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Macroseismic intensity evaluation with the «Fuzzy Sets Logic»

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Abstract
The use of a macroseismic scale often requires subjective choices and judgments which may produce inhomogeneities and biases in the resulting intensities. To get over this problem it would be necessary to formalize the decision process leading to the estimation of the macroseismic intensity but, on historical records, this is often hindered by the poorness and incompleteness of the available information and by the intrinsical ambiguity of the common language. Moreover, all the intensity scales have always been created and updated to be used «in the field» on contemporary earthquakes and then it may happen that even detailed historical descriptions often do not correspond to the descriptive frameworks of any grade. In order to face these problems, we propose in this work a computer method for the evaluation of the macroseismic intensity which makes use of the «Fuzzy Sets Logic». This approach reproduces the tasks performed by the human brain which, taking advantage of the tolerance of imprecision, is able to handle with information bearing only an approximate relation to the data. This allows to understand and make explicit some passes of the evaluation process that are unconsciously followed by the macroseismic expert.

Key words *macroseismic intensity – Fuzzy sets – MCS scale*

1. Introduction

Even if the collection techniques of historical macroseismic information have been greatly enhanced in the last few years (in fact nowadays wide use is made of computer information recording and retrieving techniques and even more frequently historians and linguistics experts are involved in these studies) the techniques of analysis had not a comparable improvement. Nowadays, macroseismic information is still interpreted using methods which are nearly the same that have been used since the nineteenth century, and consisting in the application of a macroseismic scale not really so different from the one originally defined by Giuseppe Mercalli in 1897.

Although several important improvements have been brought to the scales, up to the last version (the European Macroseismic Scale (EMS92) (Grünthal, 1993)) to increase their effectiveness and their applicability to present time data (but most of them only concern the effects on new types of buildings), little has been done to make the assessment of the intensity level more objective when ambiguous evidences occur, and to make explicit some *non written* assumption often used in practice. It is remarkable that the analysis of macroseismic data still now makes very little use of computational techniques which on the contrary represent the main tool in most of other fields of scientific research. This occurs even if the macroseismic data are irreplaceable to evaluate and mitigate the seismic risk in many seismic regions and moreover it might also be crucial for the comprehension of the generation process of strong earthquakes on long time-scales.

This lag can be partially due to a sort of estrangement from this kind of studies by most of the seismological investigators. Indeed, at the beginning of the Sixties, the availability of a huge amount of instrumental data and the apparent success of the «dilatancy theory» to model the physical phenomena occurring in the proximity of the earthquakes, convinced many to invest all the resources, both in terms of human effort and economic budget, in the analysis of instrumental data, and to neglect instead the non-instrumental ones (allegedly considered as non-scientific). On the other hand it must be noted that the obvious difficulty encountered in extracting quantitative parameters from qualitative information may also contribute to the poor development of this field up to now.

Even if both the information content and the reliability are largely superior for an instrumental recording than a single macroseismic observation, the detail on the seismogenesis of a given area obtainable by macroseismic data may be better than the one which can be inferred by instrumental ones due to the larger number and wider time span of macroseismic data.

Intensity scales had been originally compiled to classify the effects of earthquakes occurring *synchronously* to the observers. The scales are then tools which have been thought to be applied on the basis of *direct* observations. In his first proposition of the MCS scale, Sieberg (1912) stressed the importance of direct observation on his work to improve the scale formerly proposed by Mercalli (1897, 1902). Nor, either before or after Sieberg, the macroseismic scales had been conformed to do a more efficient usage with historical sources. It often arises the paradox that even a very detailed source furnishes a framework of effects which cannot be clearly addressed to the elements of the grades of the scale. In some way the possible scenarios, given by the source, are imprecise and only partially usable. Experience both in the field and on historical sources demonstrates that the observed effects of seismic events are associated, one to the others, in a way which is less *hard* (or more *fuzzy*) than the one described in the different scales. In considering this, it must be reminded that almost all the scales (even the most recent ones) had been formulated and improved almost without computational support, qualitatively comparing the different effects only on the basis of direct experience. The only work (Brazee, 1979) we know which approached the problem more systematically, seems to be completely forgotten by the following literature.

We do not face here the question of the assessment of a more or less improved macroseismic scale, but we only want to approach the problem of the arbitrariness in the evaluation process of seismic intensity. In the above mentioned recent version of the scale (Grünthal, 1993) indeed, even if many undefined aspects of previous scales were cleared and some contradictions eliminated, the compilers renounced to give a complete formalization being able to resolve univocally all the interpretative ambiguity. Even the «Guide to the use of the scale» of the EMS92 for example, which certainly represents a well-deserving effort to direct the user to a correct application of the scale and to warn him against the most common mistakes, is only a collection of recommendations without the indication of clear priorities among them and after all does not clarify all the interpretative difficulties so that the use of the scale still retains subjective elements.

In general the evaluation of the grade is not only entrusted to the canonical definitions (*written* on the scale) but also to some implicit (*non written*) assumptions, the use of which depends on personal experiences and beliefs of the investigator (see for example Ferrari and Guidoboni, 1995) and then may be not always univocally and homogeneously applied to different localities and to different times by different investigators (as an example of a *non written* rule we can mention the more or less conscious assignment, on behalf of almost all the investigators, of different weights or priorities to various effects).

In this work we try to get a formalization of the macroseismic scale, *i.e.* an algorithm which is able to give the same answer in every situation, if the basic observation is the same. This does not solve the problems of difficulties related to the interpretation of the texts which require specific contributions by historical and linguistic specialists. Our aim is only to increase the level of uniformity in the choices performed during the process of intensity assessment.

To reach this target we adopted a multicriteria decision making model (MCDM) (Xiang *et al.*, 1987), making use of «Fuzzy Logic», which was already applied in the past to different fields, ranging from the landscape planning to the evaluation of natural hazards.

2. Fuzzy sets and decision making

The need to make decisions is intrinsic in all the human actions involving a choice. Decision making is a complex, human activity which can be defined as the choosing, usually on the basis of many criteria, of a course of action among alternatives to accomplish one or several objectives. Much decision making in the real world takes place in an environment in which the objectives, the constrains and the consequences of possible actions are not known precisely. Before the introduction of the fuzzy approach (Zadeh, 1965) the only source of this imprecision was considered the randomness, while, after that, many authors have argued that the major source of imprecision is *fuzziness*, *i.e.* the real impossibility in many cases to attribute precise properties to different subjects. The basis of this contention is the so called «Principle of Incompatibility» which can be defined informally stating: «as the complexity of a system increases, our ability to make precise and yet significant statements about its behaviour diminishes until a threshold is reached beyond which precision and significance become almost exclusive characteristics. The Fuzzy sets logic partially reproduces the mental processes of the human brain taking advantages from the tolerance of the imprecision and so obtaining a result, anyway, even in case of lack of complete and precise data».

Coming back to the field of macroseismic studies, it is quite evident that the data are *fuzzy*. In many cases the application of the scale encounters ambiguous evidences when for example effects that the scale attributes to different grades are observed at the same time. On the basis of the Fuzzy philosophical approach, which is not only a theoretical way of thinking but has been actually mathematically formalized in a «fuzzy sets algebra», the ambiguous evidences which can be encountered in the application of a macroseismic scale are not due to the randomness in the appearance of certain events but to the uncertainty (or *fuzziness*) in recognizing them as belonging to different grades of the scale. Although the macroseismic scale reports a precise description of several phenomena defining the framework of each grade, effects belonging to different grades may sometimes occur at the same time. For example, it is quite obvious that the effect «Felt by everybody outdoors», which is characteristics of grade VI, can also be found being associated to effects of higher grades and, in some cases, also of grade V (for which the statement should be «Felt by many outdoors») and maybe also of grade IV («Felt by many indoors»). This occurs because the different effects of each grade are not really connected each other by physical relationships. Moreover some descriptions include terms as «few», «some», «many», «most», which have not a unique interpretation expecially when they are inferred from common language sentences.

These last difficulties should seem to be really crucial only for the application of the MCS scale (Sieberg, 1912, 1932), the one actually used for our work. The problem should be less relevant for the scales of «modern» kind, like the MM (Wood and Neuman, 1931), the MSK (Medvedev *et al.*, 1967; Medvedev, 1977) and the EMS92 (Grünthal, 1993), where some uncertain definitions have been reformulated in terms of percent. However even in these cases (see *i.e.* the discussion about «quantities» in the guide to the EMS92 scale (Grünthal, 1993)) the *fuzziness* is still present: when the observed rate falls close to the boundary of the intervals.

In the ordinary sets algebra the membership of an object X_i (belonging to a universe of objects $X = \{X_1, X_2, ..., X_i, ..., X_{na}\}$) in a set A can be defined as a characteristic function U from X to a valuation set $\{0,1\}$ such that

$$U_A = 1 \text{ if } X_i \in A$$
$$U_A = 0 \text{ if } X_i \notin A.$$

In the fuzzy algebra the membership function is not limited to only two values (1 = member, 0 = not-member) but can assume all the real values in the interval [0,1]. From this definition, the theory of the fuzzy sets can be developed defining some operations among sets in a way similar to the ordinary (*hard*) sets theory. The basic operations between fuzzy sets can be seen as an extension of the classic sets theory. We can define the fuzzy operations of union, intersection and complementation in terms of membership function as

$$\forall X_i \in X,\ U_{A \cup B}(X_i) = \mathrm{MAX}[U_A(X_i), U_B(X_i)];$$
$$\forall X_i \in X,\ U_{A \cap B}(X_i) = \mathrm{MIN}[U_A(X_i), U_B(X_i)];$$
$$\forall X_i \in X,\ U_{\bar{A}}(X_i) = 1 - U_A(X_i).$$

Note that these definitions also hold for the *hard* sets using the appropriate (two-valued) definition of membership function. These simple rules can be used to apply the fuzzy sets logic to the field of MCDM and in particular to represent the relationship between the possible alternative decisions and the objectives or attributes which must be taken into account for taking the decision. In the classic decision theory one considers a system of variables with a set of constrains which limits the choice among alternatives and some attributes or objectives which sort the alternatives on the basis of certain criteria. Bellman and Zadeh (1970) suggested that each attribute can be represented as a fuzzy subset over the set of alternatives $X = \{X_1, X_2, ..., X_i, ..., X_{na}\}$. Thus, if A_j indicates the j-th attribute, then the grade of membership of alternative X_i in A_j, $U_{Aj}(X_i)$ indicates the grade to which X_i satisfies this attribute. Two possible approaches to obtain membership function $U_{Aj}(X_i)$ are the subjective approach and the empirical approach. In the first case, the membership function is determined by an expert of the field on the basis of its proper belief while in the second one is derived in some way from the available data.

The debate is open in the literature on the effectiveness of these two options (for a more detailed discussion see Xiang *et al.* (1987)); anyway, both approaches will be considered in this work.

To combine multiple attributes to form the decision making function the «minimax» decision making procedure has been proposed by Bellman and Zadeh (1970). The decision function $D(X_i)$ that satisfies all of the attributes is obtainable as intersection of the fuzzy sets corresponding to all the attributes A_j. This corresponds to get the minimum of the membership function for each alternative:

$$\forall X_i \in X,\ D(X_i) = \mathrm{MIN}_j\,[U_{Aj}(X_i)] \quad (j = 1,2,...,m).$$

To find the «least objectionable» solution X^*, the maximum value over the alternatives in D must be calculated so that the whole procedure can be summarized by

$$D(X^*) = \mathrm{MAX}_i\{\mathrm{MIN}_j[U_{Aj}(X_i)]\},\ \forall X_i \in X.$$

This procedure does not allow to have attributes that may differ in importance but this can be effectively done by appropriate weighting of attributes.

The model we finally adopted is a slightly modified version of the one used for the assessment of landscape plan by Xiang *et al.* (1987). It allows that more than one expert or criterion may be used at the same time. This is done by the *aggregation* of the different membership U^k_{Aj} and weighting W^k_j functions relative to each k-th expert/criteria. At odd with the original model however, we take the «acceptabilities» equal to zero for every expert/criterion and alternatives and simply assume as aggregation procedure: taking the minimum of the membership functions between different experts/criteria. This means that any value of membership is acceptable by every expert/criterion for all of the alternatives. Under this assumptions, the aggregate membership, when more than one expert/criterion is considered, is given by

$$\bar{U}_{Aj}(X_i) = \mathrm{MIN}_k[U^k_{Aj}(X_i)],\ \forall k = 1,.., q$$

being q the number of different experts/criteria and $U^{k}{}_{\mathrm{Aj}}(X_{\mathrm{i}})$ the grade of membership of alternative X_{i} to the j-th attribute and relatively to k-th expert/criterion. A similar expression is given for aggregation of weights

$$\overline{W}_{\mathrm{j}} = \mathrm{MIN}_{\mathrm{k}}[W^{\mathrm{k}}{}_{\mathrm{j}}],\ \forall \mathrm{k} = 1,\dots,\ \mathrm{q}$$

where $W^{\mathrm{k}}{}_{\mathrm{j}}$ is the weight of j-th attribute on the basis of k-th expert/criterion. The least objectionable solution X^* is obtained via the decision rule

$$\mathrm{D}(X^*) = \mathrm{MAX}_{\mathrm{i}}\{\mathrm{MIN}_{\mathrm{j}}[\overline{U}_{\mathrm{Aj}}(X_{\mathrm{i}})^{\overline{W}_{\mathrm{j}}}]\},$$
$$\forall \mathrm{i} = 1,\dots,\ \mathrm{n},\ \forall \mathrm{i} = 1,\dots,\ \mathrm{m}$$

being n the total number of alternatives and m the total number of attributes.

3. Application to real macroseismic data

The data we analyse are relative to the Garfagnana earthquake of September 7th, 1920 and are mainly taken from macroseismic questionnaires (ING, 1920) and newspaper cuts. We encoded them in the form of answers, with three possible outcomes: yes, no, not known, to the questions on whether the different effects indicated in all the sentences of the MCS scale were really observed. We identified each sentence by an increasing number ranging from 1 to 106 (reported in table I of the Appendix). We also added some other sentences, with codes ranging from 107 to 155, (also reported in table I of the Appendix) which are often found on the sources but are relative to effects not included in the scale (as for example «felt by everybody outdoors», «few dead», «many wounded», «not felt», «damage», etc.), or which are referenced by the scale but with different severity (*i.e.* «*some* bridges are *slightly* damaged» while in the scale one can find only «bridges are *seriously* damaged» or «*some* bridges are *destroyed*»).

In table II of the Appendix a sample of the input data-set is reported. Different sources are given as separate records. For every felt locality the answer to the question: «was the effect described by sentence *n* effectively observed?» is given with the indication of the number *n* of the sentence with positive sign if the answer is «yes», with the negative sign if the answer is «no» and omitting the number if the source does not permit to establish the truth (but we did not actually use negative evidences in this work). Each record also reports the intensity estimated in the usual way by a macroseismic expert and the weight assigned to the source. For this latter was established a criterion which privileges the macroseismic questionnaires compared to newspapers and the contemporary sources compared to the posterior ones. The different grades of the macroseismic scale are defined as alternative scenarios of the multicriteria decision making model described above, while the effects actually observed at each given locality are the attributes. The membership function of each attribute is simply the one defined for the corresponding effect, while the actual weight is given by the product of the weight assigned to the effect by the weight of the source.

A key problem in the application of the MCDM algorithm consists in the reasonable choice of weights and of membership functions. We now introduce some possible candidate weighting and membership rating schemes which we will compare afterwards. To simplify the discussion we will assign them simple names indicated with *italic* characters.

As already cited above one possible option for the membership function is that it is established by a macroseismic expert on the basis of his own experience. In this case the choice is fully entrusted to investigator knowledge with the only constrain that the function must be 1.0 for the grade to which the effects belong in the scale and must decrease monotonically for the other grades. In fig. 1 the corresponding function for the effect «felt by everybody outdoors» is reported with closed dots. It must also be noted that for most effects there should not be, at least in theory, a decrease of the value of membership function for increasing intensity. For example: if the earthquake was «felt by everybody outdoors» for grade VI this should also hold for grades from VII to XII (at least if still somebody remains alive) and the membership function should be 1.0 for all grades greater or equal to VI (in fig. 1 with open circles). However, since larger effects tend to sway the mind from smaller ones, it may happen that some weak effects may have not been actually recorded by the observer. This may arise in particular in the case of questionnaires where the compiler often owns a certain experience in the scale use and may not consider relevant the indication of least noticeable details. It must also be considered that in certain cases a phenomenon cannot be observed whenever the corresponding felt intensity is reached or not (for example if bells do not exist, nobody can hear their sound or if there are not water flows the corresponding effects cannot be observed). A similar problem arises for the awakening: if the earthquake does not occur during the night almost nobody can be awaked (maybe only the sleepers). Anyway, since in general the absence of weak effects from the reports may, on the basis of the above arguments, be due to the occurrence of a higher grade, this can be usefully taken into account in the assessment of intensity. For this, both the membership functions with decay on higher intensities (that we will call *expert* membership) and without it (*continue* membership) will be analysed in the following. As for the weights, these should be given in the range [0,1] with higher values for more reliable effects. As in the case of the choice of the membership functions, the weight can be simply established by the expert (we will call this *expert* weighting).

Generally speaking, the effects of earthquakes on buildings can be considered more affordable since they can be better documented even on historical records. On the contrary, less reliable are usually considered the effects on environment like landslides, fissures, variations of level of wells and of the flow of springs. These effects, that have been almost completely eliminated in the EMS92, in some cases indicate high grades of the MCS scale, but it often happens that they are actually mistaken, especially when reading ancient chroni-

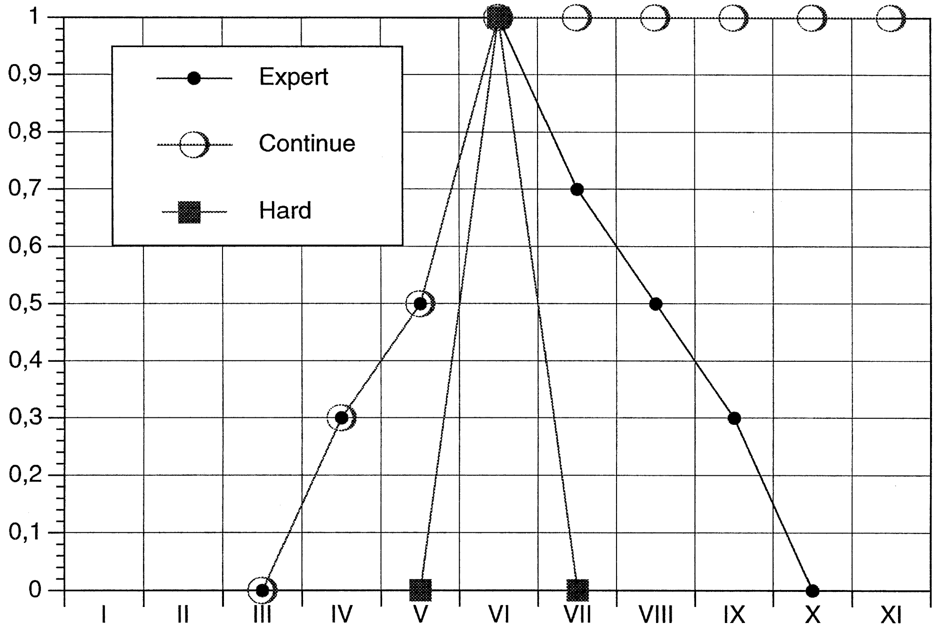

Fig. 1. Membership functions for effect «Felt by everybody outdoors» relative to *expert* (close dots), *continue* (open circles) and *hard* (squares) schemes.

cles, for weaker effects occuring at low grades. In order to take into account these priorities, one possible scheme for weighting (we will call this *damage* weighting) is to give weight equal to 1.0 to effects corresponding to damage on buildings and a lower weight (0.5) to all other effects.

Besides the ones seen above, there are some other «natural» weighting and membership schemes to be considered. One of them is corresponding to the *hard* or «non-fuzzy» scale. For this scheme the membership function (indicated with squares in fig. 1) is equal to 1.0 (member) for the grade to which the sentence belongs in the MCS scale and is equal to 0.0 (not-member) for the other grades. The corresponding weighting scheme must be chosen to give weight 1.0 (*unit* weighting) to all the sentences, since the MCS scale does not explicitly define priorities between different effects. Thus the *hard* scheme should correspond to the rigid application of the MCS scale. As we will see below, the strict use of this criterion produces undetermined results in most of cases (*i.e.* in all cases where evidences are ambiguous).

Another possible choice of membership functions is to define bell-shape functions with maximum in the grade where the sentence is reported on the MCS scale and extending on both sides a certain number k of grades. We will analyse some of these membership schemes both with symmetrical and asymmetrical behaviour. We will call them *bellxx* memberships, with *xx* indicating the particular shape. For example in fig. 2 the *bell2* (close dots) and the *bell4* (open circles) membership functions, for the effect «felt by everybody outdoors», are shown.

As noted above, another possibility, supported by some authors, is the empirical determination of membership functions and weighting. In order to follow this approach, we performed the backward estimation of the membership by the data themselves, on the basis of the expert estimated intensities. This can be done counting, for each sentence, the number

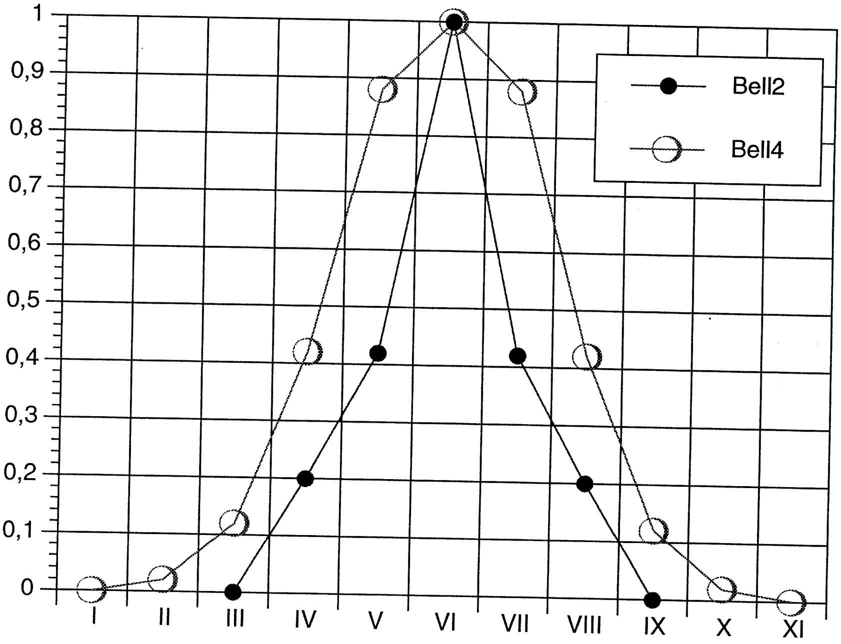

Fig. 2. Membership functions for effect «Felt by everybody outdoors» relative to *bell2* (close dots) and *bell4* (open circles) schemes.

of occurrences for each grade of intensity estimated by the expert. The fuzzy membership function can be computed normalizing these numbers by the largest one of each sentence. We will call the scheme so obtained *inverse* membership. This procedure can be seen as the definition of a different macroseismic scale since it is not granted anymore that the grade to which a sentence belongs on the MCS scale corresponds to the maximum of the membership function. It is obvious that this approach is strongly influenced by the characteristics of the data used for the «inversion» and then it could *overfit* the data themselves. This means that varying the data-set the membership function might change substantially. Even for the weights we may adopt a criterium derived from the data to privilege the most effective effects, assuming a weight inversely proportional to the standard deviation of the observed grades for each effect (*inverse* weighting).

4. Results and discussion

For each analysed locality, we can distinguish between two possible outcomes of the decision making algorithm:

a) a single grade is found to be the least objectionable solution (single determination);

b) the least objectionable grade is not unique (multiple determination).

Due to the ordinal nature of seismic intensity, even in the second case it is possible to obtain a single value as representative of the locality, by taking the median among the multiple solutions, but this is anyway a poor estimate. Among the multiple solutions we can further distinguish the case where:

c) the ambiguity is limited to two consecutive grades (double determination).

The occurrence of case c) and of case b), when the number of the equivalent solution is even, implies that the estimated intensity, being the arithmetic mean of the two central values, may be fractional. This is similar to what happens in macroseismic pratice when an intensity value, being uncertain between two adjacent grades (*e.g.* VIII-IX), is reported with a fractional grade (8 1/2). This notation, also used for other ordinal scales, like for example the Mohr scale of rocks hardness, can be interpreted in the sense that the value lies somewhere between grade 8 and 9 but not necessarily halfway (Rock, 1988).

To evaluate the effects of the choice of the membership and weighting functions we adopted some summary statistics which can be computed for the different schemes. Firstly, we reported the numbers of single, double and multiple determinations from which is possible to evaluate the ability of the membership and weighting scheme to discriminate among alternatives and univocally determine a better decision (less objectionable). Secondly, the r.m.s. between the fuzzy set intensity estimate and the one done by the expert, the average absolute difference between the two estimates and the correlation coefficient R^2 of their regression are reported to give an evaluation of the ability to reproduce the expert estimated intensities, taken as reference.

In table I the results for the *expert* membership are shown rating as a function of different weighting schemes. As it can be seen, the number of single determinations, in all cases, lies between 70 and 80 percent of the total, the double ones are about 20 percent while the

Table I. Membership: expert.

Weighting	N_{single}	N_{double}	$N_{multiple}$	r.m.s.	diff.	R^2
Unit	270	110	19	0.83	−0.35	0.85
Damage	296	84	19	0.83	−0.35	0.85
Expert	307	73	19	0.84	−0.38	0.85
Inverse	335	48	16	0.84	−0.35	0.85

multiple ones (excluding doubles) are in any case less than 5 percent. This rate between single and double determinations is lesser but not far from the one which is usually encountered in standard intensity assessment (in the expert assigned intensity data set we used the rate of double determinations is about 36 percent) and here is notably influenced by the choice of the weighting scheme. The number of double determination indeed decreases from about 30 percent for the *unit* weighting to 20 percent for both *damage* and *expert* schemes. The similarity of the results of these last two may be due to the fact that the expert, more or less consciously, assigns larger weights to effects on building than to all other effects. On the other hand the number of multiple determinations is not very sensitive to weighting and it only reduces by 3 units in the case of the inverse weighting scheme. As far as the correlation with expert intensity values is concerned, it seems that it is not very much influenced by the weighting scheme, considering that it is almost constant among alternatives. Finally, let us note the negative value (−0.35) of the average absolute difference indicating that the algorithm overestimates the intensity, when compared with the expert, by one grade in about one case over three.

In table II the same comparison of table I is done for the *inverse* membership scheme. Here the number of uncertain determinations (double plus multiple) is really small (less than 5 percent) and seems not to be affected by the weighting. The r.m.s. is about 70 percent of one grade, the correlation coefficient is about 87 percent, while the average absolute difference is still negative but with smaller amplitude than in the previous case of *expert* rating.

In table III, four membership schemes are compared, keeping fixed the weighting to the *expert* one. The *hard* membership shows very clearly its inability to resolve ambiguous data. Most cases are indeed fully uncertain and only one fourth of the total had a single or a double determination. Also the other indicators show that the *hard* rating leads to a very poor agreement with expert intensities. From the comparison of the two expert rating with and without decay, we can see that the number of single determinations is smaller for the *continue* scheme than for the *expert* one. The r.m.s. and the R^2 seem also to confirm that the decay of mem-

Table II. Membership: inverse.

Weighting	N_{single}	N_{double}	$N_{multiple}$	r.m.s.	diff.	R^2
Unit	384	7	8	0.70	−0.13	0.87
Damage	385	6	8	0.69	−0.08	0.87
Expert	386	5	8	0.70	−0.14	0.87
Inverse	386	5	8	0.69	−0.08	0.87

Table III. Weighting: expert.

Membership	N_{single}	N_{double}	$N_{multiple}$	r.m.s.	diff.	R^2
Hard	68	44	287	1.69	−0.38	0.25
Continue	299	58	42	0.99	−0.42	0.81
Expert	307	73	19	0.84	−0.38	0.85
Inverse	386	5	6	0.70	−0.14	0.87

bership function on higher grades, adopted in the *expert* scheme may increase the ability of the MCDM algorithm to reproduce expert determinations.

A complete comparison between bell-shape membership functions (in this sense the *hard* membership function can be seen as a *bell0* function) is in table IV. The weighting is the *unit* one in all the cases. The efficiency of the algorithm increases steadily with increasing width of the bell up to 3 grades but remains almost constant passing from 3 to 4. In table V we compare some membership functions having both symmetrical (*bell2*) and asymmetrical (*bell2h* and *bell2l*) patterns. For *bell2h* the values of the function are chosen a little higher in the right half of the bell (for higher grades) while for *bell2l* the values are higher at left (for lower grades). This leads to very similar results in terms of numbers of double and multiple determinations but with a preference for *bell2l* due to higher correlation and lower r.m.s. For the *bell2h* the absolute average difference is strongly negative while it is only slightly negative for the *bell2l* indicating that the first tends to clearly overestimate the intensity with respect to the expert, while the second is almost in agreement with it. In table V, the results for the symmetric function *bell2* and for the aggregation *bell2l* + *bell2h* are also reported. These last two look like very similar in terms of numbers of different types of determi-

Table IV. Weighting: unit.

Membership	N_{single}	N_{double}	$N_{multiple}$	r.m.s.	diff.	R^2
Hard	68	44	287	1.69	−0.38	0.25
Bell1	165	138	96	0.97	−0.25	0.76
Bell2	202	172	25	0.86	−0.28	0.82
Bell3	207	180	12	0.83	−0.31	0.84
Bell4	207	181	11	0.83	−0.31	0.84

Table V. Weighting: unit.

Membership	N_{single}	N_{double}	$N_{multiple}$	r.m.s.	diff.	R^2
Bell2h	330	44	25	0.96	−0.47	0.80
Bell2l	330	44	25	0.85	−0.09	0.82
Bell2	202	172	25	0.86	−0.28	0.82
Bell2l + *Bell2h*	169	205	25	0.85	−0.27	0.82

Table VI.

Membership	N_{single}	N_{double}	$N_{multiple}$	r.m.s.	diff.	R^2
Expert	307	73	19	0.84	−0.38	0.85
Expert + *inverse*	336	46	17	0.69	−0.24	0.88
Inverse	386	5	8	0.69	−0.08	0.86

nations, r.m.s. and R^2 indicating that the composition of the two asymmetrical functions acts very similarly to the symmetric one.

The results shown in table VI are reported to give an example of the effect of the simultaneous application of two different scales. In fact, as already noted, the *inverse* membership function can be seen as a really new scale which does not coincide with the MCS scale. It is interesting to note that while the numbers of the aggregate *expert + inverse* membership (taken with their corresponding weighting schemes) result intermediate between the two separate cases, the R^2 is higher than the original schemes.

5. Conclusions

The computation performed shows that the intensities resulting from the algorithm are in fair agreement with the expert estimates for all the reasonable choices of the membership functions. It makes exception only the *hard* scheme (corresponding to the strict application of the scale) which gives an unacceptably high number of uncertain cases and low values of the estimators of the agreement with the expert determined intensities.

The form of the membership function notably influences both the distribution among various types of determination (simple, double etc.) and the agreement with expert determined intensities, while the weighting seems to have weaker effects. This actually seems to be the most evident deficiency of this approach, and the development of a more effective weighting procedure of the effects could be the objective of future improvements of this method.

The comparison of bell-shape functions indicates that, on the basis of the evaluation criteria we adopted, the optimal half width of the fuzzy membership function seems to be around 3 seeing that, in that case, the number of uncertain cases is minimized and the agreement is the highest one.

The number of uncertain assignments done by the algorithm is, in all reasonable cases, lesser than that done by the expert. This is an important result of this approach. As seen above, for example, in the case of the *inverse* membership, the uncertainty in intensity assessments can be reduced by this algorithm almost to zero. It is also to be noted that due to this characteristic, the goodness of the fit with expert determined intensities can never be perfect.

All the analysed schemes, more or less, tend to overestimate the intensity with respect to the expert. Possible explanations of this behaviour can be:

a) a wrong choice of the membership and weighting schemes;

b) an intrinsic defect of the algorithm in reproducing the behaviour of the investigator:

c) an unconscious *conservative* behaviour of the expert who assigns a higher grade only when some key effects are certainly observed and so doing completely neglects other effects.

More data on different earthquakes are required to give a final answer to this question.

The intensity assessment algorithm we build up may also be seen as a useful tool to discover assignment errors and also to make explicit some implicit and often unconscious assumptions done. In fact, this analysis produced also a detailed and critical discussion among the authors (only partially reproduced here) on the choices that are usually done and the criteria commonly adopted, more or less consciously, by macroseismic investigators when assigning the intensity.

We believe that the amount of information on the physics of seismic generation that could be extracted by macroseismic data is very big and thus it is very useful to make an effort to improve the analysis techniques of this kind of data. This work is only a first step in this direction but it already showed some positive indications on the possibility of this methods.

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Appendix

Table I. The complete «disassembly» of the MCS scale (Sieberg, 1932, table 102) is reported with codes ranging from 1 to 106. The sentences are grouped by types of effect. The sentences reported also in table 103 of Sieberg (1931) are typed in **boldface**. Sentences marked with codes greater than 106 have been added to include effects often observed on sources (see text).

Code	Sentence
142.	Not felt
1.	**Felt by few nervous persons in total stillness, or by very sensitive persons, particularly on the upper floors** of buildings
4.	Some realize that it was an earthquake only after talking about it to others
2.	**Felt by few indoors**
6.	**Felt by many indoors**
29.	Felt by everybody indoors with fear
107.	Felt by everybody indoors
5.	**Felt by not many outdoors**
14.	**Felt by many outdoors, even when performing everyday jobs**
30.	Felt by everybody outdoors
13.	**Awakening of few sleepers**
109.	Awakening of few sleepers (the majority continues to sleep)
27.	**General awakening of the sleepers**
12.	Felt with not much fear except by persons who became nervous and apprehensive after previous earthquakes
28.	**ew escape outdoors**
31.	**Many escape outdoors**
116.	Everybody escapes outdoors
119.	Considerable panic, fear, fright in many persons
120.	Great panic, much fear and great fright in everybody
3.	Perceived as the passing of a fast car
15.	Felt indoors after the shaking of the whole building
117.	Caused sensations of discomfort, like dizziness, nausea, trembling, etc.
32.	Some persons lose balance
8.	Pieces of glassware and pottery collide with each other as if a heavy lorry passed on uneven ground
11.	It seems that a heavy object (sack, piece of furniture) upsets, or that chairs or beds oscillate as on a ship in rough sea
17.	**Slight oscillation of hanging objects (curtains, traffic lights, lamps, light chandeliers)**
110.	Violent oscillation of hanging objects
21.	Pictures on the walls move or bang
34.	Pictures fall from the walls
9.	**Windows tinkle; doors, beams and planks bang, ceilings creak**
26.	Windows and doors bang, glasses break
144.	Light trembling of objects, furnishings and books
23.	**Objects and furnishings may upset**
35.	**Steady objects, books and furnishings are moved or fall**
36.	Dishes and pieces of china shatter
44.	Even heavy objects upset and break
43.	Considerable damage is made to a larger number of household furnishings.
7.	Trembling or light oscillation of furniture
25.	Furniture roar
24.	Light pieces of furniture may be slightly moved
37.	Some pieces of furniture are moved or fall
58.	Part of heavy furniture is moved away; part is upset
19.	**Pendulum clocks stop, start up again or oscillate with more amplitude; clock springs ring**
18.	Ringing of bells
38.	**Stroke or toll of small bells in clocks or steeples**
45.	**Toll of big bells**
10.	In open vessels, liquids are slightly moved

Table I. (*continued*)

Code	Sentence
22.	In full open vessels, slight pouring out of liquids
33.	Liquids move violently
143.	No damage
39.	**Light damage in steadily built houses**
42.	**Some tiles and stones of chimney-tops fall down**
121.	Some chimney-tops and cornices fall down
53.	**Ruined chimney-tops upset on the roofs, damaging them**
51.	**General falling down of tiles**
52.	Many chimney-tops are damaged by cracks, by falling of tiles, by emission of stones
54.	Badly fixed decorations fall from towers and high buildings
122.	Irreparable damage and/or collapse in smokestacks
123.	Collapse of some roofs and/or ceilings
124.	Collapse of many roofs and/or ceilings
125.	Collapse of almost every roof and/or ceiling
41.	Serious but still harmless damage to badly built houses
56.	Badly built or badly maintained houses seldom collapse
59.	Shifting, rotation or collapse of statues or monuments
126.	Mild damage to towers, churches, castles etc.
127.	Serious damage to towers, churches, castles etc.
128.	Partial or total collapse of towers, churches, castles etc.
129.	Crackings open up in retaining walls and/or boundary walls
130.	Partial collapse of retaining walls and/or boundary walls
60.	Steady stone boundary walls open up and collapse
40.	Crackings and detaching of plaster in steady buildings
50.	Collapse of rather large parts of rendering, stucco and bricks
131.	Mild damage to few steady buildings: small crackings in the walls
49.	**Mild damage to many steady buildings: small crackings in the walls**
132.	Considerable damage to few steady buildings: large crackings in the walls
133.	Considerable damage to many steady buildings: large crackings in the walls
61.	**Serious destruction of approximately 1/4 of buildings**
68.	**Approximately 1/2 of stone houses is seriously destroyed**
135.	Almost every stone house is seriously destroyed
73.	**Serious destruction of approximately 3/4 of buildings**
134.	Some houses become uninhabitable
62.	Many houses become uninhabitable
70.	Most houses become uninhabitable
63.	**Some houses collapse**
69.	**More houses collapse [more than 1/4]**
74.	**Most houses collapse [more than 1/2]**
90.	**Every brickwork building collapses**
55.	In framework houses, damage to the rendering and to the framework are rather serious
64.	In framework houses most of curtain walls collapse
72.	The beams in some wooden framework are broken
71.	Framework houses are uprooted from their foundations and crushed
65.	Wooden houses are crushed or upset
91.	Single steady buildings and elastic fixed joint wooden huts may withstand
114.	The majority of steady buildings and elastic fixed joint wooden huts withstand
115.	Few steady buildings and elastic fixed joint wooden huts withstand
75.	Even steady wooden buildings are seriously damaged
136.	Some bridges are slightly damaged
76	Bridges are seriously damaged
77.	Some bridges are destroyed

Table I. *(continued)*

Code	Sentence
92.	Even the largest and safest bridges collapse owing to the breakdown of stone pillars or to the subsidence of iron pillars
137.	Banks and dams, etc. are slightly damaged
78.	Banks and dams, etc. are more or less considerably damaged
112.	Banks and dams, etc. are considerably damaged
93.	Banks and dams are totally separated, often even for long parts
79.	Railway tracks are slightly bent
94.	Railway tracks hard bent and compressed
80.	Pipes (gas, water and drain) are cut off, broken and crushed
95.	Pipes in the ground are separated and become useless
81.	In paved and asphalted roads crackings and large undulating folds
138.	Crackings and fissures open
97.	Large crackings and fissures open
83.	Soft ground slides down the slopes
84.	Rocks roll downhill
85.	Large rocks come off river banks and steep coasts
100.	Many flakings in the ground and fallings of rocks
87.	The relief of the ground sometimes changes considerably
139.	The changes in the ground are remarkable
102.	The changes in the ground are majestic
140.	Ruined buildings
141.	Irreparably ruined buildings
145.	Light damage
146.	Damage
147.	Serious damage
148.	Collapses
149.	Ruins
101.	**No human work withstands**
150.	Dead and/or wounded
151.	Few wounded
152.	Many wounded
154.	Few dead
155.	Many dead
16.	Visible shaking of thin branches of plants and bushes, as with moderate breeze
57.	Waving and uprooting of tree-trunks
66.	Crackings in steep slopes and wet grounds
67.	Expulsion of sand and slime in wet grounds
82.	Large fissures up to 1 metre are open in soft and wet grounds, in parallel to rivers
86.	Shifting of masses of sand and mud in flat coasts
96.	Remarkable changes caused by the nature of the soil occur in the ground
98.	Disorder is considerable, particularly in soft and wet grounds, both horizontally and vertically
99.	Overflowing of water carrying sand and slime with its displays
46.	Watercourses, ponds and lakes generate waves and become muddy owing to the stir of slime
47.	Some sliding of parts of sandy and gravel banks
89.	Waters are hurled against the banks in rivers, canals, lakes etc.
103.	Underground watercourses go through various changes when flowing outdoors
104.	Waterfalls form
105.	Lakes stagnate
106.	Rivers diverge
48.	The level of the water changes in wells
111.	Changes take place in wells and fountains
88.	The level of the water frequently changes in wells
113.	The level of the water frequently changes in wells and fountains
20.	Temporary suspension or interruption of electric power
118.	Interruption of telephone and telegraph lines

Table II. Sample of the data set used for this work. Expert intensities are given in decimal form (4.5 stands for IV-V). The source weight is related to the reliability of the source. Effects actually observed are reported with their codes (see table 1).

Locality	Expert Intensity	Source Weight	Answers				
Ripa	9.0	0.9	69	73	70	151	154
Seravezza	4.5	1.0	143				
Capanne	7.5	0.9	134				
Sillano	8.5	1.0	152				
Sillano	8.5	0.9	147	151	152		
Sillano	8.5	0.9	147	151	152		
Sillano	8.5	1.0	63	69			
Sillano	8.5	0.4	63				
Sillano	8.5	0.9	147	152			
Sillano	8.5	0.9	69	73	70		
Sillano	8.5	0.9	154				
Palagnana	5.0	0.9	6	9	40	48	31
Stazzema	4.5	1.0	143				
Roggio	8.0	0.4	154	62	70		
Roggio	8.0	0.9	154				
Vagli Sopra	9.0	0.9	74				
Vagli Sopra	9.0	0.9	151	154			
Vagli Sopra	9.0	0.4	152	154			
Vagli Sopra	9.0	0.9	147				
Vagli Sotto	8.5	1.0	74				
Vagli Sotto	8.5	0.9	69	73	70		
Vagli Sotto	8.5	0.4	151	154			
Vagli Sotto	8.5	0.9	152	154	70		
Vagli Sotto	8.5	0.9	151	154			
Vagli Sotto	8.5	0.9	152	154	69	70	127
Vagli Sotto	8.5	0.9	152	154	62		
Vagli Sotto	8.5	0.9	69	70			
Vagli Sotto	8.5	1.0	152	154			
Viareggio	6.0	0.4	120	127	42		
Viareggio	6.0	1.0	120	127			
Viareggio	6.0	1.0	127	42			
Viareggio	6.0	0.4	31	127	129		
Viareggio	6.0	0.9	49	133	51		
Viareggio	6.0	1.0	120	127			
Viareggio	6.0	0.9	127	42	51		
Viareggio	6.0	0.9	42	120			
Canigiano	10.0	1.0	74				
Massa	8.5	0.9	69	70			
Pianacci	9.0	1.0	63	69	74		
Pianacci	9.0	0.9	69	70			
Pianacci	9.0	0.9	69	70			
Sassorosso	8.0	0.9	63				
Sassorosso	8.0	0.9	63				

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