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A single-station spectral model of the monthly median *F*-region critical frequency

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Abstract
A new single-station monthly median model of the *F*-region critical frequency has been obtained. A goniometric functional approximation of the monthly median hourly values of f_0F_2 was accepted. The diurnal course of f_0F_2 was described by a constant component and four «tides». Only the influence of the annual and semiannual waves in the seasonal course of f_0F_2 was taken into account. The solar activity was depicted not only by the well known R_{12} – index, but also by the parameter K_R, expressing the linear trend of the monthly mean values of the sunspots for the previous 12 months. The use of this new parameter gives an opportunity for the solar cycle investigated not to be divided into falling and rising parts. The main advantage of the presented monthly median spectral model is that the same approach could be used for the short-term prediction.

Key words *F-region critical frequency – single station model – monthly median hourly values of f_0F_2 – long-term prediction – diurnal f_0F_2 «tides»*

1. Introduction

Monthly median models of ionosphere parameters from a single ionosonde station have been described by several authors in the PRIME community (Sizun, 1991; Moraitis *et al.*, 1991; Stanisławska *et al.*, 1991; Apostolov *et al.*, 1992). The basic advantages of the single-station model are: i) more accurate results for a particular ionosonde station than the global one, and ii) the current updating of a single-station model is usually a very easy process. That is why each ionosonde station where current observations are available has to have its own single-station model.

The problem for the choice of a monthly median *F*-region critical frequency model over Sofia was closely connected with two basic requirements: i) to be applicable to other surrounding mid latitude stations and especially to the PRIME area, and ii) to use the same approach not only in the long-term prediction, but to adjust it to the meso- or even short-term prediction. The accomplishment of the first requirement enables the given single-station model to be used in the mapping procedures, or its space scales of application to be enlarged, in addition to the accomplishment of the second requirement which tends to extend the time scales for the predicted events.

2. Basic approach

The construction of the monthly median single-station model for f_0F_2 is closely related to the problem for an interpolation of time se-

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ries of measurements with an analytical function at a given criterion for the best fit. Fourier analysis provides a tool for a decomposition of the time varying monthly median hourly values of f_0F_2, at given solar activity. Four harmonics of the basic periodic component with period 24 h in the diurnal course of f_0F_2 are included, which are called in this paper f_0F_2 «tides»: diurnal (24 h), semidiurnal (12 h), terdiurnal (8 h) and quarterdiurnal (6 h). Consequently, the diurnal course of the monthly median hourly values of f_0F_2 can be presented as:

$$f_0F_2(\mathrm{LT}, \text{month}, R, K_R) = \sum_{i=0}^{4} A_i\,(\text{month}, R, K_R)$$

$$\cos\left[\frac{2\pi}{24}\, i\,(\mathrm{LT} - F_i\,(\text{month}, R, K_R))\right]$$

where A_i are the amplitudes of the «tides» (A_0 is the constant component), and F_i the tidal phases. They are obtained on the basis of a new method described by Mukhtarov and Pancheva (1993). The parameters R and K_R are characteristic of solar activity and will be clarified below. The basic characteristics of the diurnal f_0F_2 «tides» A_i and F_i are subjected to amplitude and phase modulation by the seasonal components. In this model, only the influence of annual and semiannual waves is taken into account, or the amplitudes and phases of the diurnal «tides» can be expressed as:

$$A_i(\text{month}, R, K_R) = \sum_{j=0}^{2} AA_{ij}(R, K_R)$$

$$\cos\left[\frac{2\pi}{12} j\,(\text{month} - FA_{ij}\,(R, K_R))\right]$$

$$\text{where } i = 0, 1, 2, 3, 4$$

and

$$F_i(\text{month}, R, K_R) = \sum_{j=0}^{2} AF_{ij}(R, K_R)$$

$$\cos\left[\frac{2\pi}{12} j\,(\text{month} - FF_{ij}\,(R, K_R))\right]$$

$$\text{where } i = 1, 2, 3, 4.$$

The next indications are used in the above expressions: AA_{ij} are the seasonal amplitudes of the diurnal f_0F_2 amplitudes, FA_{ij} the seasonal phases of the diurnal f_0F_2 amplitudes, AF_{ij} the seasonal amplitudes of the diurnal f_0F_2 phases, and FF_{ij} the seasonal phases of the diurnal f_0F_2 phases. All above mentioned seasonal amplitudes and phases are also subjected to amplitude and phase modulation by the solar activity. An important advantage of this model is the new description of the solar activity. It is represented by both the well-known R_{12}-index (designated in this paper as R) and the parameter K_R, describing the linear trend of the monthly mean values of the sunspots for the previous 12 months. Therefore, the following dependencies are accepted:

$$AA_{ij}\,(R, K_R) = \sum_{k=0}^{1} aa_{ijk}R^k + \sum_{k=2}^{3} aa_{ijk}K_R^{k-1}$$

$$i = 0, 1, 2, 3, 4; \quad j = 0, 1, 2$$

$$FA_{ij}\,(R, K_R) = \sum_{k=0}^{1} fa_{ijk}R^k + \sum_{k=2}^{3} fa_{ijk}K_R^{k-1}$$

$$i = 0, 1, 2, 3, 4; \quad j = 1, 2$$

$$AF_{ij}\,(R, K_R) = \sum_{k=0}^{1} af_{ijk}R^k + \sum_{k=2}^{3} af_{ijk}K_R^{k-1}$$

$$i = 1, 2, 3, 4; \quad j = 0, 1, 2$$

$$FF_{ij}\,(R, K_R) = \sum_{k=0}^{1} ff_{ijk}R^k + \sum_{k=2}^{3} ff_{ijk}K_R^{k-1}$$

$$i = 1, 2, 3, 4; \quad j = 1, 2.$$

An important peculiarity of the above expressions is the linear dependence of the seasonal components on the R_{12}-index and the second degree one on the coefficient K_R.

This combination of f_0F_2 wave components is a natural one, because it corresponds to the existing processes: the Earth's rotation around its axis and around the Sun, as well as the variation of the solar activity. The parameters of the model are determined uniquely by the condition for a minimum of the mean square deviation (Mukhtarov and Pancheva, 1993). A favourable circumstance is that the periods of the modulating fluctuations differ from one another considerably (11 years, 12 months, 1 day) and this fact allows the method of the instantaneous frequency and phase to be applied.

We would like to underline that our above described approach has two important distinctions from the similar monthly mean model, presented by Stanisławska *et al.*, (1991): i) it obeys the natural sequence of the modulating mechanisms, and ii) the solar activity is marked here by both parameters – level of solar activity and its tendency.

3. Monthly median model of f_0F_2 over Sofia

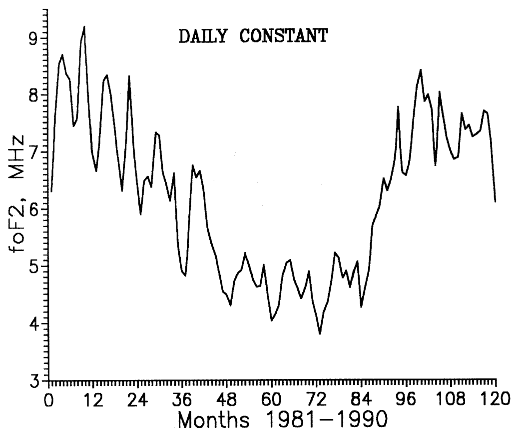
The time series of monthly median hourly values of f_0F_2 for station Sofia (42.7°N, 23.4°E) and for the time interval 1981-1990, or 10 years, was processed. The last 1991 year was used separately for an application of this model to long-term prediction. The method of the instantaneous frequency and phase allows us to assume that in the frame of one month all fluctuations with essentially longer periods (6 or 12 months, 11 years) have stationary amplitudes and phases. Figure 1 presents the courses of the studied 10 years of the constant f_0F_2 component, mean square deviation, as well as the amplitudes and phases of the four diurnal f_0F_2 «tides». All these parameters demonstrate expressed seasonal and solar cycle dependences very well. Further, the time series of the monthly values for the above mentioned diurnal parameters were interpolated by the components of the seasonal course (annual and semiannual waves). Because of the introduction of the parameter K_R in the solar activity description, the investigated solar cycle does not have to be divided into falling and rising parts. The expressions for the dependence of the seasonal components on the solar activity automatically give an account of this fact. Furthermore, on the basis of this new parameter K_R we can model and predict well the course of f_0F_2 during the years when the solar activity changes very rapidly. Figures 2a,b demonstrate the behaviour of the seasonal components during the investigated solar cycle. Their peculiarities at sharply changing solar activity are clearly visible.

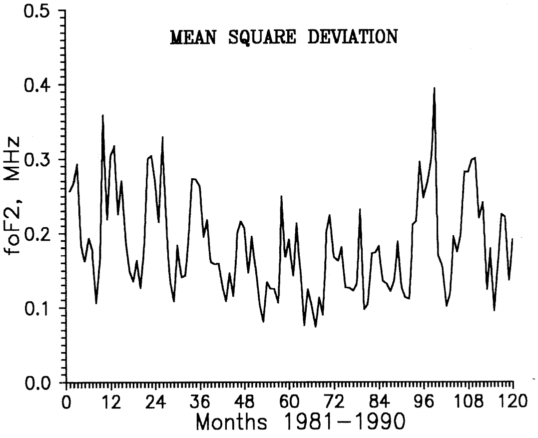
On the basis of the obtained dependencies a time series of f_0F_2 was generated for the period 1981-1991. The corresponding parameters R_{12} and K_R are used for each month of every year. Figure 3 presents all the investigated years, as the model is shown by dashed line and the measured f_0F_2 values by continuous line. At the bottom of the graph for every year the following information is also given: i) the monthly Standard Deviation (SD) (continuous line), and ii) the geomagnetic *aa*-index. It was clearly visible that in many cases the increase in the monthly SD coincides with the increase in the geomagnetic activity. The application of this method to long-term prediction is demonstrated on the last year-graph, for 1991 (this year was not included in the analysis). The result is satisfactory, bearing in mind the very high geomagnetic activity in June, October and November 1991.

Table I demonstrates the statistical assessments of the above described single-station spectral model: standard deviations, average values of percentage deviations, scatter errors and systematic errors (their mathematical expressions are given in Stanisławska and Juchnikowski, 1994).

4. Conclusions

The presented single-station spectral model of the monthly median *F*-region critical frequency describes the most important features of the diurnal, seasonal and solar cycle courses of f_0F_2, including the peculiarities of the seasonal course at very rapid changes in solar ac-


DAILY CONSTANT
foF2, MHz
3
4
5
6
7
8
9
0 12 24 36 48 60 72 84 96 108 120
Months 1981–1990


MEAN SQUARE DEVIATION
foF2, MHz
0.0
0.1
0.2
0.3
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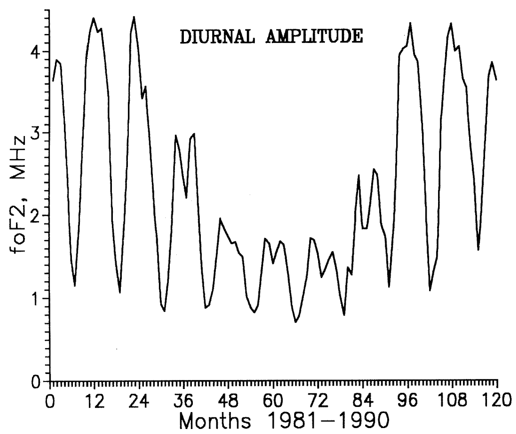
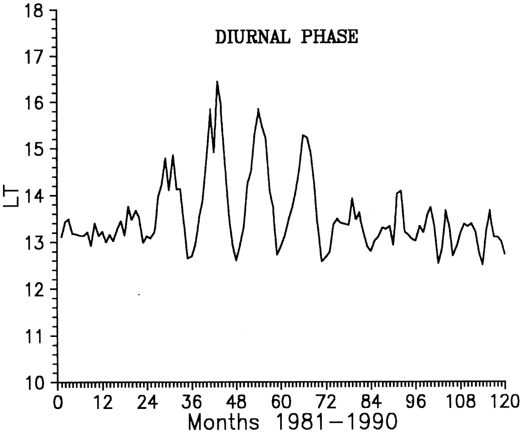
0 12 24 36 48 60 72 84 96 108 120
Months 1981–1990


DIURNAL AMPLITUDE
foF2, MHz
0
1
2
3
4
0 12 24 36 48 60 72 84 96 108 120
Months 1981–1990


DIURNAL PHASE
LT
10
11
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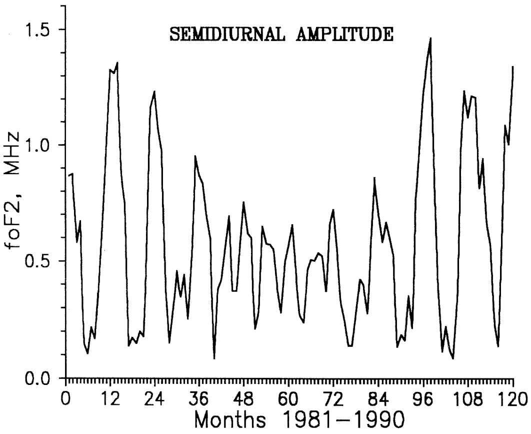
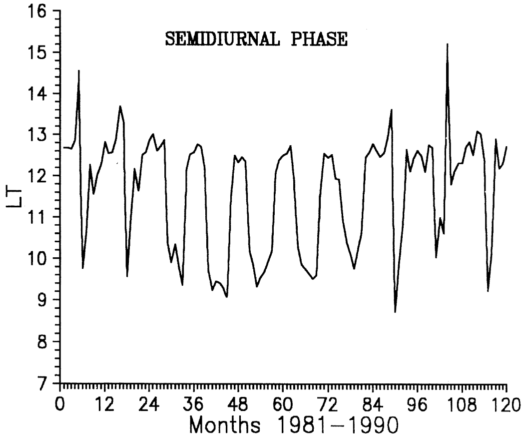
17
18
0 12 24 36 48 60 72 84 96 108 120
Months 1981–1990


SEMIDIURNAL AMPLITUDE
foF2, MHz
0.0
0.5
1.0
1.5
0 12 24 36 48 60 72 84 96 108 120
Months 1981–1990


SEMIDIURNAL PHASE
LT
7
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0 12 24 36 48 60 72 84 96 108 120
Months 1981–1990

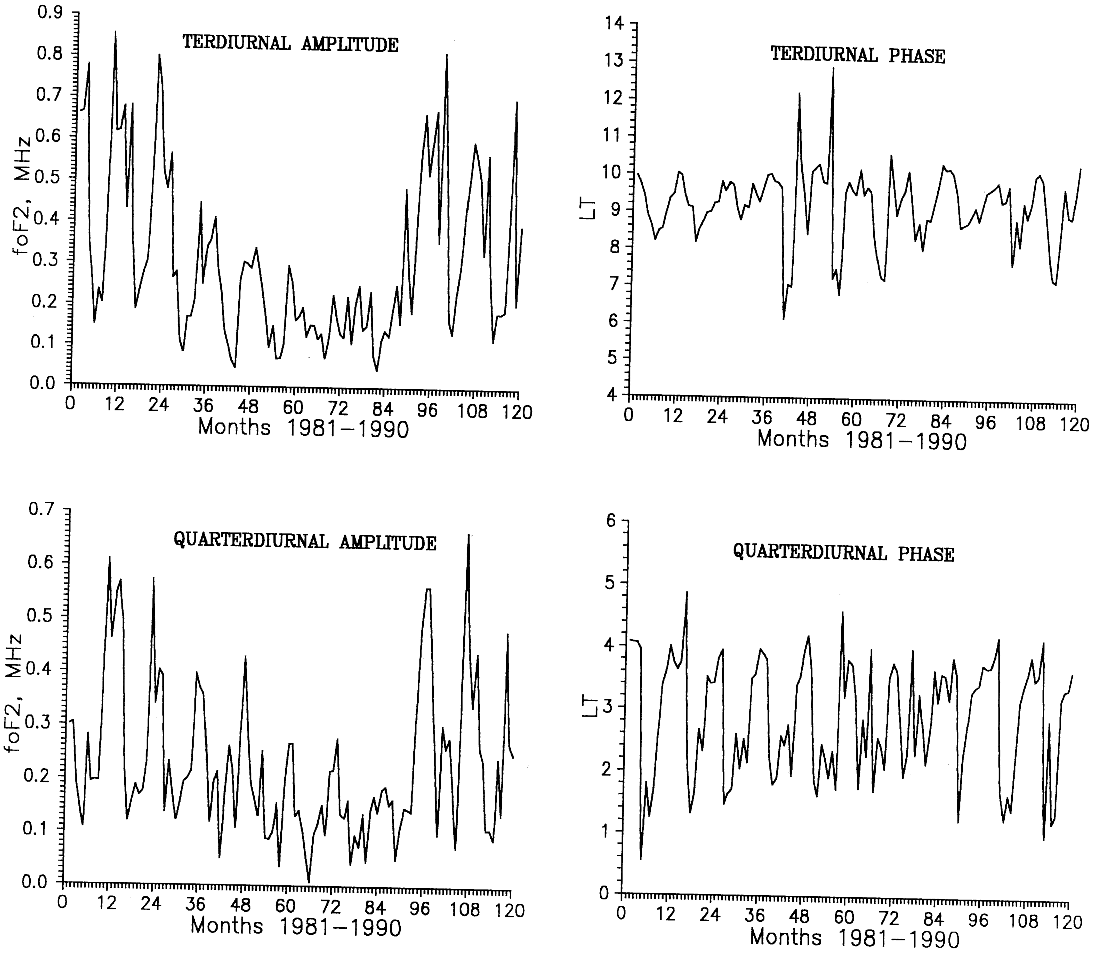


Fig. 1. The variations of the f_0F_2 daily constant component, the mean square deviations, as well as the amplitudes and phases of the four f_0F_2 «tides» (diurnal, semidiurnal, terdiurnal and quarterdiurnal) for the ten years studied, 1981-1990.

tivity. The input parameters for the model are: hour, month and the respective R_{12}. The results produced in fig. 3 show that the accuracy of this monthly median model is satisfactory. The next step in improving the model should be the inclusion of the geomagnetic activity!

The obtained regular courses of the investigated diurnal components of f_0F_2, performed in fig. 1, afford an opportunity to correct the monthly mean model on the basis of an extrapolation accomplished on the deviations of the model parameters observed during the last 12 months. Some attempts have been made recently to model the daily course of f_0F_2 over Sofia. The preliminary results indicated that this approach is applicable to shorter periodical variations of solar (for example 27-day variations) or inter-atmospheric origin.

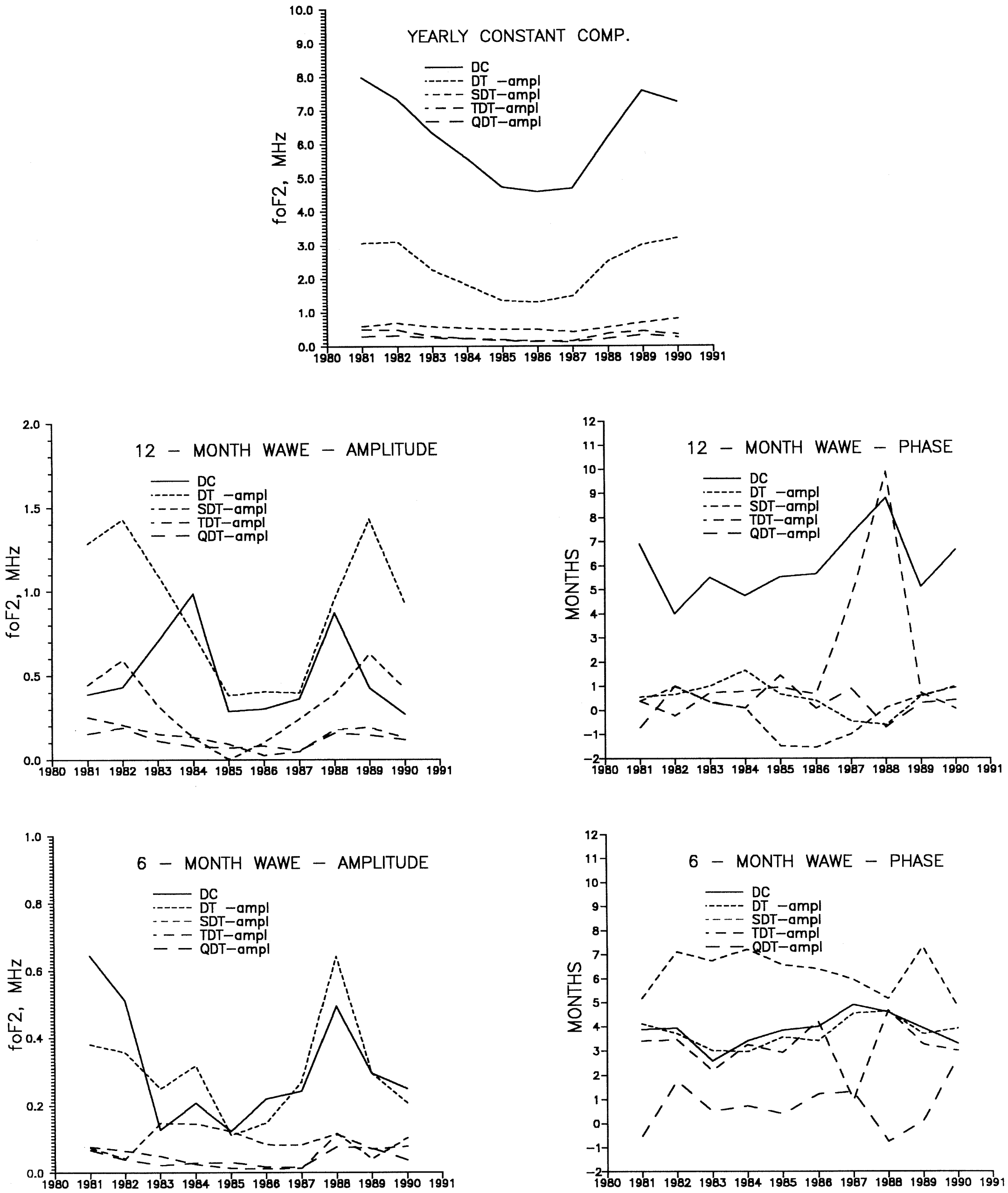


Fig. 2a. The variations of the yearly constant components, as well as the amplitudes and phases of the annual (12-month) and semiannual (6-month) waves attended in the Diurnal Constant Component (DC) and in the amplitudes of the four f_0F_2 «tides» (DT-Diurnal Tide, SDT-Semidiurnal, TDT-Terdiurnal and QDT-Quarter-diurnal) for the investigated 1981-1990 period.

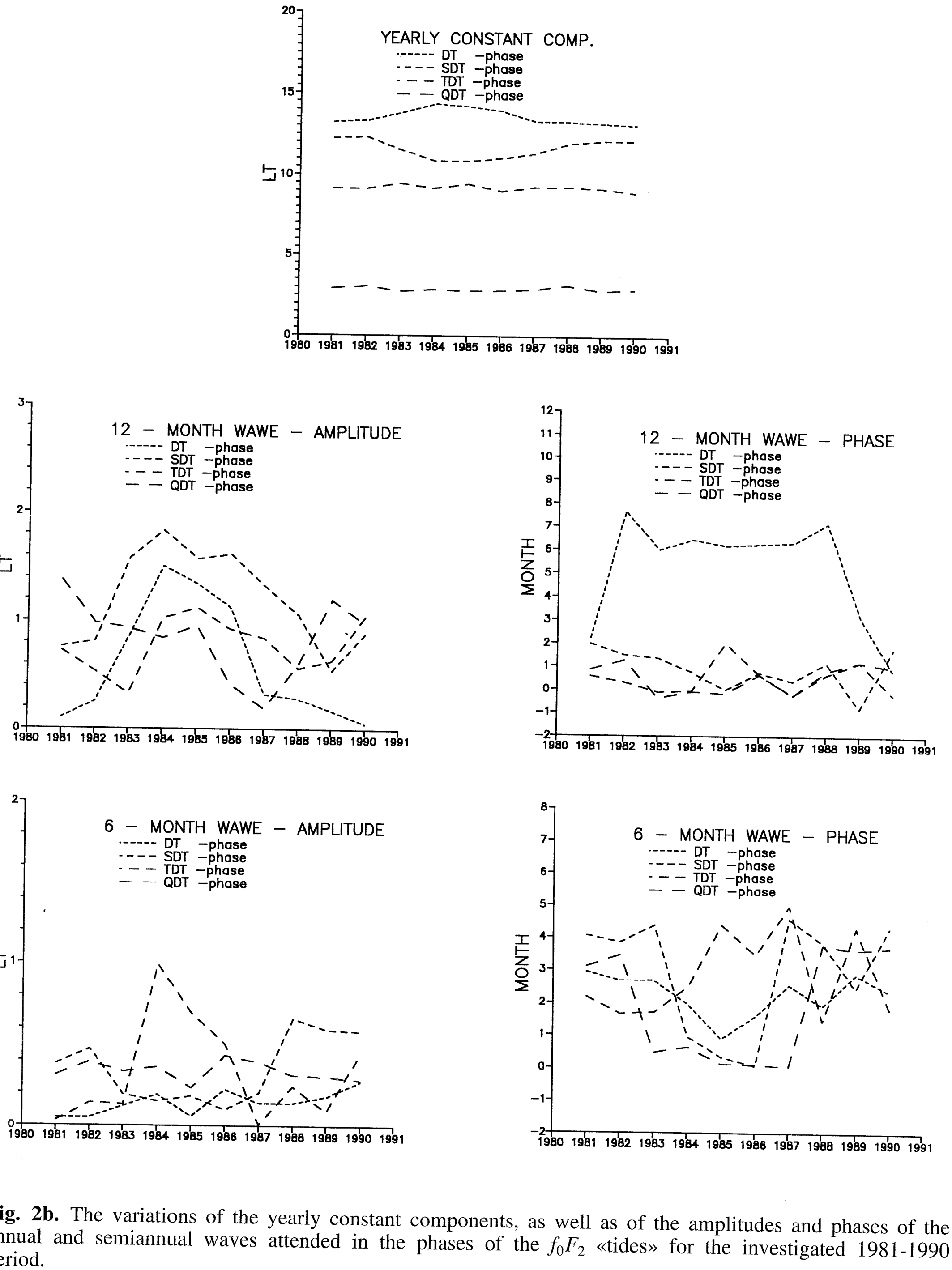


Fig. 2b. The variations of the yearly constant components, as well as of the amplitudes and phases of the annual and semiannual waves attended in the phases of the f_0F_2 «tides» for the investigated 1981-1990 period.

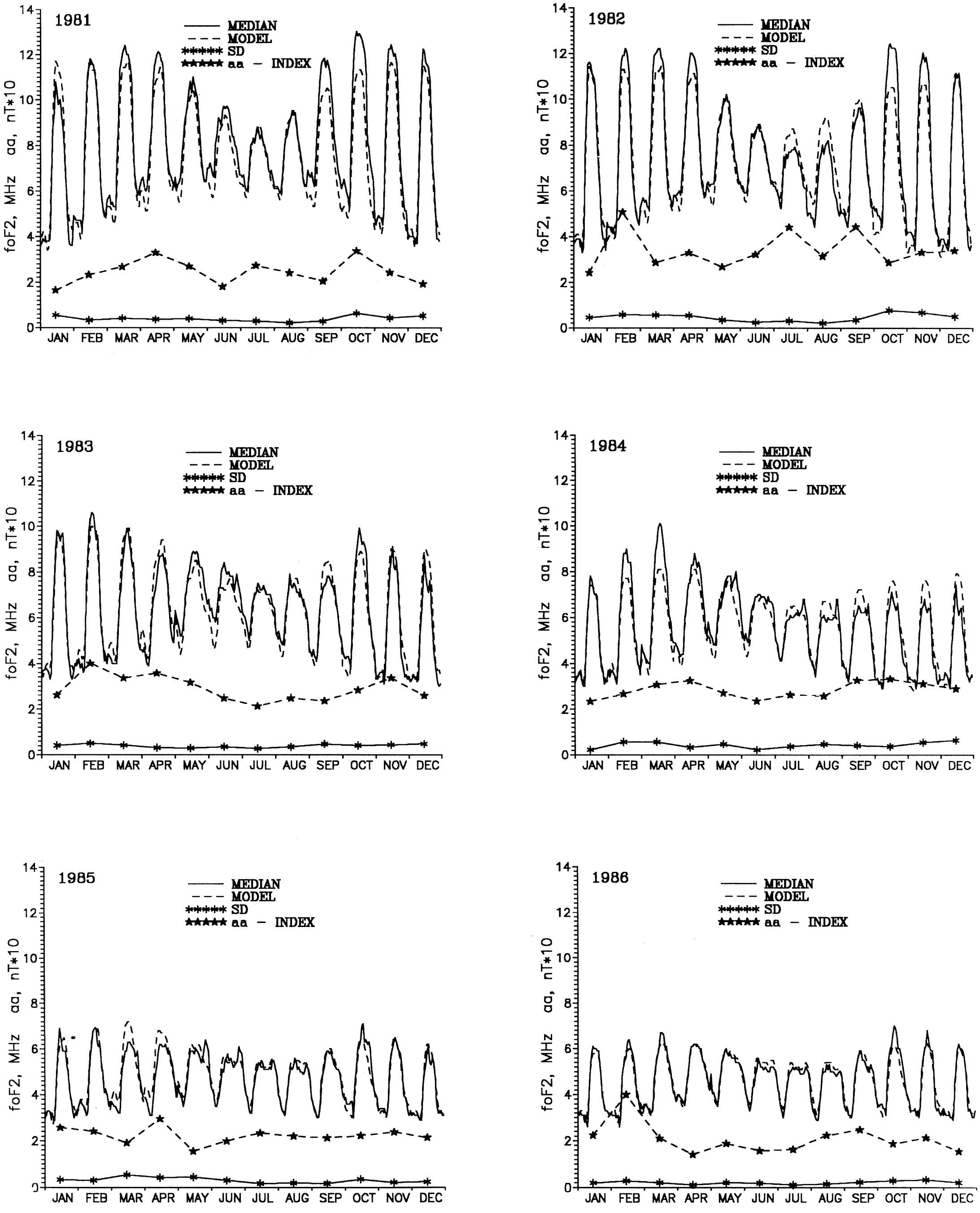
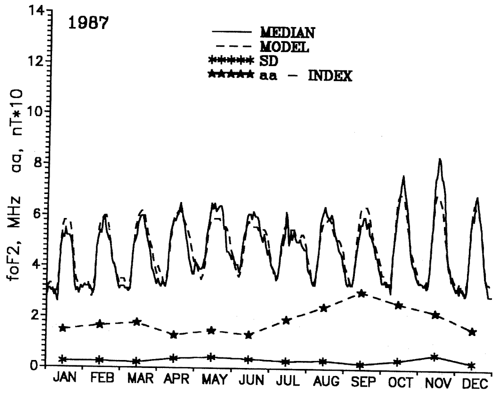
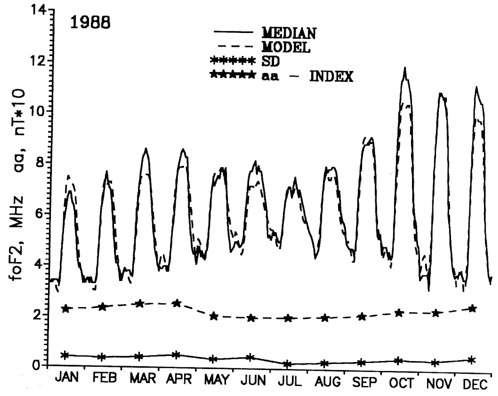
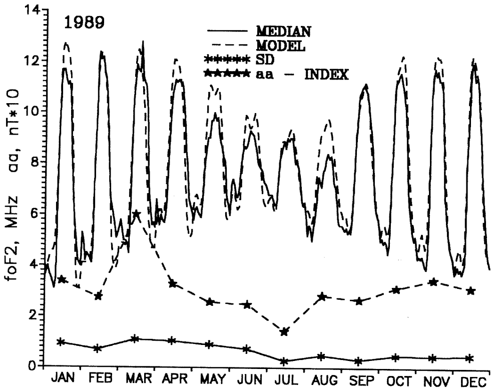
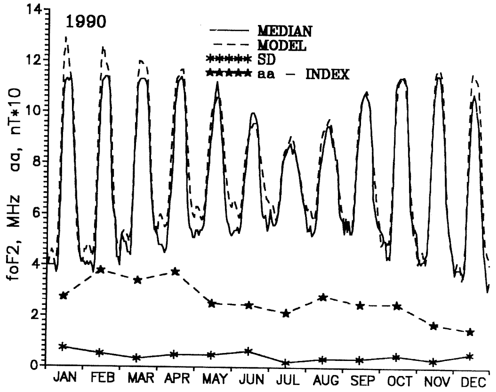


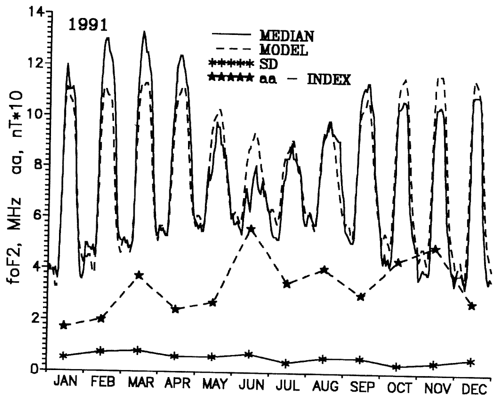
Fig. 3. The generated monthly median hourly values of f_0F_2 on the basis of the model (dashed line), compared with the measurements in Sofia station (continuous line) for the interval 1981-1991, as the year 1991 is











an example of a pure long-term prediction. At the bottom of the figure for each year the following information is also presented: Standard Deviations SD (continuous line), and the geomagnetic *aa*-index (dashed line).

Table I. Statistical assessments of the model, given for each month of every investigated year: standard deviations; average values of percentage deviations; scatter errors and systematic errors.

Month	Standard dev. MHz	Average %	Scatter MHz	Systematic error MHz	Standard dev. MHz	Average %	Scatter MHz	Systematic error MHz
			1981				**1982**	
1	0.543	11.0	0.824	−0.607	0.446	7.4	0.464	−0.125
2	0.325	4.7	0.430	0.276	0.573	5.3	0.612	0.210
3	0.407	8.4	0.810	0.686	0.555	8.1	0.784	0.541
4	0.356	7.5	0.716	0.608	0.532	6.2	0.658	0.379
5	0.389	5.7	0.606	0.455	0.344	5.0	0.447	0.280
6	0.296	9.2	0.812	0.740	0.235	2.6	0.247	0.075
7	0.280	3.5	0.309	0.129	0.306	6.8	0.568	−0.468
8	0.202	2.5	0.218	0.079	0.202	17.4	1.102	−1.060
9	0.275	14.0	1.268	1.212	0.345	4.8	0.420	−0.235
10	0.625	16.9	1.690	1.538	0.766	16.1	1.563	1.334
11	0.416	8.1	0.775	0.640	0.674	11.6	0.980	0.697
12	0.515	6.9	0.527	0.111	0.495	8.1	0.495	0.020
			1983				**1984**	
1	0.413	6.9	0.430	0.119	0.226	4.2	0.226	−0.002
2	0.504	7.2	0.505	0.017	0.563	8.7	0.696	0.400
3	0.421	6.7	0.521	−0.300	0.574	11.8	1.069	0.883
4	0.314	9.4	0.655	−0.563	0.334	5.4	0.482	0.340
5	0.296	8.1	0.662	0.580	0.475	8.8	0.701	0.505
6	0.355	10.7	0.849	0.755	0.229	6.6	0.467	0.399
7	0.271	5.1	0.395	0.281	0.368	4.9	0.371	−0.048
8	0.348	4.9	0.357	0.079	0.476	7.6	0.533	−0.235
9	0.476	6.1	0.497	−0.141	0.416	6.8	0.504	−0.279
10	0.409	10.1	0.768	0.636	0.367	8.3	0.533	−0.379
11	0.440	10.0	0.549	−0.322	0.545	13.0	0.684	−0.404
12	0.481	11.2	0.743	−0.553	0.642	12.5	0.767	−0.411
			1985				**1986**	
1	0.333	5.9	0.333	−0.010	0.194	3.7	0.196	0.029
2	0.297	4.8	0.312	−0.094	0.287	6.2	0.288	−0.031
3	0.540	11.6	0.662	−0.375	0.211	3.5	0.239	0.109
4	0.423	12.7	0.653	−0.487	0.118	2.2	0.133	0.060
5	0.439	6.7	0.446	−0.077	0.205	3.5	0.207	0.030
6	0.299	4.7	0.303	−0.050	0.180	4.6	0.252	−0.173
7	0.174	3.5	0.199	−0.095	0.112	4.0	0.205	−0.168
8	0.184	3.9	0.237	−0.145	0.152	6.3	0.295	−0.248
9	0.162	2.7	0.183	−0.083	0.230	4.1	0.230	−0.006
10	0.350	9.2	0.556	0.423	0.289	8.5	0.510	0.411
11	0.216	4.2	0.222	0.050	0.326	6.0	0.330	0.050
12	0.238	5.1	0.289	−0.161	0.210	4.4	0.210	−0.003

Table I (*continued*).

Month	Standard dev. MHz	Average %	Scatter MHz	Systematic error MHz	Standard dev. MHz	Average %	Scatter MHz	Systematic error MHz
			1987				**1988**	
1	0.247	8.5	0.388	−0.293	0.401	7.3	0.481	−0.259
2	0.240	5.1	0.282	−0.145	0.352	5.7	0.381	−0.143
3	0.209	7.4	0.362	−0.290	0.398	5.7	0.450	0.205
4	0.358	8.0	0.432	−0.237	0.490	7.6	0.512	−0.145
5	0.420	7.5	0.450	0.159	0.350	5.8	0.415	−0.219
6	0.352	6.3	0.377	0.134	0.447	7.5	0.562	0.334
7	0.280	4.9	0.309	−0.130	0.212	3.4	0.268	0.160
8	0.315	5.2	0.316	0.022	0.259	3.8	0.324	0.190
9	0.212	7.3	0.412	−0.345	0.327	4.3	0.341	0.096
10	0.348	6.0	0.350	−0.035	0.427	8.3	0.810	0.674
11	0.581	7.4	0.693	0.370	0.372	7.1	0.448	−0.245
12	0.259	5.5	0.279	−0.102	0.512	7.8	0.694	0.459
			1989				**1990**	
1	0.943	17.2	1.050	−0.451	0.740	10.1	0.881	−0.469
2	0.702	10.9	0.856	0.480	0.521	13.0	0.931	−0.755
3	1.097	12.0	1.195	0.464	0.335	6.2	0.545	−0.421
4	1.042	10.8	1.049	0.114	0.492	15.1	1.068	−0.928
5	0.907	10.7	1.055	−0.528	0.484	11.3	0.836	−0.667
6	0.743	8.1	0.748	−0.080	0.655	11.5	0.848	−0.527
7	0.269	2.9	0.270	−0.017	0.209	3.6	0.319	−0.236
8	0.492	14.2	1.116	−0.980	0.348	7.3	0.549	−0.416
9	0.316	3.3	0.316	−0.006	0.350	4.9	0.451	−0.278
10	0.481	8.6	0.730	−0.537	0.489	6.4	0.496	−0.081
11	0.473	9.4	0.701	−0.507	0.317	4.0	0.370	−0.186
12	0.487	7.6	0.532	−0.210	0.573	14.4	1.011	−0.815
			1991					
1	0.528	7.7	0.537	0.097				
2	0.752	13.9	1.465	1.231				
3	0.801	8.5	1.124	0.771				
4	0.601	7.5	0.934	0.700				
5	0.602	7.0	0.707	−0.363				
6	0.736	12.7	1.149	−0.863				
7	0.398	9.1	0.656	−0.510				
8	0.591	5.3	0.684	0.338				
9	0.602	6.4	0.657	0.256				
10	0.350	14.0	0.965	−0.880				
11	0.443	18.9	1.214	−1.106				
12	0.595	13.0	0.906	−0.669				

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