Fault geometry and earthquake mechanics

Dudley J. Andrews

U.S. Geological Survey, Menlo Park, CA, U.S.A.

Abstract

Earthquake mechanics may be determined by the geometry of a fault system. Slip on a fractal branching fault surface can explain: 1) regeneration of stress irregularities in an earthquake; 2) the concentration of stress drop in an earthquake into asperities; 3) starting and stopping of earthquake slip at fault junctions, and 4) self-similar scaling of earthquakes. Slip at fault junctions provides a natural realization of barrier and asperity models without appealing to variations of fault strength. Fault systems are observed to have a branching fractal structure, and slip may occur at many fault junctions in an earthquake. Consider the mechanics of slip at one fault junction. In order to avoid a stress singularity of order $1/r$, an intersection of faults must be a triple junction, and the Burgers vectors on the three fault segments at the junction must sum to zero. In other words, to lowest order the deformation consists of rigid block displacement, which ensures that the local stress due to the dislocations is zero. The elastic dislocation solution, however, ignores the fact that the configuration of the blocks changes at the scale of the displacement. A volume change occurs at the junction; either a void opens or intense local deformation is required to avoid material overlap. The volume change is proportional to the product of the slip increment and the total slip since the formation of the junction. Energy absorbed at the junction, equal to confining pressure times the volume change, is not large enough to prevent slip at a new junction. The ratio of energy absorbed at a new junction to elastic energy released in an earthquake is no larger than P/μ, where P is confining pressure and μ is the shear modulus. At a depth of 10 km this dimensionless ratio has the value $P/\mu = 0.01$. As slip accumulates at a fault junction in a number of earthquakes, the fault segments are displaced such that they no longer meet at a single point. For this reason the volume increment for a given slip increment becomes larger. A junction with past accumulated slip u_0 is a strong barrier to earthquakes with maximum slip $u_m < 2 (P/\mu) u_0 = u_0/50$. As slip continues to occur elsewhere in the fault system, a stress concentration will grow at the old junction. A fresh fracture may occur in the stress concentration, establishing a new triple junction, and allowing continuity of slip in the fault system. The fresh fracture could provide the instability needed to explain earthquakes. Perhaps a small fraction (on the order of P/μ) of the surface that slips in any earthquake is fresh fracture. Stress drop occurs only on this small fraction of the rupture surface, the asperities. Strain change in the asperities is on the order of P/μ. Therefore this model predicts average strain change in an earthquake to be on the order of $(P/\mu)^2 = 0.0001$, as is observed.

Key words *fault geometry – earthquake mechanics*

1. Introduction

This paper will sketch some ideas suggesting a speculative model of fault dynamics in which earthquake instability arises not from an unstable friction law but from fresh fracture on a small portion of the surface that slips in any earthquake. The fresh fracture is required because of geometric incompatibility between displaced crustal blocks. Slip at fault junctions regenerates the incompatibility and associated stress concentrations. This model was suggested by an analysis of the mechanics of fault junctions (Andrews, 1989). The analysis is limited to two dimensions, but the results point toward a promising new field of fault modeling.

To accommodate general finite deformation in a solid there must be sets of faults with several different orientations. The faults will meet at triple junctions. King (1983) suggests that slip approaches zero at junctions and that large finite deformation is accommodated on a fractal array of subfaults around each junction. I suggest that a limited amount of slip can occur at a junction, but that eventually fresh fracture forms a new junction.

A fault system has a fractal branching structure. Slip in an earthquake occurs on a fractal subset of the fault system that contains many junctions. The following two sections will summarize the analysis of Andrews (1989) of slip at a single junction.

2. Junction kinematics

Consider first the elastic solution for stress due to infinitesimal slip at a junction. To isolate the contribution of the junction, consider that slip is uniform on each fault segment meeting at the junction. Then stress is the sum of dislocation solutions, one for each segment. A dislocation has a stress singularity of order $1/r$, larger than the singularity at a crack tip and not physically realistic at macroscopic scales. Therefore the Burgers vectors of the dislocations must sum to zero. This means that to lowest order deformation at the junction is rigid body displacement. A corollary is that on the line where two fault segments meet there must also be a third to relieve the stress singularity. Because the plane of the third fault segment contains the intersection of the other two, the assumption of two-dimensionality in this work may not be as restrictive as it first appears.

Figure 1, top, shows a triple junction labeled with the notation that will be used in this paper. The three fault segments, A, B, and C, have opposite angles α, β, and γ respectively. Each of the angles is less than 180° in the case shown. In the bottom of the figure the three blocks bounded by the fault segments are displaced as rigid bodies. Displacement on each fault segment is assumed to be pure slip, that is, there is no opening displacement. Slip dis-

placements u_A, u_B, and u_C on the three segments are defined to be positive for right-lateral slip. The requirement of rigid-body displacement is equivalent to the statement that the ratio of slip to the sine of the opposite angle is the same for all three segments,

$$\frac{u_A}{\sin \alpha} = \frac{u_B}{\sin \beta} = \frac{u_C}{\sin \gamma} = U \qquad (2.1)$$

and this common ratio is designated by U in the following equations. In the case shown in fig. 1, the sines of all the angles are positive, so slip is in the same sense (right lateral) on all segments.

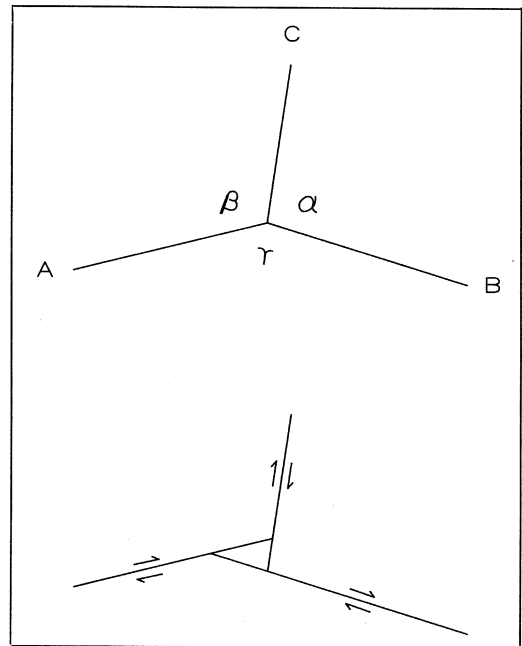Figure 1, bottom, shows that a void opens at the junction as a result of the rigid-body dis-

Fig. 1. Top: a junction of fault segments A, B, and C. The opposite angles α, β, and γ are each less than 180°. Bottom: rigid-body displacement at the junction consists of slip in the same sense on the three fault segments, and a void opens.
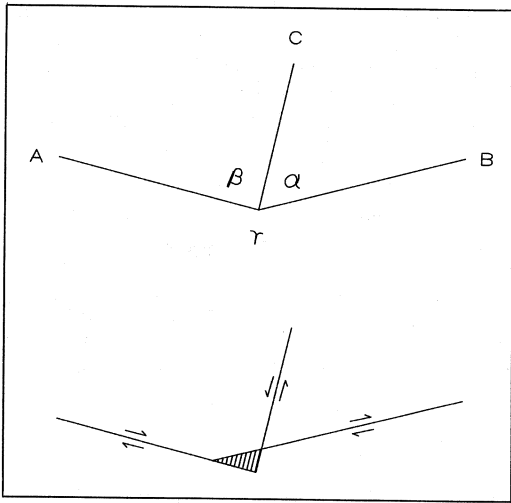
Fig. 2. Top: a junction of fault segments A, B, and C. One of the angles γ is greater than 180°. Bottom: in rigid-body displacement slip on segment C is opposite in sense from slip on segments A and B. Material overlap requires deformation and compression in the neighborhood of the junction.

placement. The volume of the void per unit length out of the paper, the area of the triangle in the figure, can be written in a symmetric form

$$V = \frac{1}{2} U^2 \sin \alpha \sin \beta \sin \gamma \qquad (2.2)$$

or in terms of slip on one of the segments,

$$V = \frac{1}{2} u_A^2 \frac{\sin \beta \sin \gamma}{\sin \alpha} \qquad (2.3)$$

If the medium were not subject to any confining stress, the rigid-body displacement could occur with no stress change. Where there is a confining stress, however, such as in the earth, there must be a stress change in the vicinity of

the void in order to satisfy the boundary condition of zero normal stress at the void surface.

The sum of the dislocation solutions of the fault segments predicts that there is no stress change associated with the rigid-body displacement. The solution for the stress field of a dislocation is based on the assumptions of linear elasticity, including the assumption that displacements are small compared to the length scales of interest, so that the configuration does not change. The opening of the void at the junction and the stress change around the void are not accounted for in linear elasticity.

At a dilatational junction, where a void opens, the three angles α, β, and γ are all less than 180°. Figure 2, top, shows a compressive triple junction where one of the angles γ is greater than 180°. Since $\sin \gamma$ is negative, (2.1) predicts that u_C has an opposite sign from u_A and u_B, and (2.2) predicts a negative void volume. In the displaced configuration, fig. 2, bottom, slip on segment C is left lateral, and there is an overlap of material at the junction. The overlap must be accommodated by deformation and compression in a zone around the junction.

3. The energetics of slip at a junction

Energy is absorbed due to the volume change accompanying slip at a triple junction. The object of this section is to show that the energy absorbed is proportional to the slip increment times the total accumulated slip at the junction. This conclusion follows from the assumption that energy absorbed is proportional to the volume change times a resisting stress that is dependent on depth. Only a rough approximation of the proportionality factor is needed for later considerations in this paper.

Consider first a dilatational junction. Because the material near the triple junction is displaced, to a first approximation, as rigid blocks, the net expansion of the region containing the junction is V, given by (2.2), and work is done against the confining stress.

For the sake of an order of magnitude estimate, ignore the dependence on orientation in an anisotropic stress field. The work done

against the mean compressive stress P is

$$PV = \frac{1}{2} PU^2 \sin \alpha \sin \beta \sin \gamma \quad (3.1)$$

This estimate of the work required to open the void needs to be corrected by the frictional work associated with the altered stress field in the immediate vicinity of the void. Nevertheless, (3.1) will be taken as an estimate of the energy absorbed at a dilatational junction, and it will be compared below with an estimate of energy released by slip on the fault system.

At a compressive junction a volume V_0, larger than the overlap V, must be compressed and deformed by microcracking. Despite the microcracking, if there is no open pore volume in the rock, the volume change is elastic, so the stress needed to accomplish this compression is kV/V_0, where k is bulk modulus. The deformed volume V_0 will be large enough that this compressive stress is resisted by the yield stress of the surrounding material, which for a brittle rockmass is proportional to the mean compressive stress P. Therefore the volume of the intensely deformed zone is $V_0 \propto (k/P)V$, and its radius is proportional to $\sqrt{k/P}$ times displacement at the junction.

Energy absorbed in the deformed zone is equal to work done on the boundary of the zone. The component of that work due to volume change in a compressive stress field is an energy release PV where V is negative. A much larger energy is absorbed by shear on microcracks. The surface area of the deformed zone is proportional to $\sqrt{k/P} \sqrt{|V|}$, the displacement on the surface is proportional to $\sqrt{|V|}$, and stress on the surface is proportional to P. Therefore, work done on the intensely deformed zone is proportional to $\sqrt{Pk|V|}$.

In general the energy absorbed at the junction may be estimated as

$$cP|V| = \frac{1}{2} cPU^2 |\sin \alpha \sin \beta \sin \gamma| =$$

$$\quad (3.2)$$

$$= \frac{1}{2} cPu_A^2 |\sin \beta \sin \gamma / \sin \alpha|$$

where c is of order 1 for $0 < \alpha, \beta, \gamma < 180°$,

and c is on the order of $\sqrt{k/P}$ if one of the angles is greater than 180°. The essential feature of the estimate is that energy absorbed is proportional to the square of slip at the junction.

Energy absorbed at a junction will now be compared to energy released by slip on the fault system in an earthquake. For this purpose the earthquake is approximated as a simple plane-strain shear crack. The elastic strain energy released by a crack with length $2L$ and stress drop $\Delta\tau$ minus the work done against the sliding friction stress is

$$E = \frac{3\pi}{8\mu} (\Delta\tau)^2 L^2 \quad (3.3)$$

where μ is shear modulus and Poisson's ratio is taken to be 1/4. A formula more useful for the present purpose expresses this quantity in terms of the maximum slip on the crack u_m,

$$E = \frac{\pi}{6} \mu u_m^2 \quad (3.4)$$

This equation shows that the available energy released is proportional to the square of slip on the crack. Although an earthquake is not in general a simple crack, the energy released will be proportional to (3.4) on average.

The ratio of energy absorbed at the junction (3.2) to the available energy released (3.4) is

$$c \left(\frac{P}{\mu} \right) \left| \frac{\sin \beta \sin \gamma}{\sin \alpha} \right| \left(\frac{u_A}{u_m} \right)^2 \quad (3.5)$$

If the triple junction is near the center of the fault system ruptured in an earthquake, the last factor, the ratio of junction slip to maximum slip, will be near 1. Near the ends of rupture or on subsidiary branches the ratio is smaller. If u_A is the largest of the set u_A, u_B, u_C at a junction, the combination of trigonometric factors lies between 0 and 1. Aside from these geometric factors, the magnitude of (3.5) depends on cP/μ, which is P/μ at a dilatational junction and is proportional to $\sqrt{k/P}(P/\mu) \propto \sqrt{P/\mu}$ at a compressive junction.
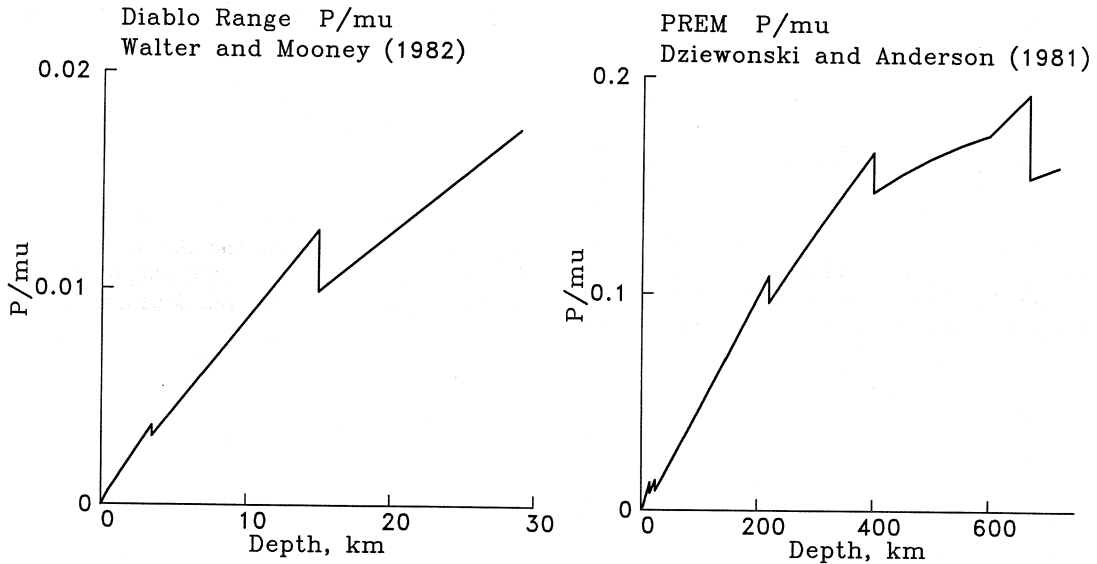
Fig. 3. Mean compressive stress P (taken to be equal to the vertical lithostatic stress) divided by shear modulus μ as a function of depth. Left: data at crustal depths from Walter and Mooney (1982), μ found from P-wave velocity assuming Poisson's ratio of 0.25. Right: preliminary reference earth model (Dziewonski and Anderson, 1981).

The dimensionless ratio of mean compressive stress to shear modulus P/μ is plotted as a function of depth in fig. 3. The ratio is zero at the surface, is 0.01 at 10 km depth, and ranges up to 0.2 at 650 km, the depth of the deepest earthquakes. Therefore, energy absorbed at the junction is less than the available energy released, so it is possible for slip to occur at a triple junction in an earthquake. A fault branch may branch in turn, and because of the diminishing size of the slip (the last factor in (3.5)), branching may continue down to an infinitesimal scale, yielding a fractal structure. The remainder of this paper is concerned with shallow faulting at depths of about 10 km, where $P/\mu = 0.01$. The analysis focuses on the dependence on slip and on powers of P/μ and ignores proportionality constants.

I assume that it is easier for slip to occur on an old rupture surface than to form a fresh fracture. This assumption is implicit in our identification of faults on which earthquakes occur. Since an earthquake rupture will tend to follow old slip surfaces, it is necessary to consider triple junctions at which slip has accumulated in earlier events. The available energy released by an earthquake with maximum slip u_m (eq. (3.4)) is the same on average, but the energy absorbed at the triple junction is larger, because it depends on the total accumulated slip. Figure 4 shows a triple junction at which previous slip has opened the dark-shaded void, which is now filled with fluid or remineralized. If the previous slip is u_A^0, the volume of this filled void is

$$\frac{1}{2}(u_A^0)^2 \sin \beta \, \sin \gamma / \sin \alpha$$

Because of the previous displacement, the three fault segments no longer intersect at a single point, and for this reason an increment in slip will produce a larger increment in void opening.)
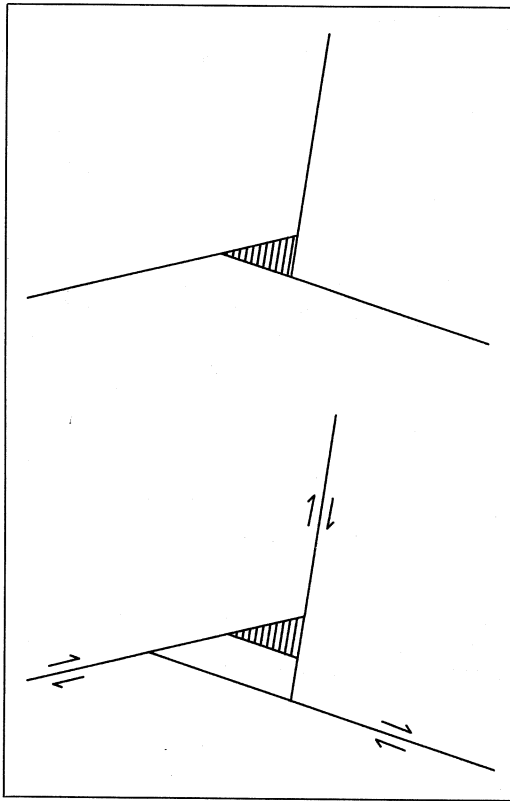
An additional earthquake with slip u_A will

Fig. 4. Top: the triple junction shown in the bottom of fig. 1 with the void filled with fluid or remineralized. Bottom: another increment of slip produces a larger increment of void volume.

increment the void volume by

$$V = \frac{1}{2}[(u_A^0 + u_A)^2 - (u_A^0)^2] \frac{\sin\beta\sin\gamma}{\sin\alpha} =$$

$$= (u_A^0 + u_A/2) u_A \frac{\sin\beta\sin\gamma}{\sin\alpha}$$

Note that this volume increment is proportional to the product of the slip increment u_A times the total slip that has occurred at the junction, averaged before and after the current event, $u_A^0 + u_A/2$. The ratio of energy absorbed at the junction to available energy released, eq. (3.5), is modified to become

$$c\left(\frac{P}{\mu}\right)\left|\frac{\sin\beta\sin\gamma}{\sin\alpha}\right|\frac{(2u_A^0 + u_A)u_A}{u_m^2} \qquad (3.6a)$$

The relative energy cost of slip at a junction increases as slip accumulates at the junction. For the sake of some order of magnitude estimates, consider only dilatational junctions, set $c|\sin\beta\sin\gamma/\sin\alpha| = 1$, and consider that the slip increment u_A at the junction is near u_m. Then the energy ratio is

$$\frac{P}{\mu}\left(1 + 2\frac{u_0}{u_m}\right) \qquad (3.6b)$$

where u_0 has been written for u_A^0. The junction is a strong barrier to slip in small earthquakes for which $u_m < 2(P/\mu)u_0 = u_0/50$, while some slip can occur at the junction in earthquakes for which $u_m > 2(P/\mu)u_0$. The slip value for which a compressive junction is a strong barrier is proportional to $2\sqrt{(P/\mu)}u_0 = u_0/5$.

4. A proposed model of fault mechanics

As slip accumulates at a junction, the junction becomes a stronger barrier to slip; a larger earthquake is required to increment slip there. As slip continues to occur elsewhere in the fault system, a stress concentration will grow at the junction. After sufficient slip has accumulated, a junction will become a strong enough barrier to require that slip bypass it on a fresh fracture. The location of the fresh fracture remains an unanswered question. The most highly stressed region will be the neighborhood of the old junction, so the fracture is likely to be initiated there. The change in configuration must extend beyond the locality of the junction, however, because the reason that there is a large increment of volume change accompanying a slip increment is the fact that the three fault segments at an old junction no longer meet at a single point (see fig. 4). At least one of the fault segments must jump to an

alignment such that the junction is again a single point.

A fresh fracture could be the initiating event of an earthquake. Even if there is no instability in the friction law on old fault surfaces, fresh fracture at old junctions could provide the energy release of an earthquake. Although most of the rupture in an earthquake will occur on old slip surfaces, a small fraction of the rupture area will consist of fresh fracture.

An earthquake is determined by the collective effect of many junctions, with most acting as barriers and with those newly formed by fresh fracture acting as asperities. To model the collective effect one must face the challenge of dealing with widely different length scales. The configuration of a junction changes at the scale of the displacement, which on average is 10^{-4} times the length of an earthquake rupture. Accumulated slip at an old junction is about 10^{-2} times the rupture length for which it is a strong barrier. There is no preferred length scale in these considerations, so there may be earthquakes of all sizes in a fractal fault network.

A finite element or boundary element method using the small displacement approximation (no change of configuration) might be augmented to account for energy absorbed at junctions by incorporating an effective generalized concentrated force proportional to accumulated displacement. Fresh fracture at or near an old junction could be represented by resetting the effective restraining force to zero. Since the initiation of fracture depends on microstructure near the old junction, which is essentially unknowable in the earth, fracture will occur randomly with probability dependent on the accumulated slip at the junction. Slip events could be modeled by this mechanism without appealing to a change of friction on any pre-existing fault segment.

A question to be answered by such a collective model is what is the size of an event generated by fracture at a particular junction? I conjecture that the slip that occurs in an event generated by fresh fracture near an old dilatational junction with accumulated slip u_0 is likely to be on the order of $u_m = 2\,(P/\mu)\,u_0$, because the stress concentration was generated

by impeding events up to this size. Therefore the active lifetime of a junction is limited to $n = \mu/(2P) = 50$ events of the size of the event that eliminates the junction. The geometry of the fault network governing events of this size will be completely changed after this number of events at any point. Therefore I suggest that the fraction of a rupture surface that is fresh fracture is on the order of P/μ.

The stress drop in an asperity will be on the order of the shear strength of the rock, which is proportional to confining pressure, P. Therefore the strain drop in an asperity is on the order of P/μ. If asperities cover a fraction P/μ of a rupture surface, then the average strain drop in an earthquake is on the order of $(P/\mu)^2$. The average value of observed strain drops in shallow earthquakes is 10^{-4}. This is also the value of $(P/\mu)^2$ at 10 km depth. This coincidence, which is remarkable considering that factors of 2 have been ignored at many points in this paper, provides a motivation for more careful analysis.

5. Discussion

Inversions of observed ground motion to infer the distribution of slip in an earthquake generally show irregular slip distributions with isolated peaks that could be explained by stress drop confined to asperities. Since inversions are less constrained at shorter wavelengths, data do not contradict the idea that stress drop is confined to asperities covering only a very small fraction of the rupture surface. King and Nábělek (1985) and Andrews (1989) review evidence that the slip concentrations as well as the starting and stopping points of earthquake rupture are associated with geometric complexities in a fault system.

Robert Wallace (personal communication, 1989) notes that observations do not preclude fresh fracture on a small percentage of a rupture surface, even on a well-developed fault such as the San Andreas, where there is a remarkable tendency for slip to follow old slip surfaces. There is little geomorphic expression of fault junctions (Robert Sharp, personal communication, 1988), which is consistent with the

idea that slip can occur at a triple junction in only a limited number of events before at least one of the fault segments moves to a different location.

Earthquake slip is complex at all scales in both space and time. Bak and Tang (1989) propose that the complexity arises naturally from the dynamics in a process of self-organized criticality. Rice (1993) reviews models of seismic slip that exhibit self-organized criticality and shows that they are all inherently discrete. Rice (1993) shows that slip on a planar fault with a smooth constitutive law is complex only when the numerical discretization is too coarse. With adequate resolution the solutions become simply repeating cycles of large earthquakes. Therefore the complexity of seismic slip cannot be explained with a smooth laboratory-based friction law on a planar fault.

Rice (1993) concludes that over-sized numerical cells may be mimicking the effect of geometric discontinuities in a fault system. In this paper I have examined the mechanics of a fault junction, and I have proposed an inherently discrete model of fault mechanics that is naturally based on the geometry and physics of a faulted continuum. I have not specified the geometry of the fresh fracture that occurs when a junction becomes a strong barrier to further slip. With such a specification one would have a dynamic model not only of earthquakes but

also, as envisioned by Sornette and Sornette (1989), of the evolution of a fault system.

Acknowledgements

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