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Earthquakes and large block monumental structures

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Abstract
A review of a research program developed in the last few years into the dynamics of large block structure is presented. The many aspects which characterize the structural behaviour and determine the complexity of the consequent dynamics are briefly discussed. The preliminary mechanical models and first results obtained are also analyzed; they allow to define the direction of the main object of the research in progress.

Key words *earthquakes – ancient monumental structures – non linear dynamics*

1. Introduction

The problem of conserving our architectural heritage and protecting it against the risk of earthquakes has in recent years stimulated much research in the field of structural engineering, with the aim of describing and understanding the seismic behaviour of monumental structures. The multiplicity of the research and the variety of methodologies proposed are signs of a debate and exchange of views among workers that stem not only from specific disciplinary fields, but also from epistemological questions.

The history of technique reveals that the absolute supremacy of theory over technique is debatable. In fact, in many cases theory has been sparked off by technique, and a scientific explanation has been reached at the end of a lengthy evolution of construction techniques, whose origins are lost in time.

Theory has often been presented as rational knowledge of what was put into practice even though it was not understood: it evolved not because a deeper understanding of the techniques was needed, but because of intellectual and individual curiosity (Benvenuto, 1991).

St. Peter's basilica was built without the benefit of theory. Technique therefore preceded theory but did not surpass it; in fact, the serious damage to the dome that seemed to make its collapse likely in the 18th century was repaired only thanks to the work of three mathematicians and G. Poleni, whose improvements enabled the building to survive to the present day.

Modern structural mechanics has its origins in the well-known theory of elasticity, formulated only in the last century, when the large monumental structures had already been built. This theory lends itself well to the description of the behaviour of continuum media such as steel and concrete.

The study of the mechanical behaviour of historical monumental works is a task for our times, for the building techniques have long been forgotten and the theoretical questions concerning their behaviour are still not clear. Through a mathematical model of a structure the simulation of behaviour obtained is expected not only to be in keeping with actual behaviour, but also to respect the structural concept governing the work.

2. The Greek and Roman concept of space

Since «every constructive evolution is called for and determined by the evolution of a precise architectural language: that is, by the evolution of spatial forms that are the vocabulary used by architecture to express itself artistically» (Bettini, 1948), it is to the concept of space in Greek civilisation that we must refer when facing the problem of the simulation of large block monumental structures, such as temples, which were the manifestation of this concept.

According to Bettini, «for a Greek, figurative space is not identifiable with nature, which he considers irrational; its artistic image can be portrayed only within the limits of a space reduced from nature to reason, that is, geometrically defined» (Bettini, 1948).

A temple is based on a system of geometric relationships among the major structural elements, made evident by their functional value: «the horizontal and vertical elements» are «counterbalanced to obtain great impact in the play between the heavy and supportive parts»... «in Greek architecture columns are the perfect figurative expression of support, the rationality of which is accentuated by the entasis and swelling movement in the capital»; «...the columns, capitals and lintels the composition of trilithic schema, are the elements of spatial syntax, that is plastic expressions of tectonic connections, contained within the limits of a rationalized space» (Bettini, 1948).

Such spatial awareness was extremely different from that of the Romans; in fact, if with the Greeks art had reached the absolute expression of a 'Platonic' space, with the Romans it became the expression of a space intended as a place for men's actions, characterized by a relationship with the subject that operated within it, and measured by this. «The Roman sense of space was expressed in a totally different way from that of the Greeks, with its original syntax of walls, arches and vaults. The space that such language expressed was not at all a composite of trilithic elements, but rather a unified form, closed and compact; it was a decidedly *internal* space; a continuum defined by the continuity of the perimeter walls and vaults» (Della Seta, 1939).

This is exactly what provides the most concrete explanation – if one can speak of explanation – of the difference between Greek and Roman art. For instance, it explains why Greek architecture is expressed only by the *external* space of a building, while Roman architecture expresses itself in an *internal* space. «Right from the beginning, the emphasis of Roman architecture is placed not on the single *element*, in the Greek manner, but on the unit as a whole. The «classic» Greek construction using blocks of stone without any form of mortar, that nevertheless obeyed and also resisted the law of gravity, as clearly shown by their plastic form and the constructive scheme of the whole, based its static architectonic language on the plastic-static significance of a single element».

«Roman construction, by reversing the technique of a structure, and substituting the use of large blocks held in place by their own weight with that of small stones or bricks united by a strong mortar that almost cancelled out both the plastic form and its weight, created an architectonic language whose emphasis was no longer on the single element, but on the syntactic connexion that guaranteed unity as a whole to the elements» (Della Seta, 1939).

Large block structures were therefore an obstacle to the development of Roman architecture, and to the achievement of its aim, which was to cover as much internal space as possible in the various types of buildings. A solution could be reached only by adopting a very different structural technique, whose evolution coincides with a process of progressive liberation from the principle of *weight* of the construction. To express the Roman expansion and contraction of spaces, a new technical means had to be found that would allow a building to be considered as a single spatial image. «This means was cement, which was therefore not only a great technical invention, but also a linguistic novelty for Roman builders. The technical and artistic result of using cement is the attenuation, if not the abolition, of the *weight* of the materials used in construction. Cement becomes the 'means' of the structure, the fundamental element, the syntactic connection of the architectonic language»... «In fact, the technical evolution of Roman construction can be traced by examining concretionary cores, rather than surfaces» (Della Seta, 1939).

Conversely, the basis of the Greek concept of space and its rationalization is the size of the blocks. «The size of the blocks was both obedience and resistance to the law of gravity; what determined the construction was the principle of stability of a stone» (Bettini, 1948).

3. Large block structures and mechanical models

Our work deals with monumental structures of large stone blocks, such as the Greek temples, within the framework of the Greek concept of space. The temples, which are stable structures resistant to their own weight, lose the apparent formal integrity of their rectilineal elements during seismic movement. The intrinsic characteristic of these structures is thus revealed; they are made up of discrete *heavy* elements: blocks, simply supported by each other.

The complexity of the consequent dynamics has occupied scientists for many years: stress discontinuity, the law of contact, structural damping, impact, contact friction and the nonlinear character of the structural response have all become new objects of research.

According to the entity of the excitation and the elastic characteristics of the elements and the support, the actual structural behaviour varies in time from that which is purely elastic to that which is purely rigid, or to a combination of the two.

It is certainly questionable whether criteria typical of continuous structures subjected to bilateral constraints should be used to work out the causes of their collapse. For example, in the case of stability of columns, to refer to the resonance frequencies typical of a beam with a clamped end, and therefore to the theory of Eulero-Bernoulli, in which the ground is considered as a bilateral constraint, is highly debatable.

In the absence of binding materials, both the ground and the contacts between adjacent elements are monolateral. We refer to a study in

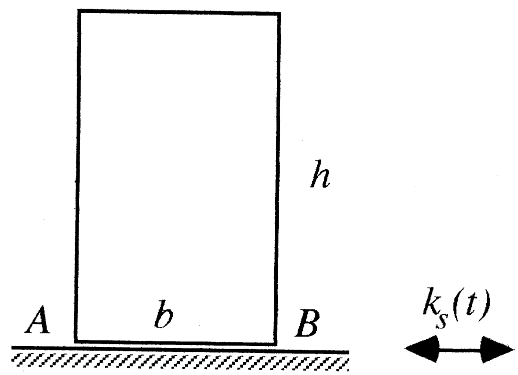

Fig. 1. Two dimensional scheme of a monolithic column.

preparation for a detailed analysis of this problem (Sinopoli, 1995c); for the moment it should be sufficient to mention some preliminary considerations, which have motivated the hypotheses used in research carried out so far.

Let us consider the simplest problem, that is, a thin block simply supported on a unilateral ground (fig. 1); let b and h be the base and the height respectively of the block in its equilibrium configuration and subject to its own weight. Let us also suppose that the ground is characterized by a given horizontal movement, the first simulation of a seismic occurrence, and that $k_s(t)$ is the instantaneous intensity of the seismic acceleration, in g units. For values of $k_s(t)$ lower than critical value, which for a rigid ground equals b/h, a motion of combined compressive and bending vibrations comes into play, characterized by a reduction of the bearing surface due to the inability of the ground to counterbalance tensile stress. The area of the resistant bearing section depends on the elastic properties of the block and support; it also depends on $k_s(t)$, and therefore on time.

Because the seismic acceleration oscillates between positive and negative values, the localization of the support surface varies from an area covering corner A to that covering corner B, as shown only qualitatively in fig. 2. In these conditions it is arbitrary to consider resonance phenomena similar to those of linear elastic theory with bilateral constraints. Even

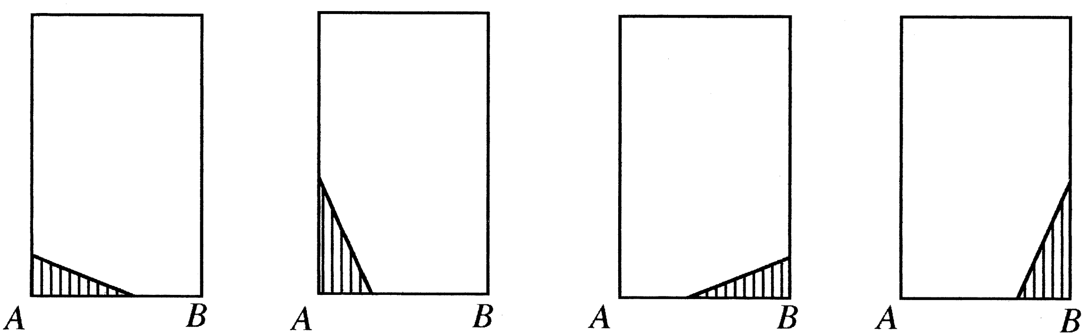

Fig. 2. Qualitative compressive stress distributions due to an oscillating horizontal ground, acceleration of small intensity.

assuming small deformations, the compressive and bending vibrations lead in fact to a system of differential equations with partial derivatives, time-dependent coefficients and unilateral boundary conditions.

However, it is sufficient for $k_s(t)$ to reach its critical value in order for the area of the resistant section to fall to zero, and the support is then concentrated on corner *A* or *B*. The boundary conditions of the block become equal to the ones with free ends as the contact area measures practically zero; the elastic behaviour of the block is reduced to free vibrations, rapidly damped by internal friction, while the energy supplied by the ground movement causes rigid body dynamics only.

Also when $k_s(t)$ is lower than its critical value, the situation can be the same if the solution is unbounded and the stresses tend to rise. In this case too, at a given instant the dynamic behaviour coincides with a rigid body motion. Once the rotation mechanism around a base corner is activated, elastic behaviour becomes negligible; though it can occur again when the block passes through its initial equilibrium configuration, hitting the ground. Furthermore, only the amount of elastic energy dissipated by internal damping and contact friction is determined by elastic behaviour.

In short, elastic behaviour is reduced to time intervals in which: a) the vibrations are of small amplitude and correspondingly the stress level has low value; b) the block hits the support. Analogous qualitative considerations can be made for multi-block structures with reference to each single contact between contiguous elements. The collapse of this kind of structure requires as a necessary condition the formation of a mechanism. This is in accordance with Heyman (1966), who thus identified a lower stability boundary value. On the other hand, the higher boundary value is defined by the stability of the dynamics following the formation of the mechanism, *i.e.* by comparing the energy supplied by external force with that lost during the movement.

For many years now there has been a lively debate among scientists regarding the most suitable instruments or models to describe the behaviour of large block monumental structures. The many studies carried out can be divided directly or indirectly into two main branches of research. The first consists in the study of the dynamics of rigid elements using both analytical and numerical methods, in which possible elastic behaviour is taken into account by means of coefficients of restitution or dissipation. The second consists of mainly numerical studies, in which elasticity is either concentrated in the ground (Winkler's ground) or described through finite element codes where elastic behaviour is governed by appropriate laws of contact and is examined using penalty methods.

It is extremely difficult to provide briefly full information on the advantages and limits of these two branches of research. The methods using models of elastic behaviour, and especially those with finite elements, are of doubtful reliability with respect to the definition of the constitutive law, and above all the convergence of the solution that depends on the finite elements mesh for each structural element. In fact, the possibility of tension discontinuity concerns points *a priori* unknown, since possible sliding between contiguous elements during movement alters the position of points and surfaces that may enter into contact or maintain it.

Methods based on modelling each structural element as a rigid one come up against other difficulties. Above all, it must be emphasized that the dynamics of even a single rigid element with a unilateral contact are extremely complex; many analytical and numerical studies have been carried out on this subject during the last few years (Augusti and Sinopoli, 1992).

In addition, there are at present no reliable computing codes available for rigid elements and unilateral contact, except those worked out by researchers for themselves (Jean and Moreau, 1991); this explains why studies concerning the characteristics and law of unilateral contact, which are indispensable for building up such codes, are currently being carried out (Sinopoli, 1995b). Moreover, some aspects of the problem, such as schematization of possible elastic behaviour that can occur in the activation of mechanisms and during shocks, are still open questions and also objects of study (Sinopoli, 1995c).

In the process of researching into the dynamics of large block structures, I have so far assumed that their behaviour is similar to that of a rigid-labile structure; by this I mean that elastic energy during movement has been neglected, since it is negligible with respect to that of rigid movement. So far the effects of elasticity during the setting in motion of the mechanisms and during collisions, which require closer specific study, have been ignored but will be the subject of future research. The aim of research carried out up to now has, in fact, been to analyse aspects that determine the dynamics of one or more rigid elements, in order to resolve some mechanical problems and thus succeed in defining a general model.

The next sections are devoted to some of the numerous questions arising when the dynamics of multi-block rigid structures are tackled. Only those that fit in with, and help to define the direction of the main object of the research in progress, are mentioned here. These deal with:

– the results of studies that have already been concluded (for detailed review see Augusti and Sinopoli, 1992), showing that, even in the simplest well-defined mechanical model, analytical investigations cannot provide complete information regarding the stability of the system, due to a kind of unpredictability, so that numerical methods are mandatory; this is the case of the forced rocking of a block discussed in section 4.1;

– the analysis of some aspects (section 4.2) of questions still not completely clear in the scientific community, like the definition of the conditions to start a given mechanism and the identification of the mechanism after an impact as a function of the friction performance and geometry. The definitive answer to these questions can come only from the solution of questions closely connected with the need for of a general formalism (Sinopoli, 1995b) able to describe rigid body dynamics in the presence of unilateral constraints;

– the main results of numerical investigations into the dynamics of a trilith, the first multi-block structure to be analysed, although restrictive assumptions and an approximate mechanical model are used; this is the subject of section 4.3.

Special reference is made to the case of the single block because the most important and general properties defined for it, such as contact law and the resolution of the impact problem, can be extended relatively easily to the case of multi-block structures.

In any case, the results obtained should be considered an initial contribution to the general question of the stability of these structures: the problem is in fact so complex that there is still much more research to be carried out in this field.

4. Rigid body dynamics with unilateral constraints

Consider a rigid block, of mass m, simply supported on a horizontal plane rigid ground in its static equilibrium configuration (fig. 1). Assume, for simplicity, that the block is a parallelopiped and let b and h be, respectively, the base width and the height of the block. This system can be considered the two-dimensional scheme of a monolithic stone pillar or column on its support.

The dynamics of such an apparently simple system, in presence of the unilateral constraint of impenetrability with respect to the ground, are actually very complicated. In fact, during the dynamic evolution, the possibility of transition from one mechanism to another, among all those allowed by the constraints, induces the fact that the motion can be governed by different systems of differential equations matched at given instants; they depend on the initial conditions, on the geometry of the system, on the intensity of the active forces and on the value of the friction coefficient between the ground and the block. Therefore, the dynamic behaviour is in general non linear.

4.1. *Forced rocking motion*

In most papers of the scientific literature, oriented to different applicative purposes (Housner, 1963; Hogan, 1989; Sinopoli, 1991; Lipscombe and Pellegrino, 1993), a mechanical model of the Housner type (Housner, 1963) has been assumed to describe the dynamics of a block simply supported on rigid ground; *i.e.*, the dynamic analysis has been restricted to the motion of the block induced by a given ground acceleration and characterized by only one degree of freedom, namely the rotation around either base corner edge A or B, described respectively by either positive or negative value of the rotation angle θ (fig. 3).

Let us consider the forced dynamics induced by a harmonic horizontal ground acceleration: $\ddot{x}_g = K_s\, g \sin(wt + \phi)$, where g is the gravitational acceleration.

The harmonic excitation obviously cannot be considered a simulation of a seismic motion; nevertheless, it is a first necessary step toward the knowledge of the dynamic behaviour of the system, according to the standard techniques of non-linear dynamics. Furthermore, in the case of multi-block structures as will be shown in the following, this kind of stationary excitation characterized by a zero mean value can identify the disposition of the system to exhibit a dynamic response with mean value different from zero; with respect to the safety of the system, this means irreversible variations of the initial geometric configuration and, then, probability of collapse.

If the block can move according to two possible mechanisms (fig. 3), the equation describ-

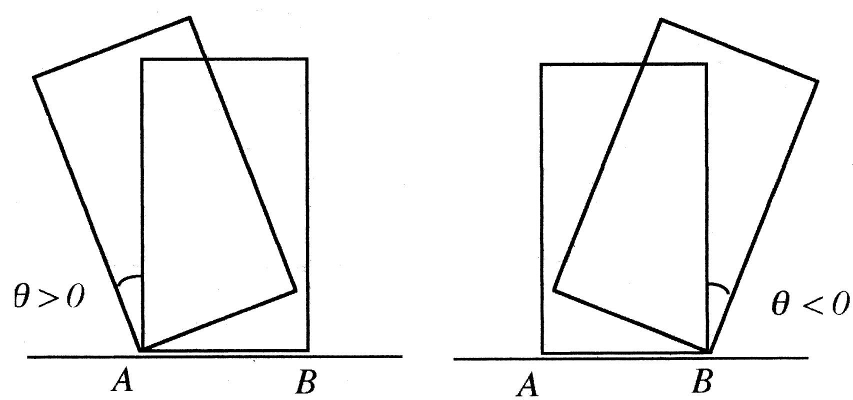

Fig. 3. Anti-clockwise and clock-wise rotation mechanisms.

ing the dynamics is, for $\theta > 0$ (Sinopoli, 1991):

$$\ddot{\theta} - \alpha^2\left[1 + K_s \frac{b}{h} \sin(\omega t + \phi)\right] \sin\theta +$$

$$+\alpha^2\left[\frac{b}{h} - K_s \sin(\omega t + \phi)\right] \cos\theta = 0 \tag{4.1}$$

where:

– b/h is the ratio between the base width and the height of the block;

– $\alpha^2 = \dfrac{3g/2h}{1 + b^2/h^2}$;

– g is the acceleration of gravity;

– ϕ is the initial phase required to start the motion at $t = 0$; it is such that: $K_s \sin\phi > b/h$.

Equation (4.1) is not linear. Under the assumption of small angles and changing the time scale by putting:

$$\omega t = \tau$$

and:

$$\varepsilon = -K_s \frac{b}{h} \frac{\alpha^2}{\omega^2}$$

$$\delta = -\frac{a^2}{\omega^2}$$

eq. (4.1) becomes:

$$u'' + [\delta + \varepsilon \sin(\tau + \phi)]u = -\delta[-b/h + K_s \sin(\tau + \phi)] \tag{4.2}$$

where the variable θ has been substituted by u to take account of the change in the time variable. An equation similar to eq. (4.2) can be obtained, if the mechanism B is started (fig. 3):

$$u'' + [\delta - \varepsilon \sin(\tau + \psi)]u = -\delta[-b/h + K_s \sin(\tau + \psi)] \tag{4.3}$$

with: $K_s \sin\psi < -b/h$.

A special time instant during the dynamic evolution is when the system, coming from the motion governed by eq. (4.2) or (4.3), reaches its static configuration and hits the ground. Then an impact occurs, and because a finite area is involved in the contact, the subsequent motion, depending on the values of the friction coefficients and the sizes of the block, can follow any mechanism in addition to that of rocking, *i.e.*: sliding, sliding-rocking motion, block at rest.

Under Housner's assumption (valid for slender blocks) that only rocking around point A or B is allowed, the angular velocity after each impact can be expressed as a function of the corresponding velocity before it, as:

$$u^+ = \beta u^- \tag{4.4}$$

where: $0 \le \beta \le 1$ and, according to the Housner model of inelastic impact (Housner, 1963):

$$\beta = \frac{(2 - b^2/h^2)}{[2(1 + b^2/h^2)]}. \tag{4.5}$$

Thus the value of β depends only on the ratio b/h. In order to maintain the sign of the angular velocity over the impact, it must be $\beta > 0$, whence $b/h < \sqrt{2}$, which is satisfied by Housner's slenderness requirement.

Because the motion is governed by either eq. (4.2) or (4.3) until the block comes back to $u = 0$ and hits the ground, the dynamics exhibit as many discontinuities as impacts and the motion is obtained by matching alternately the solutions to eqs. (4.2) and (4.3), at each impact instant.

Let τ_1^* be the instant of the first impact, assuming that the motion comes from the former mechanism (eq. (4.2)); for $\tau > \tau_1^*$, the rotation occurs around point B and eq. (4.3) can be written as:

$$u_1'' + [\delta - \varepsilon \sin(\tau + \phi)]u_1 = -\delta[b/h + K_s \sin(\tau + \phi)]. \tag{4.6}$$

Likewise, let i represent the counter of the impacts and τ_i^* the time span between an impact and its predecessor; after the k-th impact and therefore for $\tau > \tau_k$, where:

$$\tau_K = \sum_{i=1}^{k} \tau_i^* \qquad (4.7)$$

if k is even, the motion is an oscillation according to the first mechanism (rotation around point A); its equation is:

$$u_k'' + [\delta + \varepsilon \sin(\tau + \phi)]u_k = -\delta[-b/h + K_s \sin(\tau + \phi)] \qquad (4.8)$$

After an impact of odd $k+1$ order, on the contrary, the motion follows the mechanism B and the equation, for $\tau > \tau_{k+1}$, is:

$$u_{k+1}'' + [\delta - \varepsilon \sin(\tau + \phi)]u_{k+1} = -\delta[b/h + K_s \sin(\tau + \phi)] \qquad (4.9)$$

The resulting motion of the block is obtained by matching alternately the solutions to eqs. (4.8) and (4.9), at the instants τ_K, increasing k; the matching conditions are:

$$u_{k+1}(\tau_K) = u_k(\tau_K) = 0 \qquad (4.10)$$

$$k = 1, 2, \ldots, \infty$$

$$u_{k+1}'(\tau_K) = \beta u_k'(\tau_K). \qquad (4.11)$$

As a consequence the general expression of the dynamics equation is:

$$u'' + p(\tau)\, u' + q(\tau)\, u = f(\tau) \qquad (4.12)$$

where $q(\tau)$ and $f(\tau)$ are the functions obtained by matching the corresponding terms of eqs. (4.8) and (4.9) at the instants τ_K, increasing k from 1 to ∞. The function $p(\tau)$ takes into account the discontinuity of the angular velocity at each impact according to relation (4.11); therefore, until the instant t_k, $p(\tau)$ is a succession of impulses defined by:

$$p(\tau)\, u'(\tau) = \sum_{i=1}^{k} [(\beta - 1)/\omega]\, \delta^*(\tau - \tau_i)\, u'(\tau) \qquad (4.13)$$

where $\delta^*(\tau - \tau_i)$ is the Dirac delta density.

The functions $q(\tau)$, $f(\tau)$ and $p(\tau)$ in eq. (4.12) admit as parameters the features of the excitation K_s and ω, and the geometrical features of the block, while the details of their analytical expression depend on the succession of impacts and, therefore, on the initial conditions. Any motion is described by eq. (4.12); but different analytical expressions of $q(\tau)$, $f(\tau)$ and $p(\tau)$ correspond to different motions; therefore, an interplay exists between the motion which generates the equation and the equation by which the motion is governed.

The motion can be either aperiodic or periodic. In this latter case, if k is the number of impacts per period, a periodic succession of impacts characterizes the motion; it is such that:

$$\tau_K = \sum_{i=1}^{k} \tau_i^* = 2n\pi \qquad (4.14)$$

whatever the values of the integer numbers n and k. The functions $p(\tau)$, $q(\tau)$ and $f(\tau)$ are then periodic of period $2n\pi$ and eq. (4.12) becomes a forced damped Hill's equation, that is a linear differential equation with periodic coefficients.

No analytical expressions can be found for the solutions of Hill's equation like eq. (4.12), even if criteria of existence and stability for periodic motions are provided by the well known theory of Floquet. Namely, periodic motions of period $2n\pi$ exist only inside the parametric regions of plane (K_s, ω), where the solutions of the homogeneous associated equation are stable; furthermore, given the linearity of the equation, the stability of the general forced solution is determined by the stability of the corresponding homogeneous one, so that conditions of existence and stability coincide.

As a consequence, a stability and existence analysis can be performed for each periodic motion, that is for given values of n, k and τ_i^* ($i = 1, \ldots k$), and therefore for given expressions of the periodic functions $p(\tau)$, $q(\tau)$ and $f(\tau)$; the regions where the given periodic motion exists and is orbitally stable can then be identified in the space of the parameters.

However as it is necessary to satisfy condition (4.10), the start of such a particular periodic motion, originating a given Hill's equation, requires specific initial conditions; thus, the conditions of existence and stability of a given periodic motion are satisfied in given regions of the parametric space only in correspondence to specific *a priori* unknown initial conditions.

Furthermore and unfortunately, the same existence and stability analysis should must be performed for all the periodic motions which can be obtained by varying the values of n, k and τ_i^* ($i = 1, \dots k$); but it is pratically impossible to do this. In any case, it must be expected that different periodic and orbitally stable motions, corresponding to different initial conditions, in some regions of the parametric space exist. Such a result was also obtained by Hogan (1989), who studied the stability of the periodic motion with $n = 1$, $k = 2$, $\tau_2^* = \pi$ and $\tau_2^* = 2\pi$.

The observations above allow us to state that the system, even in the simplest mechanical model of Housner, is characterized by a sort of unpredictability which does not permit meaningful results to be obtained by means of analytical investigations.

Anyhow, whatever the periodic motion may be, it is governed by a Hill's equation for which conditions of existence and orbital stability coincide. Therefore, for pratical purposes, the initial conditions corresponding to the block at rest can be considered and a systematic numerical investigation can be performed to obtain the regions of periodic and, consequently, stable motions. This analysis was carried out by Sinopoli (1991), who also discussed it in comparison with the results obtained by other authors (Augusti and Sinopoli, 1992).

These motions are not generally symmetric, and are characterized by different values of period and of number of impacts per period. Nevertheless, some systematic trends can be observed. The number of impacts per period and the periods generally increase with K_s. The maximum amplitude of oscillation decreases rapidly with ω, while it increases with K_s, for a given ω, until overturning of the block; furthermore, only symmetric subharmonic responses of the period equal to $2\pi/\omega$ are relatively numerous.

The interesting result from a physical point of view is that the block is generally characterized by very small amplitude oscillations, if excited by a harmonic motion of amplitude and frequency comparable respectively with standard peak acceleration and frequency component of the Fourier spectrum of a seismic occurrence.

However in the presence of actual seismic recorded motions, it must be expected that the systematic behaviour, identified by the deterministic analysis above, can be destroyed because the difference in the details of the seismic motion and the high sensitivity of the system to the initial conditions. The next step, therefore, should be an investigation by means of a statistical treatment of the earthquake, with a correct and well-defined mechanical model for the structure.

Regarding this last observation it must also be said that the restrictive assumption of only one degree of freedom is valid only inside well-bounded regions of the parametric space, depending on the features of the excitation. The Housner model can be assumed as correct for very thin blocks and large values of the friction coefficient, but the corresponding values for geometrical size and friction coefficient are not clearly defined.

Furthermore, as such an assumption neglects any evaluation of the value and particularly of the sign of the ground vertical reaction, which must always be greater than or equal to zero, nothing can be stated about the correctness of the Housner mechanical model, unless the hypothesis derives from the results of a general model.

4.2. *Multidegrees of freedom and mechanisms starting*

Some papers have taken into account the possibility of activation of other mechanisms, in addition to the one of simple rocking, during the dynamics of a single or multi-block structure (Ishiyama, 1982; Sinopoli, 1987; Shenton and Jones, 1991; Sinopoli and Sepe, 1993). With the question of stability in mind, we cannot ignore the fact that the system can exhibit sliding with respect to a contact surface, which represents, specially in the case of many blocks, a temporary or probably permanent alteration to the geometric form of the structure.

In order to understand the difficulty of carrying out a complete analysis, let us recall some aspects of the problem again considering a single block simply supported on a rigid ground.

Let x_G, y_G and θ be the Lagrangian coordinates, describing the degrees of freedom for plane motion of the block. There are therefore five possible mechanisms: rocking and sliding-rocking (taking into account that the contact can be maintained in either point A or B), sliding, uplift and block at rest.

The general form of the equations system for free dynamics, in the case of positive values of the angle θ, is (Sinopoli, 1987):

$$m\,\ddot{x}_G = \lambda_1 \tag{4.15}$$

$$m\,\ddot{y}_G = \lambda_2 - m\,g \tag{4.16}$$

$$I_G\,\ddot{\theta} = \lambda_1\,(b/2\,\sin\,\theta + h/2\,\cos\,\theta) + \\ + \lambda_2\,(h/2\,\sin\,\theta - b/2\,\cos\,\theta) \tag{4.17}$$

where λ_1 and λ_2 represent, respectively, the horizontal reaction due to friction and the vertical reaction due to the ground, m is the mass of the block, g the acceleration of gravity and I_G the moment of inertia with respect to the centre of the mass; further, let f_s and f_k be, respectively, the static and kinetic dry friction coefficients for the materials in contact.

The starting of a given mechanism or the transition during the motion to another, among all the possible ones, depends on the values of both the velocity of the point in contact and the reactive forces λ_1 and λ_2; and thus on the ratio b/h, on the friction coefficients f_s and f_k, and mainly on the effects of the impacts which occur every time some point of the block, characterized by a negative value of the vertical velocity, suddenly comes into contact with the ground.

When the block is in contact with the ground in a point, it can maintain such a contact (no uplift) only for admissible values of the contact reactive force, that is if the relationship $\lambda_2 \geq 0$ is verified. However the value of λ_2 is unknown; it depends on the interplay between possible accelerations and admissible velocities, and therefore on the dynamic evolution.

In order to understand the difficulty of recognizing the maintenance of or the transition from one mechanism to another, let us assume that the block starts rocking, from given initial conditions; in this case, the initial contact at point A without sliding is expressed by the constraints equations:

$$\dot{x}_A = \dot{x}_G + (b/2\,\sin\,\theta + h/2\,\cos\,\theta)\,\dot{\theta} = 0 \tag{4.18}$$

$$\dot{y}_A = \dot{y}_G + (b/2\,\sin\,\theta + h/2\,\cos\,\theta)\,\dot{\theta} = 0 \tag{4.19}$$

The motion can be a rocking governed by eqs. (4.15)-(4.19), if and until eqs. (4.18) and (4.19) are verified also in the following motion. This means that the generalized accelerations must be such that:

$$\lambda_1 \leq f_s\,\lambda_2 \tag{4.20a}$$

and:

$$\lambda_2 \geq 0 \tag{4.20b}$$

where f_s is the static friction coefficient. Assume that the constraints eqs. (4.18) and (4.19) are valid also in the subsequent motion; thus, λ_1 and λ_2 can be obtained as a function of the generalized accelerations and velocities of the block, according to the method of the Lagrange multipliers, by deriving eqs. (4.18) and (4.19) and substituting the expressions of $\ddot{x}_G$ and $\ddot{y}_G$ in the system of differential eqs. (4.15)-(4.17).

It is only after the integration of the differential equations that it is possible to verify if relations (4.20a,b) are satisfied; if they are not, other mechanisms must be considered. As an example, if sliding-rocking is assumed, then its motion is governed by eqs. (4.15)-(4.17), (4.19) and by:

$$\lambda_1 = -f_k\,\lambda_2\,\mathrm{sgn}\,(\dot{x}_A) \qquad \dot{x}_A \neq 0 \tag{4.21}$$

which substitutes the constraint eq. (4.18); f_k is in this case the kinetic friction coefficient.

The expression of the dynamic equations can be obtained by rearranging eqs. (4.15)-(4.17), (4.19) and (4.21). For positive values of angle θ, they are:

$$m\,\ddot{x}_G = -f_k\,\lambda_2\,\mathrm{sgn}\,(\dot{x}_A) \tag{4.21}$$

$$I_G\,\ddot{\theta} = -f_k\lambda_2\,\mathrm{sgn}\,(\dot{x}_A)\,(b/2\,\sin\,\theta + h/2\,\cos\,\theta) + +\lambda_2\,(h/2\,\sin\,\theta - b/2\,\cos\,\theta) \tag{4.22}$$

where:

$$\lambda_2 = m\,[g - (h/2\,\cos\,\theta + b/2\,\sin\,\theta)\,\dot{\theta}^2 - -(h/2\,\sin\,\theta - b/2\,\cos\,\theta)\,\ddot{\theta}]. \tag{4.23}$$

In the presence of a given horizontal ground acceleration $\ddot{x}_g$, eq. (4.21) must be substituted by:

$$m\,(\ddot{x}_G + \ddot{x}_g) = -f_k\,\lambda_2\,\mathrm{sgn}\,(\dot{x}_A) \tag{4.24}$$

and the forced dynamics are described by eqs. (4.22)-(4.24). Obviously, during the continuous dynamic evolution, the condition $\dot{x}_A \neq 0$ or $\dot{x}_A = 0$, with $m\,|\,\ddot{x}_G - \ddot{x}_g\,| \leq f_s\,\lambda_2$, decides, respectively, the permanence of the sliding-rocking mode or the transition to rocking mode.

A similar procedure is followed if it is assumed that the motion occurs according to other mechanisms. Therefore, the method is extremely laborious as, at each time step, the starting or maintenance of an assumed mechanism can be verified only after the dynamics equations have been integrated; in any case, only one mechanism must actually be activated, while all the other possible ones must be impossible, according to the condition of existence and uniqueness of the dynamics solution. In this respect the situation can sometimes seem unclear, as in the case of starting forced sliding-rocking outlined in Shenton and Jones (1991).

In fact, the amount of analytical calculations to evaluate the expressions of the reactive forces, particularly in the case of a multi-block structure, may be so enormous that mistakes cannot be avoided or excluded.

4.3. *Preliminary trilith dynamic analysis*

The above difficulty motivated the assumption of only five degrees of freedom to investigate the dynamic response of a trilith to a harmonic horizontal ground excitation (Sinopoli and Sepe, 1993); the model had the same geometric features of the columnade of the E3 Temple at Selinus (Sicily).

The plane analysed structure was made of three rigid blocks, namely two columns and the architrave simply supported on the columns, in presence of dry friction.

The possibility of sliding displacements and uplift of the two columns with respect to the ground support has not been taken into account in the evaluation of the degrees of freedom of the system; such an assumption, which quantitatively reduces the complexity of the problem, is acceptable from a practical point of view because relative sliding motions at the top of the columns are more probable and therefore more important than the corresponding ones at the bottom, in the failure mode due to the loss of the geometrical configuration.

The Lagrangian coordinates q_k (with k = 1, 2,...5) describing the instantaneous configuration were therefore: $q_1 = \theta_1$ and $q_2 = \theta_2$, that is the rotations of the left and right columns, and: $q_3 = x_3$, $q_4 = y_3$ and $q_5 = \theta_3$, respectively, the horizontal and vertical displacements, and the rotation of the architrave.

In the presence of a given harmonic horizontal ground motion, the dynamics are governed by a set of differential equations, the number and the form of which change depending on the started or maintained mechanisms.

For example, if all the contacts are efficient and persistent without sliding displacements and the dynamics are smooth, referring to the Lagrangian formalism, the differential equations describing the motion are:

$$d/dt\,(\partial L/\partial\,\dot{q}_k) - (\partial L/\partial q_k) = Q_k \qquad k = 1, 2, \ldots 5 \tag{4.25}$$

together with the differential kinematic relationships, expressing the constraints imposed on the normal relative velocities $\dot{u}_i^N$ of the contact points P_i:

$$\dot{u}_i^N = \sum_{k=1}^{5} a_{ik}^N \dot{q}_k = 0 \qquad i = 1, 2 \qquad (4.26)$$

and on the tangential relative velocities $\dot{u}_i^T$:

$$\dot{u}_i^T = \sum_{k=1}^{5} a_{ik}^T \dot{q}_k = 0 \qquad i = 1, 2. \qquad (4.27)$$

In eq. (4.25) L is the Lagrangian of the system, which takes the effects of ground motion and gravity into account, and Q_k is the generalized force, representing the effects on the k-th degree of freedom of the interactions between the parts of the system.

In general, the expression of Q_k depends on the kind of contact: absence or presence of relative sliding motions, or uplift. If any i-th contact is absent, the corresponding i-th constraint equation and the i-th term in the expression of Q_k disappear.

A further difficulty in such an analysis is represented by the problem of the impacts between columns and ground or lintel and columns; the problem of the impact, in presence of a finite area of contact and dry friction, is in fact still an open problem (Sinopoli, 1987; Augusti and Sinopoli, 1992, Sinopoli, 1995a) in the scientific community. Therefore, an approximate model, in which the friction performance was neglected during the impulsive motion, was used for the trilith (Sinopoli and Sepe, 1993).

Following the dynamic evolution, it was difficult to give a systematic interpretation to the results obtained. Different failure modes have been identified: overturning and failure due to the loss of geometrical configuration for either eccessive slidings or coupled effects between excessive slidings and rotation.

The purpose of the investigation was to analyze the coupling between the different degrees of freedom and its effect on the dynamic response. Let us illustrate in comparison some results concerning the response of different degrees of freedom, in order to verify the role of the coupling in the loss of geometrical configuration responsible for the failure; loss of the geometrical configuration means not only a variation of the configuration due to the dynamics, but especially an increased distance between the support points of the lintel.

The maximum rotation of either column θ_{max} (adimensionalized with respect to the angle corresponding to the unstable static equilibrium configuration) is shown in fig. 4, as a function of the angular frequency ω, for different values of the ground acceleration amplitude

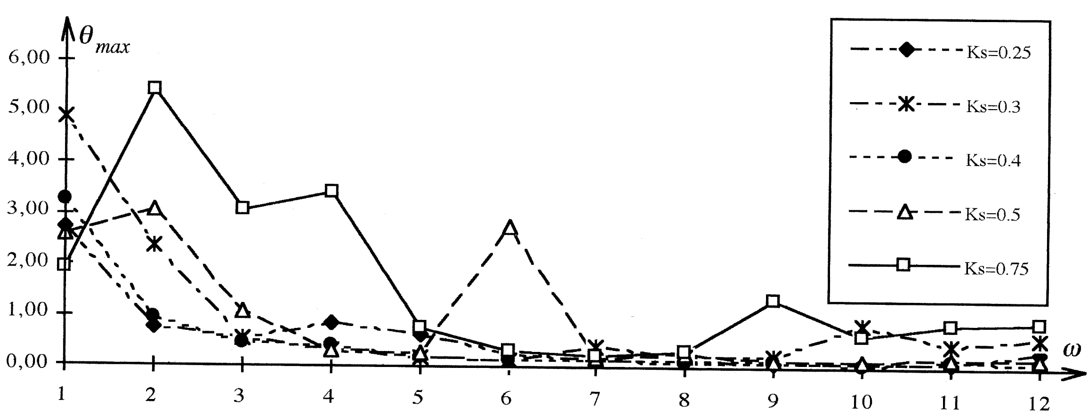

Fig. 4. Maximum non-dimensional rotation θ_{max} of either column up to 5 s.

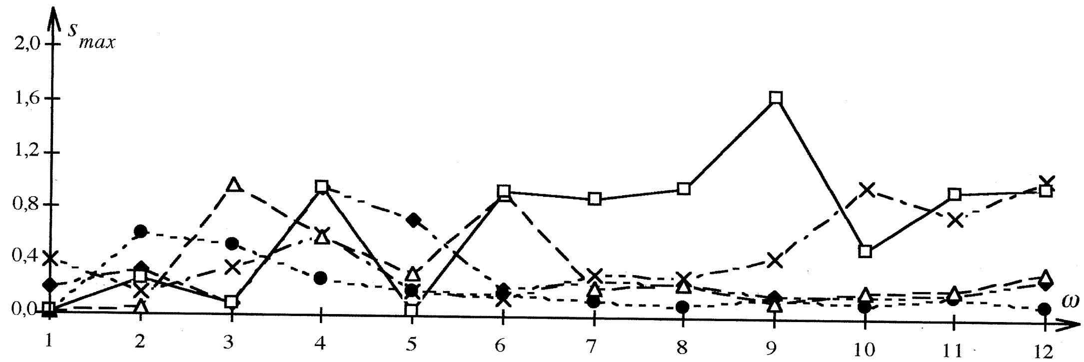

Fig. 5. Maximum relative sliding $s_{\max}$ on either contact point up to 5 s.

K_s; only the motion relative to five seconds of integration is considered in fig. 4. The maximum values of the relative slidings at the top of either column $s_{\max}$ (adimensionalized with respect to the width of the column) are shown in fig. 5.

The maximum rotation for the case $K_s = 0.4$ decreases strictly exponentially with ω: this is a typical behaviour of the simple rocking model; the same occurs only on average for the other cases: this is the first evidence of the coupling influence on the dynamics response.

By comparison of the two figures, it can be said that for small values of ω and K_s, the rotations and slidings have a dual behaviour as a function of the angular frequency; in particular, the rocking is dominant for small values of the frequency, whereas the sliding prevails in the remaining range analysed.

From a deterministic point of view, no systematic trend for the rocking can be identified by increasing the amplitude of the excitation; the maximum rotation increases with K_s only on average, inducing increasing values of the slidings, above all for large ω.

By following the dynamics evolution over a long time interval, the mean behaviour outlined above becomes more and more evident. In fact, the same variables of figs. 4 and 5 exhibit the behaviour shown in figs. 6 and 7 respectively, after a time span of 1000 s; the maximum rotation and the cumulative sliding are increased so much that it is sufficient to wait long enough for failure of the structure to occur.

It is worthwhile observing that the alteration of the geometric form of the structure induced by the relative slidings persists even if the dynamics are stopped at a given instant and the system comes back to its equilibrium configuration. Therefore, permanent relative slidings can be considered a measure of the damage induced by a given ground motion.

5. Conclusions

The results obtained from preliminary studies aimed at quantifying the stability of monumental structures, by means of investigations carried out on the effects of sinusoidal ground movements, have revealed not only a wide variety of dynamic behaviour and the need to use numerical methods, but also that, in the case of a trilithic structure, slidings between columns and architrave are the probable cause of collapse.

In particular, the action of ground motion (even under the unlikely hypothesis that it has some kind of periodicity) leads to permanent variations in the structure's geometric configuration. These reveal therefore the seismic damage suffered, the extent of which depends on the characteristics of the earthquake and the type of structure involved.

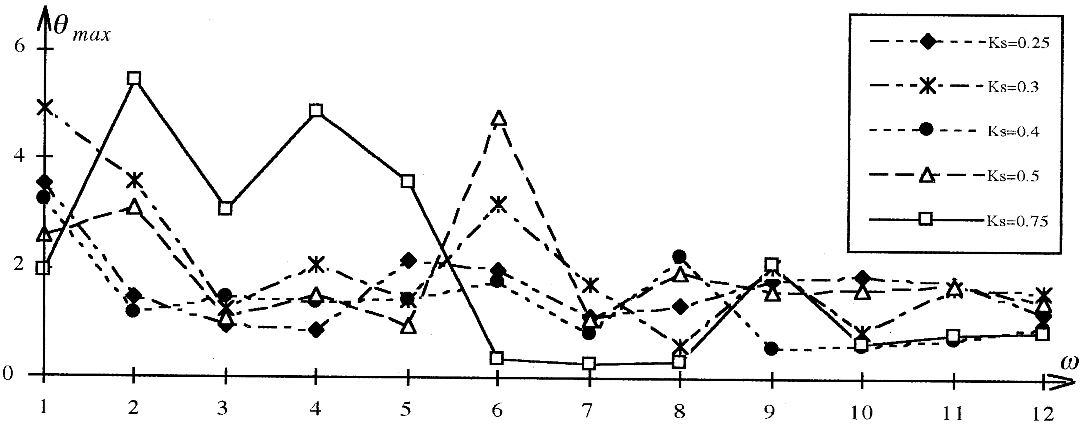

Fig. 6. Maximum rotation θ_{max} of either column up to the collapse or 1000 s.

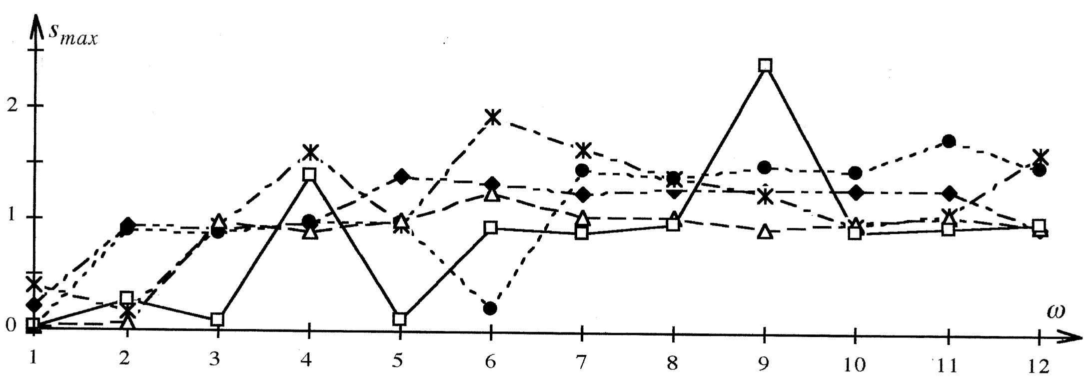

Fig. 7. Maximum non-dimensional relative displacement s_{max} on either contact point up to the collapse or 1000 s.

Bearing in mind the limitations and approximations of these preliminary investigations, one fact does emerge: after an indeterminate time span including one or more earthquakes, multi-block structures with unilateral constraints will in any case collapse. How stable a monumental structure is with respect to a seismic event therefore depends on:

a) the capacity to simulate seismic events that the structure will undergo during its life (which is not at present an object of our study);

b) the correctness of the mechanical model on which both the estimate of the amount of energy lost during the motion and the evaluation of the extent of the damage are based.

Besides, once it has been decided to use numerical methods, a new difficulty emerges, in connection with the kind of analytical calculations required to write equations of motion and with problems entailed in recognizing the only suitable mechanism among various possibilities, in order to satisfy the conditions of existence and uniqueness of the solution.

This is particularly true as far as multi-block structures are concerned; in fact, in a preliminary analysis of the dynamics of a trilith (Sinopoli and Sepe, 1993), the slipping at the foot of the columns was overlooked, and the integration was interrupted in the case of the architrave rising with regard to the columns. This limitation has stimulated further research with the aim of defining laws of general nature able to ensure a correct but simpler management of the problem. A new study has therefore recently been proposed (Sinopoli, 1995b), in which the dynamics of rigid bodies in the presence of unilateral constraints have been postulated as a problem of linear complementarity, for details of which you are referred to the bibliography.

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