Physical modelling of lava flows

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Abstract
Lava flows are not only a fascinating scientific problem, involving many branches of continuum mechanics and thermodynamics, but are natural events having a strong social impact. A reliable evaluation of volcanic hazard connected with lava flows depends on the availability of physical models allowing us to predict the evolution of these phenomena. In this regard, the rheological properties of lavas are of major importance in controlling the dynamics of lava flows. Lava is a multi-phase and chemically heterogeneous system. This entails a characteristic, non-Newtonian behaviour of lava flows, which is emphasized by the fact that the rheological parameters are strongly temperature dependent and are therefore affected by the progressive cooling of lava after effusion. Physical models of lava flows show us the complex relationships between the many quantities governing this process and in the near future they may allow us to predict the dynamics of lava flows and to take effective measures for the reduction of volcanic risk.

Key words *lava flows – rheology – non-Newtonian fluids – volcanic hazard*

1. Introduction

Aetnaeos apices solo cognoscere visu, / non aditu temptare licet. Pars cetera frondet / arboribus, teritur nullo cultore cacumen. So admonished the Latin poet Claudianus, with reference to the volcanic hazard of Mt. Etna and the difficulty of land use for cultivation or pasture. Apart from seismic activity, the major threat to human life and property is brought by lava flows. Even if flows are relatively slow, they may be fed long enough to threaten towns and property by direct invasion and engulfment. Phenomena such as the formation of lava tubes and the opening of secondary vents allow the flows to cover much longer distances than it would be possible otherwise, thus increasing volcanic hazard. In the proximity of a lava flow, most danger derives from radiant heat due to high lava temperatures, from degassing of lava and from transient processes, such as channel overflows, levée instability and avalanching of surface debris (Kilburn and Guest, 1993).

Since lava flows may have a remarkable social and economic impact, they have recently stimulated a strong demand for mitigation of the connected risk. This need has to face with the fact that, up to recent times, lava flows were mainly studied from the morphological and petrological points of view, and much less from the physical one. This has implied on one hand a scarcity of data on the physical quantities characterizing moving lava flows; on the other, the absence of theoretical models allowing predictions of the dynamics of lava flows and a more effective defence against them. Up to ten years ago, the scientific literature concerning lava flow modelling was limited to very few contributions (Johnson, 1970; Daneš, 1972; Hulme, 1974, 1982; Huppert, 1982; Park and Iversen, 1984; Dragoni *et al.*, 1986; Pieri and Baloga, 1986).

Lava is a multiphase and chemically heterogeneous system, which behaves as a non-New-

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tonian fluid and, during its effusion, is subject to a cooling process which continuously changes its physical properties. The most important factors controlling the area likely to be covered by a given flow are topography, lava rheology and total erupted volume. Topography can be known in advance, while rheology requires *in situ* measurements during the eruption. As to the total erupted volume, it can be by no means predicted at present, due to our relative ignorance of the feeding mechanisms of volcanoes. Moreover, estimating how fast the threatened area will be inundated requires information about the effusion rate. In spite of these limits, it would be a great achievement if we could predict the evolution of a lava flow assuming a constant effusion rate and evaluate the connected risk as a function of the duration of eruption. This requires the solution of the appropriate thermal, rheological and dynamical equations and is the aim of physical modelling. A good knowledge of the behaviour of lava is also crucial when attempts to halt or divert a flow are necessary for civil defence purposes (*e.g.*, Barberi *et al.*, 1992).

2. Rheology of lava

The rheological properties of lava are of major importance in determining the dynamics of lava flows. Lava flows show great variations in size, shape and surface features, but in all cases they have a characteristic behaviour which is a consequence of the rheological properties of lava at the high temperatures at which effusion takes place. It is observed that lava flows construct their own levées and come to rest on a slope when the supply of fresh lava ceases. Flow fronts are often high and steep, although unconfined by topographic features. In spite of the fluid-like aspect of flowing lava, objects thrown onto the flow surface do not always sink nor remain buoyant, even if their density is greater than the density of lava, but can be supported as if they were on a rigid surface. This behaviour cannot be ascribed to solidification of lava due to cooling, which can limit the motion of the flow front to a certain distance from the effusion vent, but cannot prevent either lateral or downhill movement upstream.

The transition from liquid to solid lava occurs within a temperature interval delimited by the *liquidus* and the *solidus* temperatures, which depend on the lava's chemical composition. Since lavas erupt at temperatures close to the liquidus and cool during emplacement, the temperatures of active flows usually lie in the range between liquidus and solidus. Laboratory experiments show that igneous melts behave as Newtonian fluids above their liquidus temperature. Below the liquidus, lavas are instead non-Newtonian (Shaw *et al.*, 1968; Pinkerton and Sparks, 1978; McBirney and Murase, 1984; Dingwell *et al.*, 1993). Many complex fluids, such as suspensions and emulsions, are non-Newtonian. The reason for this change in lava behaviour below the liquidus is the presence of dispersed crystals and gas bubbles, as well as some polymerization in the silicate melt. The non-Newtonian behaviour has many implications both on flow dynamics and morphology.

3. Constitutive equations

A constitutive equation is an equation relating stress and its time derivatives with strain and its time derivatives. For a viscous fluid, the constitutive equation is a relation between viscous stress σ and strain rate $\dot{e}$:

$$\sigma_{ij} = f_{ij}(\dot{e}) \tag{3.1}$$

where f_{ij} denotes a generic tensorial function. If the components of σ are linear functions of the components of $\dot{e}$, (3.1) can be written as

$$\sigma_{ij} = V_{ijk\ell}\, \dot{e}_{k\ell} \tag{3.2}$$

where $V_{ijk\ell}$ is the viscosity tensor: in this case the fluid is called Newtonian. If the fluid is isotropic and incompressible, $V_{ijk\ell}$ reduces to a single coefficient, the viscosity η, and (3.2) can be written as

$$\sigma_{ij} = 2\eta\, \dot{e}_{ij}. \tag{3.3}$$

If f_{ij} is instead a generic function, the fluid is called non-Newtonian and a unique viscosity cannot be defined (Skelland, 1967; Böhme, 1987). If a viscosity estimate is made on a non-Newtonian fluid, as if it were Newtonian, an apparent viscosity is obtained, depending on the strain rate involved. Many non-Newtonian fluids can be described by a constitutive equation of the form

$$\sigma = \tau + 2\eta\, \dot{e}^{n} \tag{3.4}$$

where σ denotes any component of viscous stress and $\dot{e}$ the corresponding component of strain rate, while τ, η and n are constants. If $\tau = 0$ and $n < 1$, the fluid is called *pseudoplastic*. The majority of non-Newtonian fluids are pseudoplastic. In a pseudoplastic fluid, the apparent viscosity decreases with increasing strain rate. The opposite behaviour is found in *dilatant* fluids, where the apparent viscosity increases with increasing strain rate. These fluids can be again described by (3.4) with $\tau = 0$ and $n > 1$. A further possibility is that a fluid has a yield stress, *i.e.* it is deformed only if a minimum shear stress is exceeded. Such a behaviour is described by (3.4) with $\tau \neq 0$. The existence of a yield stress can be ascribed to an internal structure which is capable of preventing movement for values of shear stress less than the yield value. Above this value, the internal structure collapses, allowing shearing movement to occur. There are many examples of fluids with a yield stress: sand in water, oil well drilling muds, coal, cement, margarine, grease, toothpaste, soap slurries and others.

The simplest fluid with a yield stress is the *Bingham fluid*, which is characterized by two parameters, a yield stress τ and a (plastic) viscosity η. Its constitutive equation is given by (3.4) with $\tau \neq 0$ and $n = 1$. The Bingham fluid can be considered as an approximation to a pseudoplastic fluid characterized by a high viscosity at very small shear rates. Robson (1967) first proposed that lava has an approximately Bingham rheological behaviour, in order to explain a relation found by Walker (1967) between flow thickness and ground slope among Etnean lavas. Since lavas are subject to a cooling process after effusion, the fact that viscosity and yield stress are strongly temperature dependent has a remarkable effect on lava rheology and dynamics. The constitutive equation of the Bingham fluid can be better written as

$$\dot{e} = \frac{1}{2\eta} \begin{cases} 0, & |\sigma| \leq \tau \\ \sigma - \tau, & |\sigma| > \tau. \end{cases} \tag{3.5}$$

A consequence of (3.5) is that regions of the fluid may exist where the maximum shear stress σ_{max} is smaller than τ and no deformation takes place. These regions are called *the plug* and are defined by the condition

$$\sigma_{max} \leq \tau. \tag{3.6}$$

Since τ increases with decreasing temperature, the size of the plug increases as the flow advances. If this region is in contact with the ground, it cannot move and forms the levées of the flow. If instead it is surrounded by flowing lava, the plug is carried passively by the lava, as if it were solid. The plug may become a significant fraction of the flow thickness at distal parts of the flow and may finally completely stop the flow itself.

The assumption of Bingham rheology has proven to be useful for the interpretation of field observations and has been extensively used in flow modelling. The great advantage of using the Bingham fluid in modelling lava flows is that the stress-strain rate relation is linear when shear stress is greater than τ. Therefore the equations of motion for a Newtonian fluid can still be used, with a great simplification of mathematics.

4. Equations of motion

Let us consider a viscous liquid in the gravity field. The equation of motion can be written as (*e.g.*, Batchelor, 1967; Landau and Lifšits, 1971)

$$\rho\,(\dot{v}_i + v_j\, v_{i,j}) = -p_{,i} + \sigma_{ij,j} + \rho g_i \tag{4.1}$$

where ρ is the density, v_i is the velocity, p is the pressure, g_i is the acceleration of gravity and σ_{ij} is the viscous stress. The dot indicates a partial derivative with respect to time. Some simplifying assumptions can be introduced in (4.1). If the liquid is Newtonian, isotropic and incompressible, the viscous stress is given by (3.3), where the strain rate can be expressed in terms of velocity as

$$\dot{e}_{ij} = \frac{1}{2}(v_{i,j} + v_{j,i}). \tag{4.2}$$

Under the assumption of homogeneity and incompressibility (ρ = const), the continuity equation reads

$$v_{j,j} = 0. \tag{4.3}$$

Introducing (3.3) into (4.1) and using (4.2) and (4.3), one obtains

$$\rho(\dot{v}_i + v_j v_{i,j}) = = -p_{,i} + \eta_{,j}(v_{i,j} + v_{j,i}) + \eta v_{i,jj} + \rho g_i. \tag{4.4}$$

If the liquid is homogeneous and isothermal, viscosity is uniform (η = const) and (4.4) reduces to the well-known Navier-Stokes equation

$$\rho(\dot{v}_i + v_j v_{i,j}) = -p_{,i} + \eta v_{i,jj} + \rho g_i. \tag{4.5}$$

5. Heat equations

Thermal processes have a primary role in the dynamics of lava flows. From the time when lava effusion starts, a complex thermal interaction begins with the environment, producing the gradual cooling of lava. Thermal exchange processes include conduction toward the ground and the atmosphere, radiation into the atmosphere and convection in the atmosphere above the flow. Heat is produced in the flow as a result of viscous dissipation and latent heat of crystallization. Thermal and rheological boundary layers are formed progressively and thermically insulate an inner core of fluid lava, slowing down the cooling process. The temperature distribution in the lava has a fundamental effect on the rheology of lava which, in turn, has an effect on the dynamics, determining the duration and length of a lava flow.

If temperature is not uniform, viscosity and yield stress cannot be considered uniform either, because they are strongly temperature dependent. If viscosity is not uniform, the equation of motion is no longer the Navier-Stokes equation, but the more general equation (4.4) including the viscosity gradient $\eta_{,j}$. If the liquid is incompressible, the equation governing heat transfer is the following:

$$\rho c_p(\dot{T} + v_i T_{,i}) = -q_{i,i} + \sigma_{ij} v_{i,j} + H \tag{5.1}$$

where c_p is the specific heat at constant pressure, T is the absolute temperature, q_i is the heat flow density and H is a volumetric heat production. The terms on the left hand side of (5.1) are the total derivative of temperature, multiplied by ρc_p, while the first and the second term on the right hand side are respectively the heat flow and the viscous dissipation per unit volume. The heat production indicated by H includes the thermal effect of progressive crystallization in the liquid phase:

$$H = \rho L \dot{\phi} \tag{5.2}$$

where L is the latent heat of solidification per unit mass and ϕ is the crystallization degree. The heat flow due to conduction is given by Fourier's law

$$q_i = -\kappa T_{,i} \tag{5.3}$$

where κ is the thermal conductivity, while radiation involves a heat flow normal to the free surface of the lava flow, which has a magnitude given by Stefan's law:

$$q = \varepsilon s T^4 \tag{5.4}$$

where ε is the emissivity of lava and s is the Stefan's constant. In a lava flow, the dominant process of heat exchange is radiation into the atmosphere, owing to the dependence on T^4, at least as long as T is sufficiently high. Heat production by viscous dissipation and release of latent heat are usually negligible. Under these assumptions, if one considers a steady-state motion ($\dot{T} = 0$), (5.1) reduces to

$$\rho c_p v_i T_{,i} = -q_{i,i}. \qquad (5.5)$$

Solutions of (5.5) can be found for a thermally homogeneous or stratified liquid layer flowing dowslope (Park and Iversen, 1984; Pieri and Baloga, 1986, Crisp and Baloga, 1990; Dragoni and Tallarico, 1994).

6. Model assumptions

An objective of models is to disentangle the close net of relations linking the many physical quantities controlling the dynamics of lava flows. Considering the complexity of the problem, single aspects of lava flows have been studied separately, allowing simplifying assumptions to be introduced in the models. The evolution of a typical lava flow on Etna can be divided into the following stages. Lava erupting from a fissure initially spreads widely as a thin sheet and soon develops marginal levées, concentrating into a channel. The channel allows lava to flow for progressively greater distances from the vent and forms most of the flow extent. Distinct levée structures disappear in the frontal zone, where a relatively thick and cool crust encloses a hot core of fluid lava.

Let us consider a flow model made of a Bingham liquid flowing on a solid surface in the gravity field and introduce a rectangular coordinate system with the x-axis directed downslope and the z-axis directed upward perpendicularly to the surface. A series of common assumptions are now examined.

1) *Newtonian, isotropic, incompressible liquid* – The liquid can be assumed as Newtonian, in view of using the equation of motion (4.5) in connection with Bingham rheology. Isotropy is a common assumption for most liquids. Incompressibility is a reasonable approximation, unless great thicknesses of lava are considered, as in lava domes, in which case lava may show a small effective compressibility due to the presence of gas bubbles (Jaupart, 1991).

2) *Homogeneous liquid* – Liquid lava contains a variable quantity of solid crystals and gas bubbles. This complication is usually neglected as concerns its effect on density, viscosity and thermal conductivity, for which average values are considered.

3) *Isothermal liquid* – In a homogeneous fluid, a viscosity gradient $\eta_{,j}$ may derive from a nonuniform temperature distribution. However, the temperature gradient along a lava flow is small: once the crust has formed, the heat loss is minimal owing to the insulating effect of the crust itself. Therefore, the change in viscosity $\eta_{,x}$ is also slow. The vertical viscosity gradient $\eta_{,z}$ may reach significant values within the flow, but they are mainly confined to the thermal boundary layer which develops at the surface of the flow, where the temperature gradient is highest; deeper in the flow, temperature can be considered constant to a good approximation (*e.g.*, Archambault and Tanguy, 1976). Since the thermal boundary layer is always much thinner than the velocity boundary layer (the plug), the highest values of $\eta_{,z}$ take place within the plug: therefore, neglecting $\eta_{,z}$ does not have a large effect on flow dynamics. Isothermal models of lava flows may be a reasonable approximation in describing a limited segment of the flow, where the temperature of the inner flowing lava can be considered uniform. This approximation can be also employed for locally isothermal models, where a slow temperature variation is allowed along the flow under the assumption of thermal equilibrium (Dragoni, 1989). In this case one can neglect $\eta_{,x}$, but not the dependence of η on x through temperature.

Under assumptions 1 to 3, the equation of motion is the Navier-Stokes equation (4.5).

4) *Channel flow* – Flow is assumed to occur between solid levées. This assumption can be applied when considering flow well behind the front, where levées are cooler and have a higher yield stress than fresh lava flowing between them. Of course levée formation and processes at the flow front cannot be considered.

5) *Steady-state motion* – This assumption, implying $\dot{v}_i$ in the equation of motion, excludes the description of any transient phenomena, due to changes in conditions at the eruption vent or along the flow.

6) *Small Reynolds number* – Since lavas are very viscous liquids and flow velocities are relatively small, in many cases the condition $R \ll 1$ holds, where R is the Reynolds number. It follows that the nonlinear term $\rho v_j v_{i,j}$ in (4.5) is negligible wih respect to the viscous term $\eta v_{i,jj}$ and can be omitted. The motion is laminar.

Under assumptions 1 to 6, the Navier-Stokes equation reduces to

$$-p_{,i} + \eta v_{i,jj} + \rho g_i = 0. \tag{6.1}$$

7) *Constant flow rate* – Flow rate at any given point of a lava flow is strictly linked to the effusion rate at the vent, which can be fairly constant for relatively long time spans. The mass flow rate m at a given cross section S of a flow is calculated as

$$m = \rho \int_S v_x(y, z)\, dy\, dz. \tag{6.2}$$

8) *Uniform slope angle* – The liquid flows on an inclined plane with slope α = const.

9) *Uniform velocity* – The flow is confined in a straight channel with fixed, parallel levées. Velocity has only one nonvanishing component v_x, having uniform magnitude. No lateral spreading ($v_y = 0$) and no thickness changes ($v_z = 0$) are allowed. If $z = h(x, y)$ is the free surface of the flow, this assumption implies $h_{,x} = 0$ and $h_{,y} = 0$. Of course the assumption is not valid at the flow front, where v_z can be of the same order as v_x and the rapid decrease of flow thickness produces a pressure gradient

$$p_{,x} = \rho g h_{,x} \tag{6.3}$$

which is responsible for the advance of the front, in addition to the gravity body force.

Assumptions 7 to 9 are closely linked together, since a change in flow thickness can be produced by changes in flow rate or slope angle, as well as in viscosity or yield stress. Such assumptions can be dropped without complicating the equations of motion if one assumes that downstream changes in flow thickness are very slow, as usually happens. In this case, $v_z \ll v_x$, with

$$v_z \simeq v_x h_{,x}. \tag{6.4}$$

At the same time, v_x can be assumed to change very slowly along the flow, so that $v_{x,x}$ can be neglected with respect to $v_{x,y}$ and $v_{x,z}$.

Under assumptions 1 to 10, the Navier-Stokes equation, separated in its components, reads

$$\eta(v_{x,yy} + v_{x,zz}) + \rho g \sin\alpha = 0 \tag{6.5a}$$

$$p_{,z} + \rho g \cos\alpha = 0. \tag{6.5b}$$

10) *Infinitely wide liquid layer* – This assumption leads to a two-dimensional model, where velocity depends only on one coordinate: depth z within the flow. The liquid flows in the x-direction, while no changes in the flow are considered in the y-direction. This model neglects friction at the levées and may be appropriate to low aspect ratio flows. Equation (6.5a) reduces to

$$\eta v_{x,zz} + \rho g \sin\alpha = 0. \tag{6.6}$$

This is an extremely simplified model, which can however reproduce some gross features of observed lava flows. If one assumes a flow rate and an initial temperature of the liquid at the eruption vent, the temperature decrease due to heat radiation and the consequent change in the rheological parameters can be computed along the flow. The equations of motion can be solved in the approximation of slow downslope change in the flow parameters, yielding flow thickness and velocity as functions of the distance from the eruption vent. Such models allowed a first estimate of the sensitivity of flow dynamics to changes in the initial conditions, ground slope and rheological parameters (Park and Iversen, 1984; Dragoni, 1989; Ishihara *et al.*, 1989).

A crucial role in the solution of the equations is played by boundary conditions. At the free surface of the flow $z = h$, boundary conditions prescribe the vanishing of shear stress, while normal stress is equal to $-p_0$, where p_0 is the atmospheric pressure. The boundary condition at the contact with solid surfaces (ground and levées) is vanishing of velocity. The boundary condition at the interface with the plug is $\sigma_{max} = \tau$. A complication is that the continuous change in the size of the plug introduces a moving boundary in the problem.

7. Flow models

In this framework, some aspects of lava flow behaviour have been investigated. A review was given in Dragoni (1993). Recent contributions have regarded a better definition of lava rheology and a study of the physical processes producing the formation of lava tubes and the opening of secondary vents.

Since the effusion temperatures of lavas are usually close to the liquidus, the cooling of lava is accompanied by progressive crystallization of the different mineral components. Therefore lava can be considered a suspension of solid crystals in a liquid phase. In a study of lava rheology, the presence of crystals cannot be neglected, because it is responsible for the non-Newtonian behaviour. Both lava viscosity and yield stress can be expressed as functions of the crystallization degree ϕ, which is in turn a function of temperature (Dragoni and Tallarico, 1994). Experimental data and theoretical considerations indicate that the yield stress is zero at the liquidus temperature and reaches a limit value when crystallization saturates (Chester *et al.*, 1985). It is interesting that the increasing crystallization has two opposing effects: it produces an increase in viscosity and yield stress at the same time. But the increase in yield stress controls the cooling of the flow, because it produces a thicker plug, makes the heat loss slower and keeps the internal temperature high, thus opposing the viscosity increase. Thus lava flows are remarkably affected by the dependence of yield stress on temperature.

The establishment of a channel is the premise for the development of a lava tube (Peterson *et al.*, 1994). Due to the heat loss into the atmosphere, a crust is gradually formed on the upper surface of the flow and may eventually weld to the channel levées. A model for tube formation has been proposed under the assumption that the tube is formed when such a crust is sufficiently thick to resist the drag of the underlying flow and to sustain itself under its own weight (Dragoni *et al.*, 1995). The minimum thickness of the crust satisfying such conditions depends on the tensile strength and shear strength of the crust itself. If one assumes that the growth of the crust produces a downflow linear increase in the shear stress at the interface between flowing lava and the crust, the distance can be evaluated between the eruption vent and the point where the tube is formed. If the flow rate is constant, the thickness of the flow increases as the crust fragments grow and weld to each other, and the velocity of the crust decreases to zero. Once the lava tube is formed, the initial flow rate can be achieved by a flow thickness smaller than the vertical size of the tube, with the same viscous dissipation: this may explain why, under steady-state conditions, the lava level inside a tube is frequently lower than the roof of the tube itself.

During eruptions on Etna, it has often been observed that the front of a stationary lava flow breaks and the inner fluid lava pours out giving rise to a new flow (*e.g.,* Pinkerton and Sparks, 1976). This phenomenon is often connected with lava tubes. The opening in the front is commonly called an ephemeral vent, since the flow originating from it often has a reduced duration and length. Such vents may form silently in a few minutes and the relatively high velocity of escaping lava may be a danger for people in the neighbourhood. The opening of ephemeral vents in the solid front of a lava flow has been studied considering the front as a viscoelastic shell which is deformed by the pressure exerted on it by the inner fluid lava (Dragoni and Tallarico, 1996). The vent opens when normal stress in the front overcomes the tensile strength of solid lava. As the front cools, the isothermal surface at the solidus temperature deepens into the lava body and the crust thickness increases. However the thermal effect is negligible, if the timescale for the opening of ephemeral vents is in the order of a few days after the front has stopped, as is often observed.

8. Final remarks

In principle, a physical model should provide the evolution of a lava flow as a function of time on the basis of initial and boundary conditions. Initial conditions include effusion rate, temperature and chemical composition at the vent. Since lava is a multi-phase system, the concentrations of different phases and their chemical compositions should be known. Time variations of conditions at the vent should be taken into account. Boundary conditions include the physical properties and state of the environment (ground and atmosphere). The evolution of a flow is given by a set of coupled equations, including continuity, dynamic, constitutive, thermal and chemical equations. Density, rheological and thermal parameters must be known in order to use the governing equations. Further equations relate such quantities to temperature and chemical composition, which change as functions of time and position.

In spite of approximations, theoretical models already highlight some important aspects of lava flow behaviour and many others are waiting to be studied. The data collected so far on active lavas appear to be broadly consistent with model predictions, but the amount of data is at present insufficient for a complete check of theoretical predictions. Simultaneous measurements of several quantities, such as flow dimensions and velocity, temperature, viscosity and yield stress, taken at different points in the flow, are necessary for a comparison with models.

On the basis of sound physical models, deterministic predictions of lava flows may be feasible in the near future. This objective will be achieved provided a coordinated research on lava flows is carried out along three parallel lines: 1) development of theoretical models; 2) *in situ* measurements during eruptions; 3) laboratory experiments on lava samples at field conditions. This strategy will make it possible to develop numerical models allowing a real time evaluation of volcanic hazard connected with specific lava flows during an eruption and the adoption of possible countermeasures. A pre-eruption evaluation of volcanic hazard from lava invasion requires the additional information on possible vent locations and erupted volumes.

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