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History, observation and mathematical models in the seismic analysis of the Valadier abutment area in the Colosseum

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Abstract
The present work aimed to outline the need to investigate different fields of research to interpret the structural behaviour of a monument as complex as the Colosseum. It is shown how defining the numerical models first, then refining them, followed by interpretation of results, is strictly linked with the information gathered from historical records and observation of the monument as it is today. The study is confined to the area of the Valadier abutment, analysing its state and its seismic behaviour before and after the XIX century restoration using different numerical tools, from the elastic modal analysis to the non linear step by step time history direct integration. The procedure comparatively evaluates the reliability in the interpretation of the results and identifies future lines of research.

Key words *historical records – Roman Colosseum – seismic behaviour – numerical analysis*

1. Introduction

The paper discusses problems raised by the study of the Valadier area and the methodology adopted for the assessment of its seismic vulnerability. As shown in previous papers (Croci, 1993; Croci and D'Ayala, 1988, 1993; Croci *et al.*, 1990, 1992) the general approach is concerned with three different fields of research: historical documents of any sort that trace the history of events involving the monument from the structural point of view; an *in situ* survey aimed to assess the quality of materials and their distribution, the quality and pattern of lay-out of structural elements, cracks and failures pattern, signs of degradation, past interventions of restoration; the mechanical analysis simulating configurations historically proved in order to assess the evolution of the dynamic behaviour. Information is gathered from each field and analysed by a correlation method whose aim is to assess the structural behaviour of the monument toward past seismic events and therefore deduct a reliable forecast of future behaviour.

From the historical point of view this methodology presents a chronological problem in defining the succession and dating of documents and related facts of different nature and the problem of the structural interpretation of information written for purposes very different from the structural one, with an uneven distribution both from the space and the time point of view. From the numerical point of view, the questions are how to reduce a very complex and diversified structure to schemes that can be handled by present numerical tools without losing structural meaning, and the treatment of the results obtained from such schemes in order to be comparable with data derived from other fields.

These general strategies are here applied to the portion of the monument known as the «Valadier area» which includes the abutments and some bays along the outer ring; this area has a crucial function in the assessment of the stability of the whole and presents a set of events fully documented so that it is possible to apply the methodology outlined above.

2. Historical records and observation

The first difficulty which appears in the study of the state of conservation in time, is the total lack of pictorial or other graphics evidences which can give us an idea of the state of the monument since the initial abandonment, from the 6th up to the 13th century. In the second half of this period the historical written records and inscription are few, inaccurate when referring to the Colosseum and in general contradictory. It is therefore virtually impossible to outline with certainty the initial mechanism of damage or where and when exactly the first failure occurred and the outer wall started to crumble and what was the main cause of it. The archaeological studies (Rea and Conforto, 1993) however, compared with the historical records, prove that the first destruction occurred on a building already weakened by fires (217 A.D.), through at least two centuries, the fifth and sixth, involving the siege of Alaric, two destructive earthquakes in 443 and 508 (damage due to the second one has only recently been confirmed by the dating of epigraphs), and finally the dismantling works promoted by Theodoric.

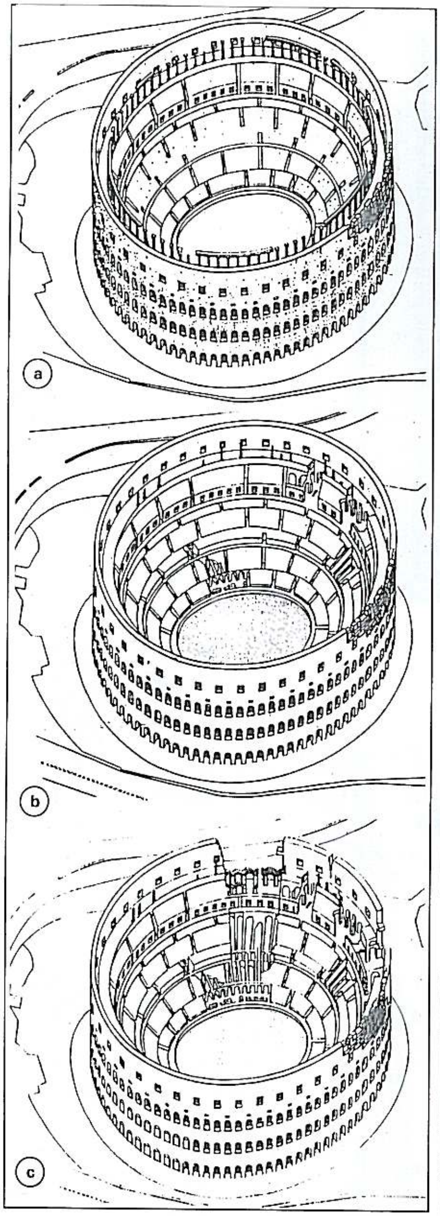
Figure 1a-c presents an attempt to spatially define a number of hypothetical mechanisms which took place during this period as a consequence of earthquakes which occurred in the area. The appearance of iconography in 1300 raises a further question of interpretation: the images in fact are often quite dissimilar, in relation to their different aims and techniques, and not always consistent with the information from other sources.

From the images shown (fig. 1d,e), among the many analysed (Croci *et al.*, 1990), a deduction can be drawn: the portion next to what will be the Valadier abutment, kept its shape almost unchanged at least from the 15th century onwards, and its state of conservation was not macroscopically modified up to 1703. This observation can be used as further proof that any man-made alteration and removal, has to be thought to be related only to the portion already collapsed and on the ground as rubble and not to what was still in place. Otherwise why not take out the blocks on this edge?

The static situation of the monument between the two images, dated 1744 (fig. 1e) and 1822 (fig. 1f), should have been slowly getting worse if the authority of the time decided to intervene first to shore, and then reinforce the outer elliptical wall. To understand the scenario before the restoration of XIX century, however, the cultural change that took place toward problems concerning stability and safety of buildings of ancient times should be borne in mind. It is in fact only around this time that the Colosseum started to be regarded as a monument and therefore studied and restored to stop the instability characterised by a slow evolution, that can be thought to be primed by earthquakes and stressed by deterioration of materials. Among these phenomena which in the previous centuries were recorded as «crolli ispontanei» (sudden and spontaneous failures), the earthquake of 1703 represented an essential event to bring the Colosseum to the limit of structural safety.

The interaction between different causes is schematically summarized in fig. 2a which plots, as a time function, the seismic documented events, the documented failures, the actual amount of missing structure as deduced from the iconography, the reduction of bearing capacity, due to the effects of the earthquake and due to the degradation of materials.

It is interesting to note how in correspondence with major earthquakes some failures are always witnessed and recorded. That dam-



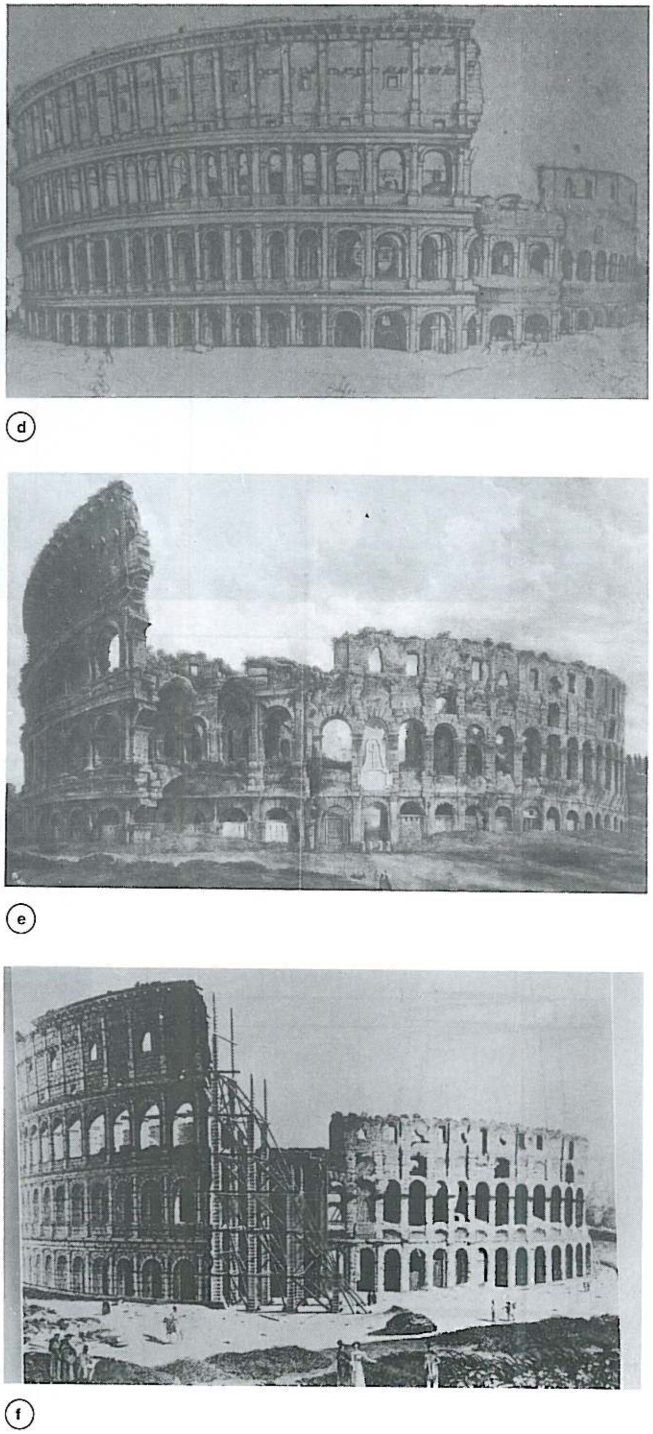
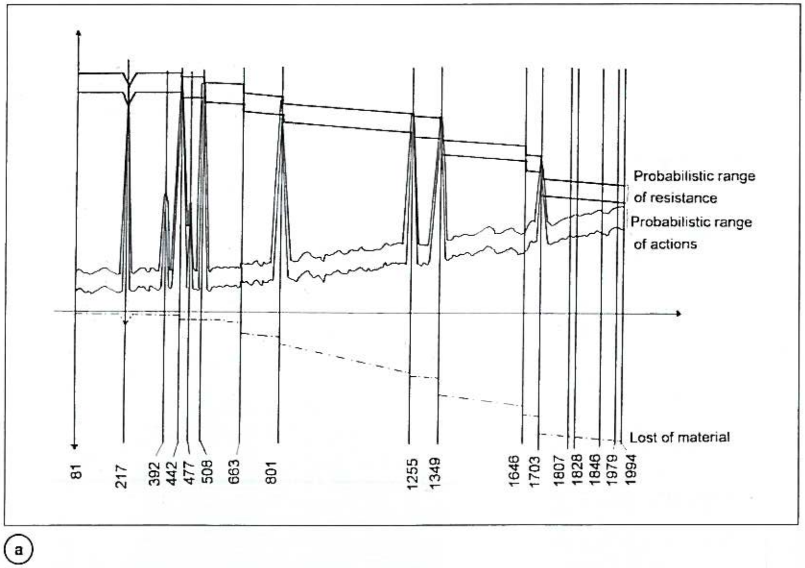


Fig. 1a-f. State of conservation: a) year 443; b) year 801; c) year 1349. d) First half of XV century (Ghirlandaio's School). e) The situation after the 1703 earthquake (Volpato, 1744). f) The shoring before the construction of the Valadier abutment (anonymous, 1822).



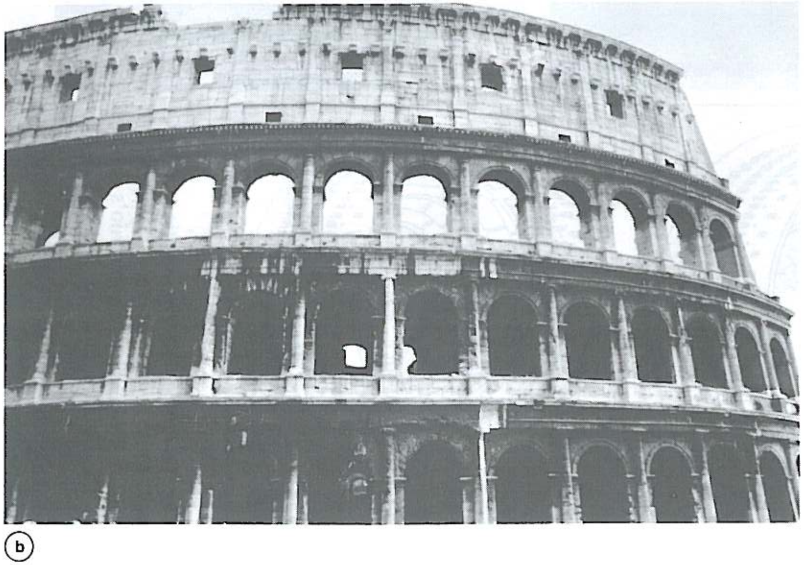


Fig. 2a-c. a) Major damaging actions and their effects on the monument in time. b) Signs of seismic damage in the studied area. c) State of conservation of the studied area today.

age, which was not followed by any repair or strengthening, produced substantial changes in the overall structural behaviour, evolving from a global three-dimensional one to a series of independent annular and radial walls, weakly connected by the annular vaults. The fact that the amount of structure collapsed is definitely greater than the portions whose failure was documented, suggests a slow evolution of a number of situations near to crisis, which in time ended in small localised collapses that nobody noticed.

3. Mathematical models

The numerical models have therefore been developed with the aim of assessing how vulnerable the structure was to earthquake and how much it is now after the last century's restorations which were designed purely for static purposes, although it has not been possible to take into account in the simulation some of the non-linear effects caused by the many previous earthquakes mentioned above.

The fact that the image following the 1703 earthquake does not present new damages on the Valadier area raises another question concerning the intensity and direction of that seismic action with respect to the previous more destructive ones. The local morphology of the valley must have influenced the propagation of the seismic wave, as proved by recent geophysical studies (Funiciello *et al.*, 1995; Moczo *et al.*, 1995), and therefore, lacking local seismological information, the choice of the accelerogramme to be used in the numerical analysis is strongly conditioned by the information gathered from historical seismology. From historical records it is known that the most destructive earthquakes originate from the Apennines and produce in Rome damage up to the IX grade of the Mercalli Scale, with a maximum expected acceleration of 0.064 g which corresponds to a return period of approximately 500 years in Rome (Giannini *et al.*, 1984). This information has been used for the seismic input of the spectrum modal analysis using the shape of the Italian Code spectrum, while for the time history analysis one of the recorded accelerogrammes of El Centro has been chosen, which had a high frequency content, and appropriately scaled. The direction chosen is that of the LIII bay axis, which appears to be the most dangerous.

The definition of a numerical model be able to simulate the peculiar non linear behaviour of the material (block stone masonry with no mortar) and the structure is obtained by a sequence of simplified models whose results are checked with respect to the historical evidence and the *in situ* survey. For the case under examination the simplified models have to meet the evidence outlined in the previous section. At the same time there is some information on the structural configuration that remains unknown, which is the level of connection between the outer and inner walls through the barrel annular vaults. From the restoration intervention which took place in the XIX century it should be expected that these constraints were very loose. Therefore the models which simulate the configuration before the restoration intervention were tested assuming the annular outer wall to be completely independent from the rest of the structure, while the models simulating the configuration of today were tested for the two limit condition of perfect connection or no connection.

In fig. 3a,b the results, obtained by modal dynamic analysis with response spectrum for the configuration before the restorations, show that there are portions whose tensional state is over the limit of the material capacity, and therefore in a critical condition: as the values summarized in table I point out the edge pillar of the first level is partialised (level of partializing being given by the ratio 3 u/h where u is the distance between the point of application of N and the compressed edge of the section, and h the geometric height of the section), while for the pillars at the upper levels the cross sections are fully compressed, which proves small flectional effects from the above mass of the attic, whose stress state is rather low, as should be expected from a free edge. The highest tensile annular stresses are found in the area between bays XLIX and LI where they reach 10 kg/cm^2, a stress level that cannot be counteracted by the frictional strength, the compres-

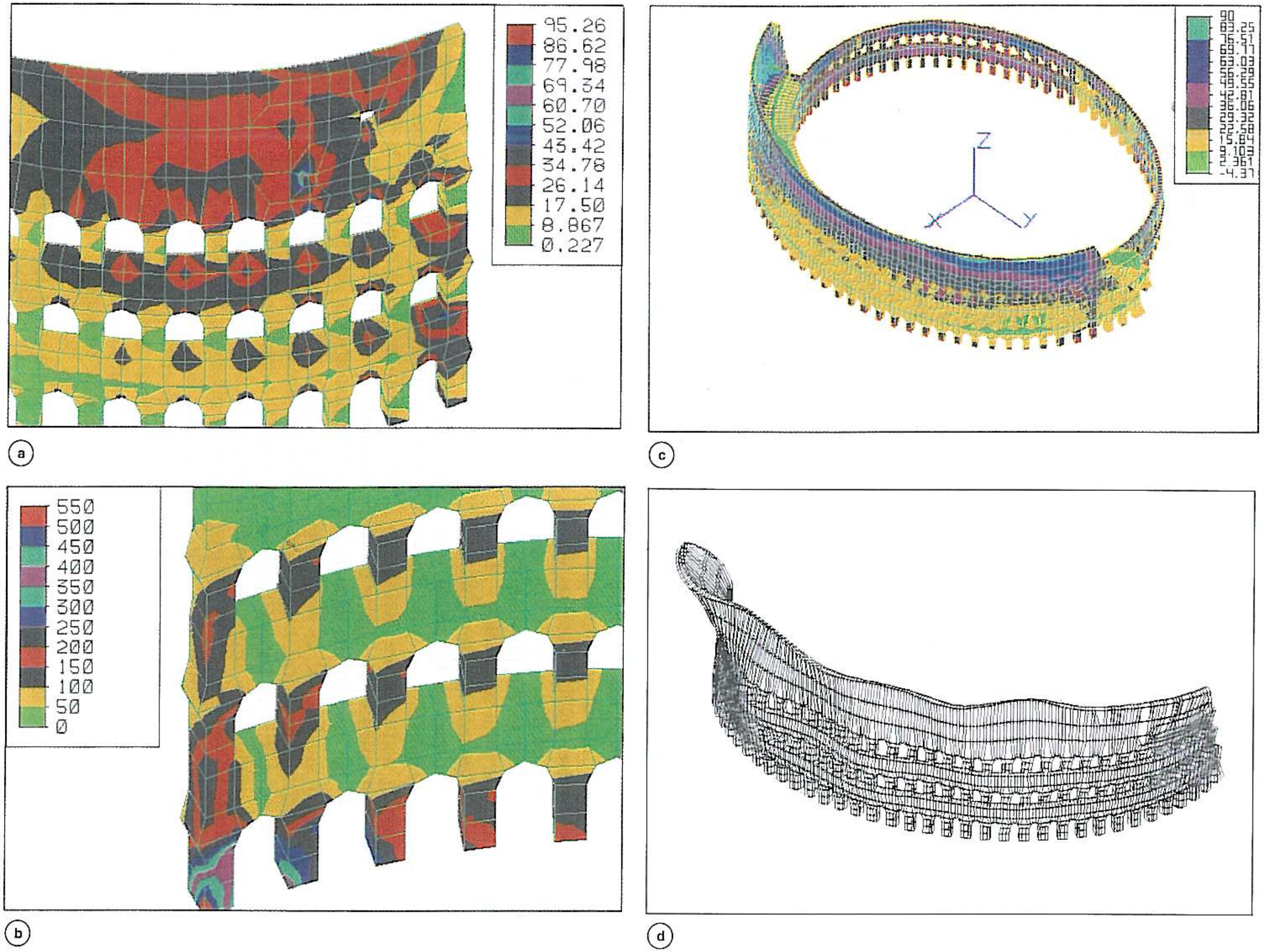


Fig. 3a-d. Elastic modal analysis of the outer wall before XIX century restorations: a) maximum principal stresses; b) vertical stresses. c) Elastic modal analysis after XIX century restorations: maximum principal stresses. d) Step by step time history analysis of outer wall: deformed state.

Tables I and II. Models related to the situation before the XIX century restorations.

Modal analysis with response spectrum			Vertical stresses (kg/cm^2)		Worse combination		
Area	Level (m)	Side	Dead load	Seismic action	Linear	Partialised	$3\,u/h$
PILLAR BAYS LIV and LV	0.4	Outer	−24	32	−56	−103.3	
		Inner	−21.4	44	+22.6	0	0.32
	12.75	Outer	−13.6	13	−0.6	−0.6	
		Inner	−18.9	18	−36.9	−36.9	1.
	24.45	Outer	−8.4	12	+3.6	0	
		Inner	−13.6	12	−25.6	−26.3	0.84
ATTIC LEVEL: horizontal stresses (kg/cm^2)				10			
Time history elastic step by step analysis							
PILLAR BAYS LIV and LV	0.4	Outer	−24	12	−36	Mech.	
		Inner	−21.4	44	+22.6	–	–
	12.75	Outer	−13.6	12	−1.6	−1.6	
		Inner	−18.9	16	−34.9	−34.9	1.
	24.45	Outer	−8.4	8	−4	−4	
		Inner	−13.6	12	−25.6	−26.5	1.00
ATTIC LEVEL: horizontal stresses (kg/cm^2)				8.5			

sion vertical stresses due to the dead load being quite low at this level. These results are confirmed by the linear time history (table II and fig. 4a,b) of the same model which actually stresses the limit of the algorithm of the modal dynamic analysis, due to the superimposing of all modes without taking into account the actual phase delay and the sign of each contribution. In fact the time history presents a situation that is more dangerous for the edge pillar of the first level from a stress point of view. Of both effects evidence can be found on site (fig. 2b,c).

However two real facts that are not simulated by the models are worth considering, to better understand the global behaviour: on the one hand the fact that at the time of the 1703 earthquake the whole of the first level was interred up to at least half of its height and had been so for the previous five centuries. While the effect of the interaction between the ground and the structure will be the subject of further research, it can be here stated that it probably improved the seismic performance of the monument, thus preventing the failure that the numerical model points out. The other fact which is not taken into account in the present models is the dumping behaviour of the structure, which would definitely modify both frequencies and modal shapes, as they are calculated with linear hypotheses, reducing peaks and delaying the structural response. These aspects will be dealt with later in the present paper. These two facts will definitely reduce the stress level calculated by the linear models. Another phenomenon that must not be considered in the present analysis is the loss of overall shape related to permanent deformations due to previous earthquakes, whose effects can only be assessed on the basis of accurate on site measurements.

Slightly different is the picture shown by the models which simulate the situation after the Valadier restoration. In particular the pillar most stressed is for these models the one at the third level, next to the Valadier abutment: as it can be seen in tables III and IV the cross-section is in this case partialised both at the first

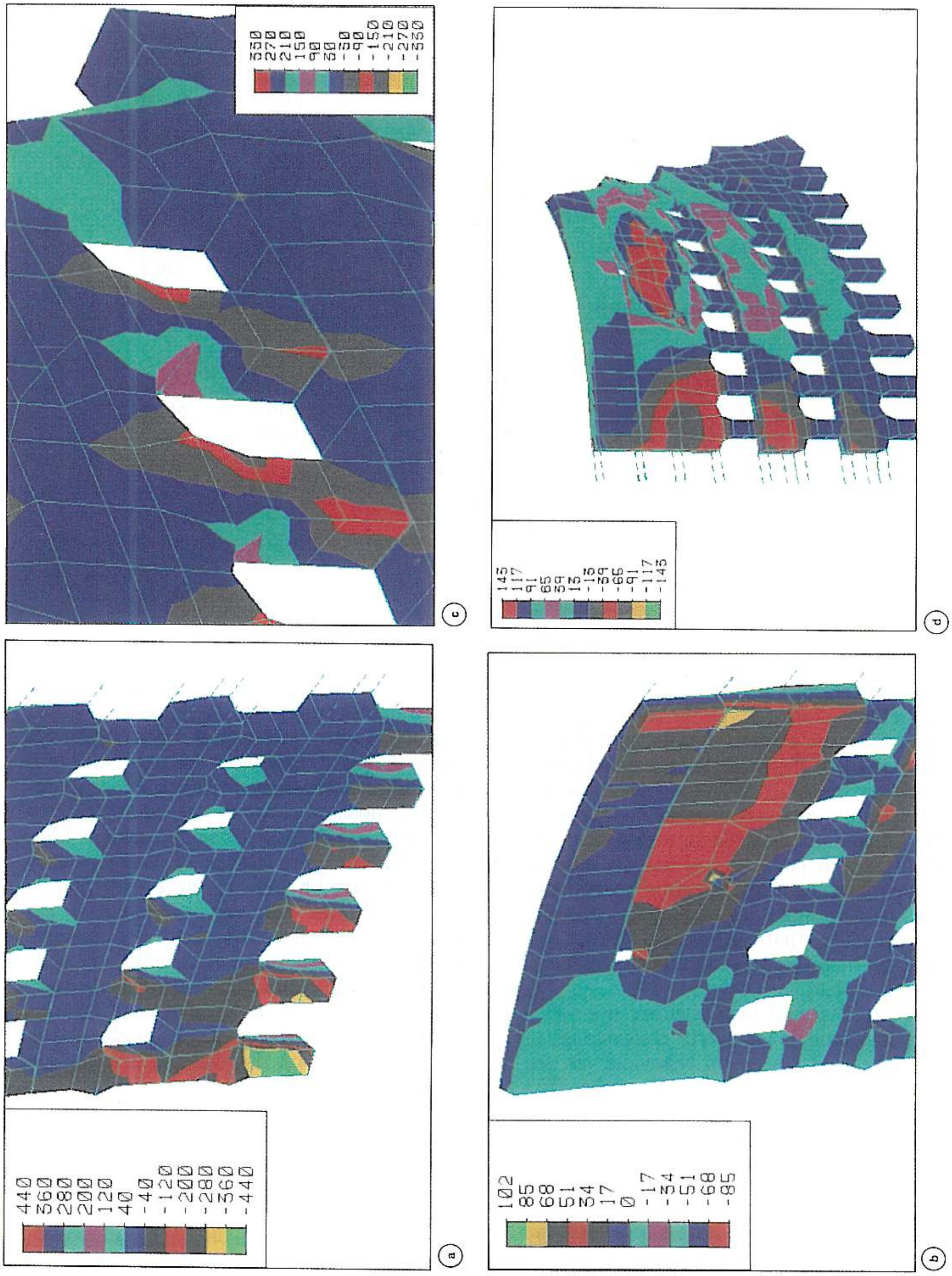


Fig. 4a-d. Time history analysis of the Valadier area: a) before XIX century restorations, vertical stresses; b) before XIX century restorations, horizontal stresses; c) after XIX century restorations, vertical stresses; d) after XIX century restorations, horizontal stresses.

Tables III and IV. Models related to the situation after the XIX century restorations.

Modal analysis with response spectrum without radial connection			Vertical stresses (kg/cm^2)		Worse combination		
Area	Level (m)	Side	Dead load	Seismic action	Linear	Partialised	3 *u/h*
PILLAR BAYS LIV and LV	0.4	Outer	−24	35	−11	0	
		Inner	−21.4	19.5	+40.9	−47.3	0.63
	12.75	Outer	−13.6	13	−0.6	−0.6	
		Inner	−18.9	10	−28.9	−28.9	1.
	24.45	Outer	−8.4	13	+4.6	0	
		Inner	−13.6	12	−25.6	−26.9	0.78
ATTIC LEVEL: horizontal stresses (kg/cm^2)				11			
Modal analysis with response spectrum with radial connection							
PILLAR BAYS LIV and LV	0.4	Outer	−24.7	5.0	−29.7	−29.7	
		Inner	−20.8	4.3	−25.1	−25.1	1.
	12.75	Outer	−15.0	3.4	−11.6	−11.6	
		Inner	−13.1	3.4	−9.7	−9.7	1.
	24.45	Outer	−7.2	10.1	+2.9	0	
		Inner	−11.1	11.7	−22.8	−23.3	0.85
ATTIC LEVEL: horizontal stresses (kg/cm^2)				9.0			

and at the third level; also a wide portion of the attic level shows tensile annular stresses (fig. 3c) which cannot be counteracted by the friction provided by the vertical load so that relative sliding takes place between adjacent blocks.

Further on, of the two elastic modal analyses elaborated with the two limit configurations, the results of the model with connection show a great reduction of flexural effects on the pillars, only the third level being slightly partialised. It also outlines, however, the presence of strong annular tensile stresses in the annular barrel vaults of the first ambulatory between the outer wall and the inner structure: this result is confirmed by the spread cracks pattern that can be seen on site. This point, together with the lack of information on the state of conservation of the last centuries' tie rods, suggest choosing a configuration which will not take into account the inner structure in the assessment of the behaviour of the outer wall for the model that is analysed with a step by step time history elastic and non linear analysis. Worthy of note is the likeness between the numerical models deformation (fig. 3d) and the shape of the deformed cornice as it can be seen today, that can be considered the result of the subsequent earthquakes.

The reliability of the results, especially from a quantitative point of view, is strongly connected to the role played by the peculiar lay out of the masonry characterised by a frictional constitutive law of coulombian type (D'Ayala and D'Asdia, 1993). At the interface between two blocks, during the earthquake, a dissipation of energy takes place. This dissipation in general has two effects: on the one hand it reduces the global deformations as it will be if a purely elastic behaviour is taken into account, on the other hand it provokes a higher concentration of stresses and localised damages on single blocks. The state of deformation therefore, and the related distribution of stresses

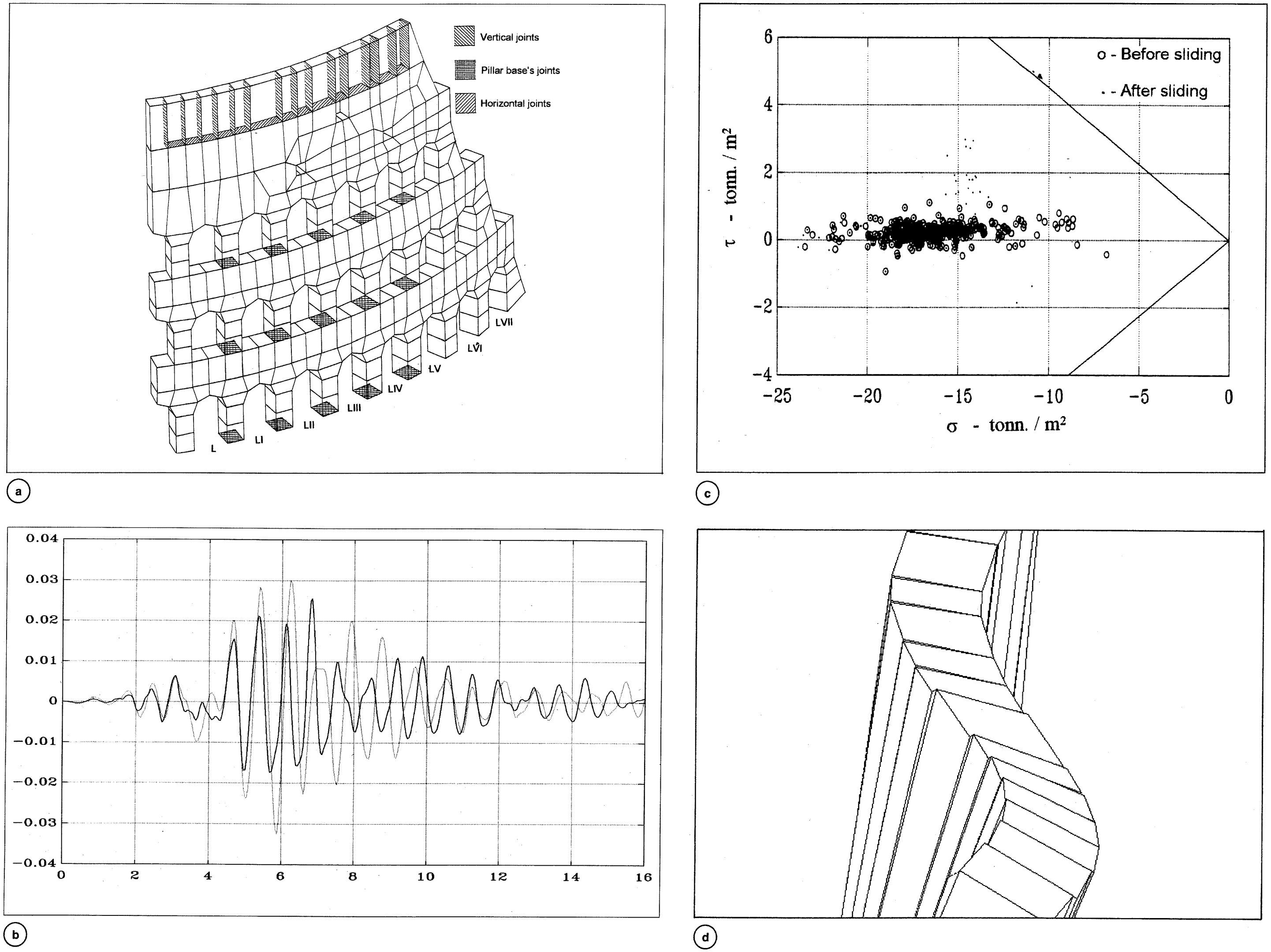


Fig. 5a-d. a) Layout of the composite finite elements. b) Comparison between the global and the partial model displacements. c) Plain tensional state of a horizontal joint before and after sliding has occurred to the vertical joints. d) Attic level: sliding between blocks.

changes every time a relative movement between two adjacent blocks takes place (sliding, rocking, etc.), so that the portion under kinematism is actually less stiff than any elastic simulation, while displacement, speed and acceleration are greater, causing the peak stress distribution to migrate to the neighbouring blocks.

In order to simulate such a non linear behaviour a procedure has been developed able to control and modify the stiffness parameters of a suitable f.e. custom-built to simulate the mechanical behaviour of the frictional joint surfaces. The composite f.e. is made up of four truss elements, perpendicular to the joint surface, which control the rotational behaviour and an 8-node brick element which simulates the sliding behaviour. This choice allows the two aspects of the problem to be separated and thus better controlled. To have a significant scheme of the portion analysed, discretized by three-dimensional brick elements, each element geometrically covers a certain number of physical blocks. The scheme therefore does not take into account the staggering between the layers of blocks and cannot correctly simulate the sliding that occurs when the frictional strength is overtaken. The extension of the Valadier area to which the procedure has been applied (fig. 5a), has been chosen with the criterion of optimising at the same time the level of discretization, to have the possible smaller finite element without losing meaning from the global dynamic point of view. To simulate the rest of the structure, boundary elements have been introduced with appropriate stiffness characteristics chosen to have comparable displacement of two corresponding points of the global and reduced model (fig. 5b).

The procedure applied basically performs two tasks: the first is to detect during the direct integration analysis the point in time and space where the limit state of the material is overtaken. On each of these surfaces, the composite finite element is introduced according to the associated kinematism. The mechanics features of such an element are then scaled according to the level of partialization or to the shear level acquired. Once the stiffness of the model has been in such a way locally modified, a new step by step analysis starts with initial conditions as deduced from the previous run. Whenever the seismic effect during the history is reduced on any joint surface, the mechanics features are re-established as for the initial elastic behaviour. This for instance allows alternate partialization on the corners of the block of the pillars (fig. 4c), a phenomenon whose signs can be found on site.

As for the frictional behaviour, for the vertical joints the results allow the limit level of shear that the joint can bear to be evaluated. The non linear procedure by reducing the shear modulus G on the vertical joint produces a redistribution of the shear stresses which will then increase the horizontal joints (fig. 5c,d). The analysis of this model (table V) confirms the results already obtained with the previous ones and illustrates the peculiar behaviour: during an earthquake the attic wall is driven by

Table V. Model related to the situation after the XIX century restoration.

Time history elastic step by step analysis			Vertical stresses (kg/cm^2)		Worse combination		
Area	Level (m)	Side	Dead load	Seismic action	Linear	Partialised	3 *u/h*
PILLAR BAYS LIV and LV	0.4	Outer	−24	23	−47	−52.6	0.67
		Inner	−21.4	33	+11.6	0	
	12.75	Outer	−13.6	9	−4.6	−4.6	
		Inner	−18.9	11	−28.9	−28.9	1.
	24.45	Outer	−8.4	15	+6.6	0	
		Inner	−13.6	18	−31.6	−34	0.74
ATTIC LEVEL: horizontal stresses (kg/cm^2)				13			

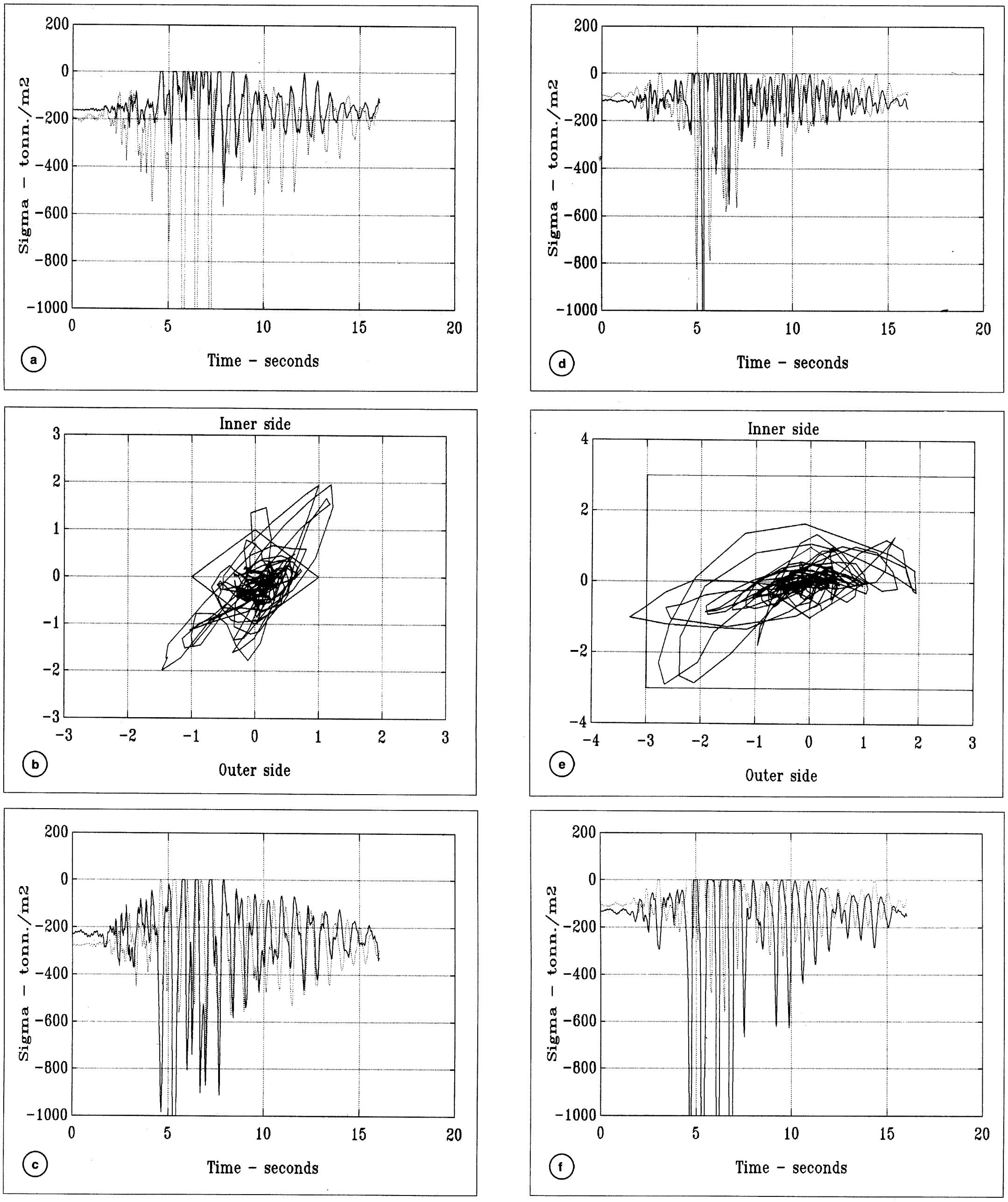


Fig. 6a-f. Non linear analysis: a) vertical stresses of pillar, bays LIV-LV, first level, inner side; b) position on the cross section of the applied stress *N*, bays LIV-LV, first level; c) vertical stresses of pillar, bays LIV-LV, first level, outer side; d) vertical stresses of pillar, bays LIV-LV, third level, inner side; e) position on the cross section of the applied stress *N*, bays LIV-LV, third level; f) vertical stresses of pillar, bays LIV-LV, third level, outer side.

a perturbation whose effects are stronger in proximity of the abutments where big deformations take place. Due to the shape and the inertia of the structure, however, the major effect of the seismic action has not necessarily the same direction as the input. The diagrams in fig. 6a,c,d,f in which the maximum level of vertical stress and the migration of the point of application of the related *N* force (fig. 6b,e) are plotted, show the relevance of flexural action in the tangent annular plane, better than in the radial one, as one should expect.

This effect is especially strong for the third level of the pillar between bays LIV and LV next to the Valadier abutment, where after the 5th second on one of the corners of the outer side the crushing strength of the material is reached in three subsequent moments. Finally, for the attic level, fig. 4d shows the distribution of horizontal stresses: tensile values, higher than in the case without abutment, are reached in an area slightly shifted with respect to the edge, acquiring a maximum of 13 kg/cm^2 in the vicinity of the windows. Therefore it can be expected that in this area over the big windows relative sliding between the blocks can take place. Such a deformation can be seen in bay LV of the attic level.

4. Conclusions

In the present work the seismic behaviour of the area of the Valadier abutment before and after the XIX century restorations has been investigated using three different types of numerical analysis: modal analysis with response spectrum, time history with accelerogramme linear analysis, step by step direct integration non linear analysis.

The results obtained by the two different types of linear dynamic simulations, modal analysis and time history, show good agreement from a qualitative point of view, and can be confirmed by the historical records and the signs of the corresponding deformation on the monument surveyed. From a quantitative point of view, however, the use of the first one is limited by the fact that it implies a perfect symmetrical behaviour of the material in tension and in compression, while this is not the case for the masonry. In addition to that, even when the time history takes into account that peculiar behaviour, if the analysis is linear elastic, it is not possible to allow partialization and sliding and therefore reduction in stiffness and variation in dumping capacity.

The procedure implemented shows the importance of such effects on the numerical values and distribution of the stresses. In particular the analysis shows that when the mechanics features of an element are reduced according to the overcoming of its bearing capacity, during the following action the maximum effects migrate to other parts of the structure, which with a linear analysis will be considered safe. At the same time, partialization and sliding, taking place at a particular section, do not necessarily mean irreversible damage, at least as far as the strength of the stone is not overcome.

Many uncertainties about the real behaviour however remain; for instance, the effects of the change in shape due to permanent deformation and sliding, and the corresponding accumulated stress levels have not been taken into account: a question still open for further analyses. As for the safety levels, the final judgement is only partially reliable; however, in the Valadier area the present study indicates that, even if the Valadier abutment were not designed for seismic purposes, it is able to improve the seismic performance of the monument, although portions at risk still remain. The improvement consists in lowering the stress level of the pillar at ground level, which according to the results of the time history analysis of the situation before the construction of the abutment, will have been otherwise subjected to a global rocking mechanism. Nevertheless, action should be taken to improve the behaviour at the third and attic levels which will still suffer major damage, being affected by composite flexural effects caused by the presence of the abutment, and so reaching the crushing level of the material at one of the corners. Of these phenomena signs can be found at the base of many pillars of the third level.

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