \Delta Z &= \sum_i (\Delta J_{xi}\,\Delta U_{5i} + \Delta J_{yi}\,\Delta U_{6i} + \Delta J_{zi}\,\Delta U_{3i}) \end{aligned} \tag{A.1}$$

where

$$\Delta U_{ji}(x,y,z,x',y',z') = \int_{z'-\Delta z}^{z'+\Delta z}\int_{y'-\Delta y}^{y'+\Delta y}\int_{x'-\Delta x}^{x'+\Delta x} U_{ji}(x,y,z,x',y',z')\,dx'\,dy'\,dz' \qquad j=1,\ldots,6 \tag{A.2}$$

and

$$\begin{aligned}
U_{1i} &= \frac{2(x-x')^2-(y-y')^2-(z-z')^2}{r^5}\\
U_{2i} &= \frac{2(y-y')^2-(x-x')^2-(z-z')^2}{r^5}\\
U_{3i} &= \frac{2(z-z')^2-(x-x')^2-(y-y')^2}{r^5}\\
U_{4i} &= \frac{3(x-x')(y-y')}{r^5}\\
U_{5i} &= \frac{3(x-x')(z-z')}{r^5}\\
U_{6i} &= \frac{3(y-y')(z-z')}{r^5}
\end{aligned} \tag{A.3}$$

Integrating the eqs. (A.2), and putting

$$\begin{aligned}
L(i,j,k) &= \tan^{-1}\frac{\Delta y_i\cdot\Delta z_k}{\Delta x_i\cdot\Delta r(i,j,k)}\\
L'(i,j,k) &= \tan^{-1}\frac{\Delta x_i\cdot\Delta z_k}{\Delta y_i\cdot\Delta r(i,j,k)}\\
M(i,j,k) &= \Delta z_k+\Delta r(i,j,k)\\
N(i,j,k) &= \frac{\Delta r(i,j,k)+\Delta y_j}{\Delta r(i,j,k)-\Delta y_j}\\
N'(i,j,k) &= \frac{\Delta r(i,j,k)+\Delta x_j}{\Delta r(i,j,k)-\Delta x_j}
\end{aligned} \tag{A.4}$$

with $i,\ j,\ k=1,2$ and

$$\Delta x_1 = x-(x'+\Delta x)$$

$$\Delta x_2 = x-(x'-\Delta x)$$

$$\Delta y_1 = y-(y'+\Delta y)$$

$$\Delta y_2 = y - (y' - \Delta y) \tag{A.5}$$

$$\Delta z_1 = z - (z' + \Delta z)$$

$$\Delta z_2 = z - (z' - \Delta z)$$

$$\Delta r(i,j,k) = [(\Delta x_i)^2 + (\Delta y_j)^2 + (\Delta z_k)^2]^{1/2}$$

we have:

$$\Delta U_{1i} = L(1,1,1,) - L(1,1,2) - L(1,2,1) + L(1,2,2)$$
$$-L(2,1,1) + L(2,1,2) + L(2,2,1) - L(2,2,2)$$

$$\Delta U_{2i} = L'(1,1,1,) - L'(1,1,2) - L'(1,2,1) + L'(1,2,2)$$
$$-L'(2,1,1) + L'(2,1,2) + L'(2,2,1) - L'(2,2,2)$$

$$\Delta U_{3i} = -(\Delta U_1 + \Delta U_2)$$

$$\Delta U_{4i} = \ln\left[\frac{M(1,1,1)}{M(1,1,2)}\cdot\frac{M(1,2,2)}{M(1,2,1)}\cdot\frac{M(2,1,2)}{M(2,1,1)}\cdot\frac{M(2,2,1)}{M(2,2,2)}\right] \tag{A.6}$$

$$\Delta U_{5i} = \frac{1}{2}\ln\left[\frac{N(1,1,1)}{N(1,1,2)}\cdot\frac{N(1,2,2)}{N(1,2,1)}\cdot\frac{N(2,1,2)}{N(2,1,1)}\cdot\frac{N(2,2,1)}{N(2,2,2)}\right]$$

$$\Delta U_{6i} = \frac{1}{2}\ln\left[\frac{N'(1,1,1)}{N'(1,1,2)}\cdot\frac{N'(1,2,2)}{N'(1,2,1)}\cdot\frac{N'(2,1,2)}{N'(2,1,1)}\cdot\frac{N'(2,2,1)}{N'(2,2,2)}\right]$$

The components of the magnetic field variation (ΔX, ΔY, ΔZ) at the point $P(x,y,z)$ are then obtained by substituting eqs. (A.6) in (A.1).