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Temporal characteristics of some aftershock sequences in Bulgaria

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Abstract
We apply statistical analysis to study the temporal distribution of aftershocks in aftershock sequences of five earthquakes which occurred in Bulgaria. We use the maximum likelihood method to estimate the parameters of the modified Omori formula for aftershock sequences which is directly based on a time series. We find that: the maximum likelihood estimates of the parameter p show a regional variation, with lower values of the decay rate in North Bulgaria; the modified Omori formula provides an appropriate representation of temporal variation of the aftershock activity in North Bulgaria; the aftershock sequences in South Bulgaria are best modeled by the combination of an ordinary aftershock sequence with secondary aftershock activity. A plot of the cumulative number of events *versus* the frequency-linearized time τ clearly demonstrates a transition from aftershock to foreshock activity prior to the second 1986 Strazhitsa (North Bulgaria) earthquake.

Key words *aftershocks – aftershock decay rate – secondary aftershock sequences*

1. Introduction

Examination of the space-time distribution of earthquakes is of fundamental importance for understanding the physics of the earthquake generation process. One challenge in applying statistical methods to study the earthquake occurrence is to distinguish objectively non-random from random earthquakes. The spatial and temporal clustering of aftershocks is the dominant non-random element of seismicity, so that when aftershocks are removed, the remaining activity can be modeled (as a first approximation) as a Poisson process (Gardner and Knopoff, 1974). The temporal decay of aftershocks is interesting because it contains information about the seismogenic process and physical conditions in the source region. It is well known that the occurrence rate of aftershock sequences in time is empirically well described by the modified Omori formula $n(t) = K/(t + c)^p$ proposed by Utsu in 1961 (Utsu, 1969). The power-low decay represented by the modified Omori relation is an example of temporal self-similarity of the earthquake source process. The variability of the characteristic parameter p value is related to the structural heterogeneity, stress and temperature in the crust (Mogi, 1962; Kisslinger and Jones, 1991). The aftershock decay rate (parameter p) may contain information about the mechanisms of stress relaxation and friction laws in seismogenic zones (Mikumo and Miyatake, 1979), but this information cannot be derived without a precise characterization of the empirical relations that best fit the data.

In this study we examine the temporal patterns of aftershock distribution of five earthquakes (surface magnitude larger than or equal to 5.3) which occurred in Bulgaria after 1900. The parameters in the modified Omori formula

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are estimated by the maximum likelihood method, assuming that seismicity follows a non-stationary Poisson process (Ogata, 1983). Two statistical models (a model without secondary aftershocks and a model including the secondary aftershock activity) are tested for goodness of fit to aftershock data. We adopt the Akaike Information Criterion (Akaike, 1974) as a measure for selecting the best among competing models, using fixed data sets.

2. Method

The frequency $n(t)$ of aftershocks at time t is well represented by the modified Omori formula (Utsu, 1969)

$$n(t) = K(t + c)^{-p} \qquad (2.1)$$

where t is the time elapsed since the occurrence of the main shock, and K, p, c are constant parameters. The most important parameter is p, which characterizes the decay of the aftershock activity.

On the assumption that aftershocks are distributed as a non-stationary Poisson process, Ogata (1983) proposed to use the maximum likelihood method for estimating the parameters K, c and p in a modified Omori formula. The intensity function of the Poisson process $\lambda(t)$ is defined by the relation

$$\lambda(t) = \lim_{\Delta t \to 0} \text{Prob}\{\text{an event in } [t, t+\Delta t]\} / \Delta t. \qquad (2.2)$$

Then the likelihood function of the aftershock sequence can be as follows

$$f(t_1, t_2, \ldots, t_N; \theta) = \prod_{i=1}^{N} \lambda(t_i; \theta) \exp\left(\int_S^T \lambda(t; \theta)\, dt\right) \qquad (2.3)$$

where the $\{t_i; i = 1, 2, \ldots, N\}$ are the occurrence times of the events in a time interval $[S, T]$ which is the available period of observation, and $\theta = (K, p, c)$.

Using the modified Omori formula, the intensity function becomes

$$\lambda(t, \theta) = K(t + c)^{-p}. \qquad (2.4)$$

If the sequence contains m secondary aftershock sequences, starting at times $T_1, T_2, \ldots, T_m$, then the intensity function can be given as

$$\lambda(t; \theta) = K(t + c)^{-p} + \sum_{i=1}^{m} H(t - T_i) K_i (t - T_i + c_i)^{-pi} \qquad (2.5)$$

where H is the Heaviside unit step function and $\theta = (K, p, c, K_1, p_1, c_1, \ldots, K_m, p_m, c_m)$. The Maximum Likelihood Estimates (MLE) of the parameters are those which maximize function (2.3) with the corresponding vector θ. The asymptotic Standard Deviation (SD) of MLE can be calculated by the Fisher information matrix

$$J(\theta; S, T) = \int_S^T 1/\lambda(t; \theta)\, \partial\lambda(t; \theta)/\partial\theta' \theta\lambda(t; \theta)/\partial\theta\, dt \qquad (2.6)$$

where the prime (′) means the transposition of the vector. The standard deviation of each parameter is obtained by taking the square root of the corresponding diagonal element of J^{-1} (Ogata, 1983).

An integration of the intensity function $\lambda(t)$ gives a transformation from the time scale t to a frequency-linearized time scale τ (Ogata and Shimazaki, 1984).

On this time axis the occurrence of aftershocks becomes the standard stationary Poisson process if the choice of the intensity function $\lambda(t)$ (*i.e.*, the parameters K, c and p) is correct.

The frequency-linearized time for an aftershock sequence can be defined as

$$\tau = \Lambda(t) = \int_0^t \lambda(s)\, ds. \qquad (2.7)$$

The time scale τ is used for testing the goodness of fit between the aftershock occurrence and the selected model. A linear dependence between the observed cumulative numbers of aftershocks

(N) and τ should be observed if an appropriate model has been selected. Anomalies in the aftershock activity are more evident on the $N(\tau)$ plot than on $n(t)$. Thus the τ time axis will be used to detect secondary aftershock activity.

To select which model fits the observations better, the Akaike Information Criterion (AIC) (Akaike, 1974) is used. This is a measure of which model most frequently reproduces features similar to the given observations, and is defined by:

$$\text{AIC} = (-2)\ \text{Max}(\ln\text{-likelihood}) + 2\ (\text{Number of the used parameters}) \qquad (2.8)$$

A model with a smaller value of AIC is considered to be a better fit to the observations.

3. Data

The aftershock sequences of five earthquakes generated in the territory of Bulgaria are analyzed (the first 3 occurred in Southern Bulgaria, and the last 2 in Northern Bulgaria). The parameters of the main events are presented in table I where M_S is surface magnitude. As aftershock, we accept each event with magnitude less than the magnitude of the main event M that satisfies the criteria introduced by Gardner and Knopoff (1974), as modified for the Central Balkans by Christoskov and Lazarov (1981):

$$\begin{aligned} &\log(R_a) \le 0.9696 + 0.1243M \\ &\log(T_a) \le -0.62 + 0.56M \quad (M < 6.0) \qquad (3.1) \\ &\log(T_a) \le -5.25 + 2.15M - 0.137M^2 \ (M \ge 6.0) \end{aligned}$$

where R_a is the distance from the main shock to the event, and T_a is the time elapsed since the occurrence of the main shock.

The aftershock sequences of the first three earthquakes (table I) are defined on the basis of macroseismic data. We use the following data sources: catalogue of earthquakes in Bulgaria and adjacent regions (Grigorova *et al.*, 1979); earthquakes in Bulgaria (Vatzov, 1904, 1905, 1906, 1907, 1908); and earthquakes in Bulgaria (Kirov, 1941, 1945). The identification of aftershocks in the initial data was performed in the following manner: first of all the approximate area and time dimensions of each aftershock sequence are defined, using empirical relations (3.1); secondly as an aftershock is accepted each event for which I_{max} (greatest reported intensity) is observed within the defined space-time window. It should be mentioned that Rila monastery is situated close to the epicenters of the first two events (table I) which occurred at the beginning of the century. This spatial closeness, and the precision of the monks in the monastery ensure reliable information about the felt aftershocks.

Nevertheless a systematic incompleteness of the data is possible for events which occurred in a certain time interval each day (for example in the night time). We chose to use random simulations to evaluate the influence of such incompleteness of the data. We assume that an event with elapsed time (since the occurrence of the main shock) greater than T_1 which occurred in the time interval from H_1 h to H_2 h is not documented with a probability P. In order to evaluate the statistical significance of the probable data

Table I. List of earthquakes.

Date (YMD)	Origin time (GMT)	Region (town)	M_S	Number of aftershocks
1903.11.25	23:16:42	South West Bulgaria (Krupnik)	5.5	170
1904.04.04	10:25:55	South West Bulgaria (Krupnik)	7.8	1098
1928.04.18	19:22:48	Central South Bulgaria (Plovdiv)	7.0	920
1986.02.21	05:39:55	North Bulgaria (Strazhitsaitca)	5.5	70
1986.12.07	14:17:09	North Bulgaria (Strazhitsaitca)	5.7	170

incompleteness the following procedure is applied:

1) A random catalog of N_a events, in a time interval of T days, is generated, which obeys the modified Omori formula (2.1) with fixed parameters p and c. As Na is the total number of events in the time interval $(0, T)$, the parameter K can be calculated by the relation

$$K = N_a(1-p)/[(T+c)^{1-p} - c^{1-p}]. \qquad (3.2)$$

2) A random number q in interval (0,1) is generated for each event with elapsed time (since the occurrence of the main shock) greater than T_1 which occurred in the time interval from H_1 h to H_2 h. The event is excluded from the catalog if $q < P$.

3) MLE of the parameters are obtained.

4) The steps 1-3 are applied N times, and the distribution of the MLE's are analyzed.

In general we put as time of the main shock $T_0 = 0$ h, then $T_1 = 24$ h, $H_1 = 0$ h and $H_2 = 5$ h in the study. We analyze two different time intervals, $T = 1000$ and $T = 300$ days, with $N_a = 1000$ and $N_a = 200$, events respectively. Our choice is based on the set of observed data. Fifty samples are generated, *i.e.* $N = 50$, for $T = 1000$ days and $N_a = 1000$ events, and $N = 100$ for $T = 300$ and $N_a = 200$. The results are presented in table II, where P, p, c, T and N_a are fixed parameters; K is calculated as a function of p, c, T and N_a; $\bar{K}$, $\bar{p}$, $\bar{c}$ are MLE's of the Omori formula parameters; and $\sigma_{\bar{K}}$, $\sigma_{\bar{p}}$, $\sigma_{\bar{c}}$ are the corresponding stand-

Table II. Results.

P	K	p	c	T	N_a	$\bar{K}$	$\sigma_{\bar{K}}$	$\bar{p}$	$\sigma_{\bar{p}}$	$\bar{c}$	$\sigma_{\bar{c}}$
1.0	127.44	1.20	0.05	1000	1000	128.35	6.72	1.241	0.021	0.058	0.010
0.5	127.44	1.20	0.05	1000	1000	127.31	4.74	1.218	0.019	0.053	0.007
1.0	177.22	1.20	0.02	1000	1000	187.40	14.98	1.255	0.026	0.231	0.038
0.5	177.22	1.20	0.02	1000	1000	182.07	13.47	1.228	0.024	0.213	0.033
1.0	90.64	0.95	0.05	1000	1000	91.93	5.17	0.996	0.017	0.061	0.018
0.5	90.64	0.95	0.05	1000	1000	90.39	5.37	0.967	0.017	0.055	0.015
1.0	102.07	0.95	0.20	1000	1000	106.15	8.73	0.999	0.020	0.237	0.058
0.5	102.07	0.95	0.20	1000	1000	103.99	7.47	0.972	0.019	0.219	0.052
1.0	39.81	0.70	0.05	1000	1000	39.91	3.51	0.740	0.019	0.074	0.041
0.5	39.81	0.70	0.05	1000	1000	41.21	3.15	0.727	0.018	0.072	0.036
1.0	40.95	0.70	0.20	1000	1000	42.05	4.18	0.745	0.022	0.247	0.097
0.5	40.95	0.70	0.20	1000	1000	41.55	3.65	0.722	0.018	0.231	0.117
1.0	26.65	1.20	0.05	300	200	27.21	2.81	1.241	0.054	0.061	0.022
0.5	26.65	1.20	0.05	300	200	27.07	2.77	1.230	0.051	0.057	0.021
1.0	37.73	1.20	0.20	300	200	41.29	8.78	1.264	0.072	0.247	0.106
0.5	37.73	1.20	0.20	300	200	38.97	7.18	1.220	0.058	0.220	0.092
1.0	21.32	0.95	0.05	300	200	22.07	2.96	1.003	0.052	0.067	0.035
0.5	21.32	0.95	0.05	300	200	22.00	2.60	0.977	0.043	0.064	0.033
1.0	24.55	0.95	0.20	300	200	27.95	6.42	1.022	0.066	0.298	0.166
0.5	24.55	0.95	0.20	300	200	25.91	5.39	0.978	0.061	0.245	0.132
1.0	11.70	0.70	0.05	300	200	12.96	2.75	0.769	0.052	0.145	0.196
0.5	11.70	0.70	0.05	300	200	12.37	2.21	0.731	0.050	0.112	0.155
1.0	12.20	0.70	0.20	300	200	13.50	3.63	0.761	0.067	0.337	0.325
0.5	12.20	0.70	0.20	300	200	13.40	3.27	0.738	0.057	0.342	0.330

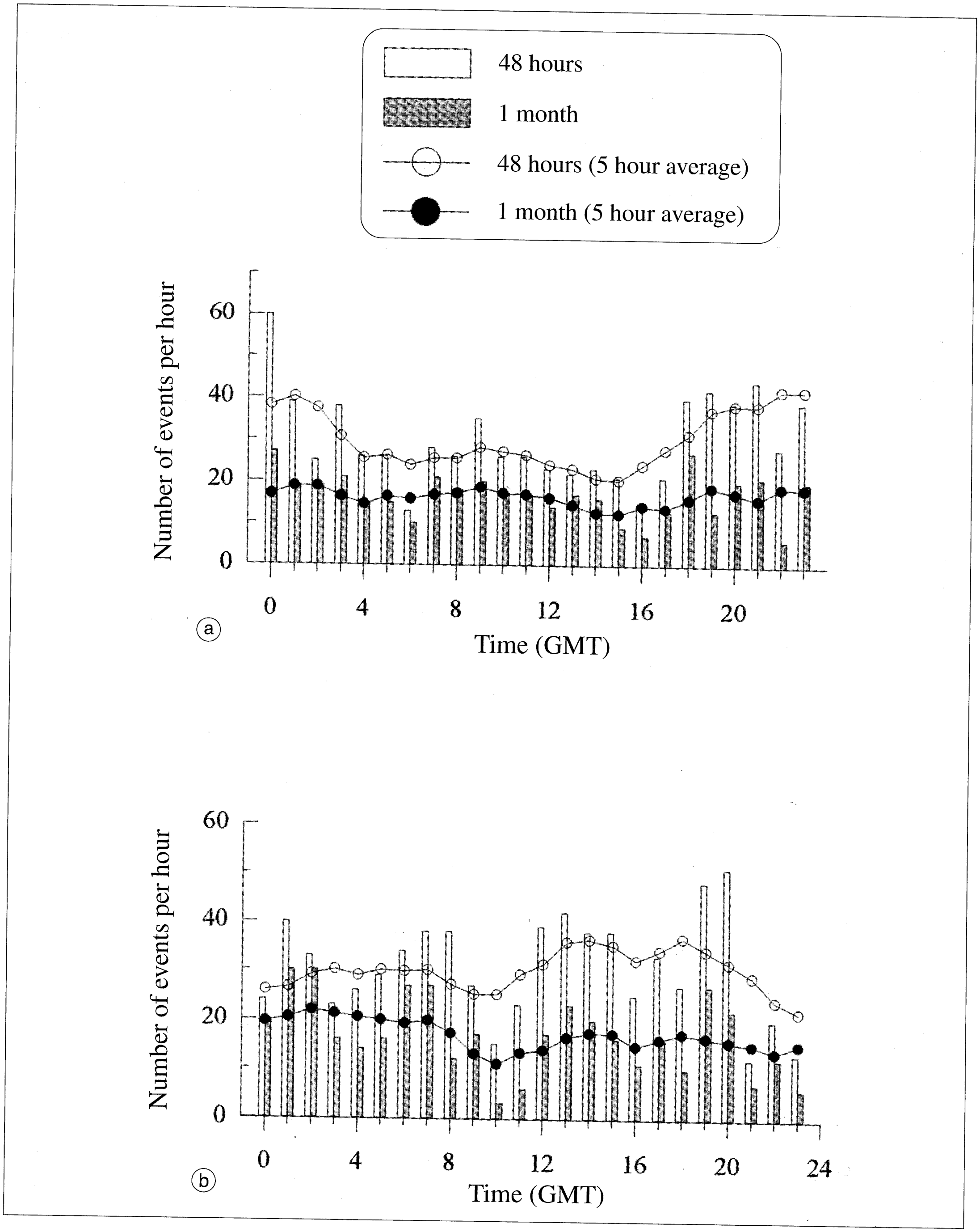

Fig. 1a,b. Aftershock hour distribution: a) 1928 earthquake; b) 1904 earthquake.

ard deviations. Hence we can summarize our observations of the overall behaviour of the estimations as follows: there is a systematic bias toward larger values for all three parameters; even in the extreme case $P = 1$ (*i.e.* excluding of all events in the time interval from H_1 h to H_2 h) the discrepancy between p and $\bar{p}$ is less than 0.061 for all the examined cases.

The observed hourly distributions of aftershocks with elapsed times greater than 2 days and 1 month after the main events for the 1904 and 1928 aftershock sequences, based on macroseismic data, are presented in fig. 1a,b. No obvious time-dependent deficiency in the number of aftershocks is observed. As expected, the time distribution of events with elapsed time greater than 48 h is governed by the origin time of the main shock (*i.e.* 10:26 and 19:23 (UT), respectively, for the 1904 and the 1928 events). This influence is strongly reduced in the distribution of events with elapsed time greater than 1 month after the occurrence of the main shock. The observed distribution for the 1903 sequence is not presented because of the small number of events. In general we can expect the same data quality for the 1903 and 1904 sequences, because the two main shocks occurred very close in time and space, and the same source of data is used.

Considering the above analysis of the possible effect of data incompleteness on the estimations and the observed distributions, we can draw the following conclusion: the use of macroseismic data for the 1903, 1904 and 1928 sequences should not cause any significant bias in estimating the parameters in Omori's formula.

We use instrumental data from the Bulgarian Seismological Report for two 1986 earthquakes. A dependence of the decay constant p on the lowest limit of magnitude above which aftershocks are counted M_a is found in some studies (Utsu *et al.*, 1995). Although this dependence does not seem to be systematic (Utsu *et al.*, 1995) and in order to ensure the homogeneity of the data a threshold magnitude equal to 2.0 was chosen for both sequences based on instrumetal data.

4. Results

We analyzed the five aftershock sequences from 0 to T_a days after the main shock (T_a is determined by relation (3.1)), by fitting them to the modified Omori formula (2.1). The parameters obtained, and the corresponding AIC (eq. 2.8), are presented in table III. We should mention that there are various magnitude estimates for the 1986 Strazhitsa earthquakes. Therefore for each sequence, two values of $T_a(M)$ were used. For the first sequence $T_a = 151$ days corresponded to magnitude $M_d = 5.0$ (duration magnitude calibrated against the MLH scale (Solakov and Simeonova, 1993)), and $T_a = 289$ days corresponded to $M_S = 5.5$ (ISC, 1989). For the second sequence $T_a = 253$ days corresponded to magnitude $M_d = 5.4$ and $T_a = 373$ to $M_S = 5.7$.

Table III. MLE's of the Omori formula parameters, and the corresponding AIC, assuming only main aftershock activity.

Earthquake	T_a	K (SD)	p (SD)	c (SD)	Max (ln$L$)	AIC
1903, $M = 5.5$	122	17.47 (2.24)	0.87 (0.05)	0.02 (0.02)	130.17	– 254.34
1904, $M = 7.8$	927	94.26 (5.24)	0.94 (0.01)	0.04 (0.01)	1474.70	– 2943.40
1928, $M = 7.0$	1221	780.69 (30.61)	0.99 (0.02)	0.19 (0.05)	780.69	– 1555.38
1986, $M_d = 5.0$	151	2.60 (0.71)	0.71 (0.09)	0.01 (0.02)	– 56.05	118.1
$M_S = 5.5$	289	2.08 (0.57)	0.52 (0.07)	0.001 (0.01)	– 137.79	281.58
1986, $M_d = 5.4$	254	18.78 (3.62)	0.93 (0.06)	0.17 (0.11)	– 16.05	36.1
$M_S = 5.7$	374	18.76 (3.79)	0.93 (0.06)	0.17 (0.11)	– 53.04	112.08

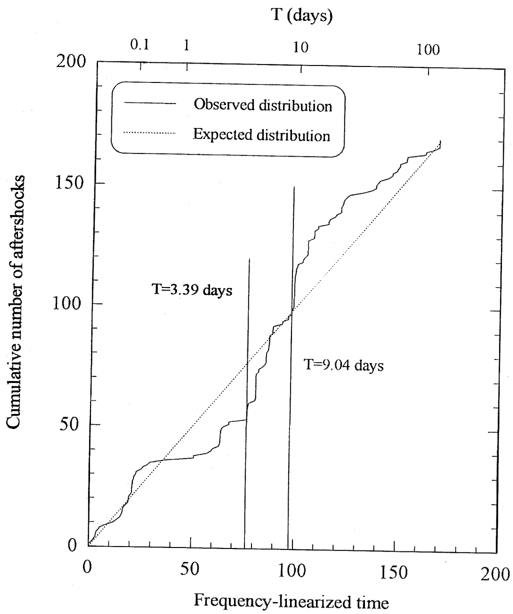

Fig. 2. Plot of the cumulative number of events *versus* the frequency-linearized time τ for the 1903 aftershock sequence (an ordinary aftershock sequence).

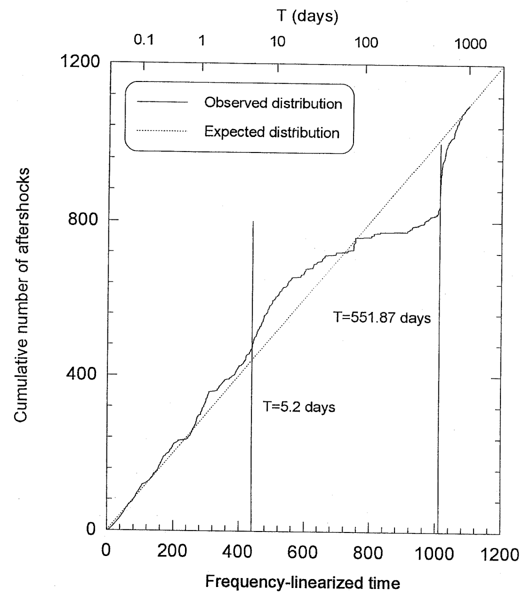

Fig. 3. Plot of the cumulative number of events *versus* the frequency-linearized time τ for the 1904 aftershock sequence (an ordinary aftershock sequence).

(These magnitudes are from the same sources respectively.) Table III shows that the accepted duration of aftershock activity, *i.e.* T_a, significantly affects the estimated decay rate (parameter p) for the aftershock sequence of the 21.02.1986 Strazhitsa earthquake, but not for the other Strazhitsa aftershock sequence.

By using the estimated parameters K, p, c the cumulative number of events was plotted against the frequency-linearized time τ, as defined in eq. (2.7) and presented in figs. 2 to 6. If the modeling of aftershock sequences is appropriate, the cumulative number of events analyzed should increase linearly with τ.

Figures 2 to 4 show that significant discrepancies between observed and expected distributions exist (*i.e.* the modified Omori formula does not fit the observational data). Figure 2 shows bumps in the plot, caused by a rapid increase in events roughly 3 and 9 days after the 1903 main shock. The time of the first anomaly, which is clearer, coincided with the occurrence of the largest aftershock ($M_S = 4.9$), and of the second one coincided with the occurrence of several strong aftershocks with magnitude higher than 4.0. For the aftershock sequence of the 1904 earthquake (fig. 3) anomalous increases in the number of events were observed roughly 5 days and 550 days after the main shock. The time of the first anomaly coincided with the occurrence of several relatively strong aftershocks (magnitude higher than 5.0) within a short time interval (of about five days), and the time of the second, which is very well expressed, coincided with the occurrence of the largest aftershock (magnitude 6.4). Figure 4 suggests the existence of increased aftershock activity about 8 days after the 1928 main shock. This anomaly could be related to the occurrence of some strong aftershocks (magnitude greater than 5.0) in a time interval of about two days. An apparent decrease of the observed cumulative number from the calculated one can be seen before $T = 3.4$ days (fig. 2), $T = 551$ days (fig. 3) and $T = 8.1$ days (fig. 4). It should be noted that such decreases may be caused by the simplicity of the model used (modified Omori relation) which suggests a constant decay rate of the activity during the whole period examined. So, more complex models should be used to examine whether there are any anomalous changes in aftershock activity before the occurrence of large aftershocks.

Our results show that the aftershock activity of these three earthquakes (1903, 1904, Krupnik and 1928, Plovdiv) can be modeled by a combination of main and secondary aftershock sequences. Furthermore, two types of secondary aftershock activity can be recognized in the observed temporal distribution of aftershocks. The first is connected with the occurrence of the largest aftershock in the sequence (3 days after the main shock for the 1903 earthquake, and 550 days after the main shock for the 1904 earthquake) and is more obvious in the linearized temporal distribution (figs. 2 and 3). The second type appears to be related to the occurrence of several strong aftershocks in a time interval of a few days (9 days after the main shock for the 1903 earthquake, 5 days for the 1904 earthquake, and 8 days for the 1928 earthquake). This type we call «swarm» aftershock activity.

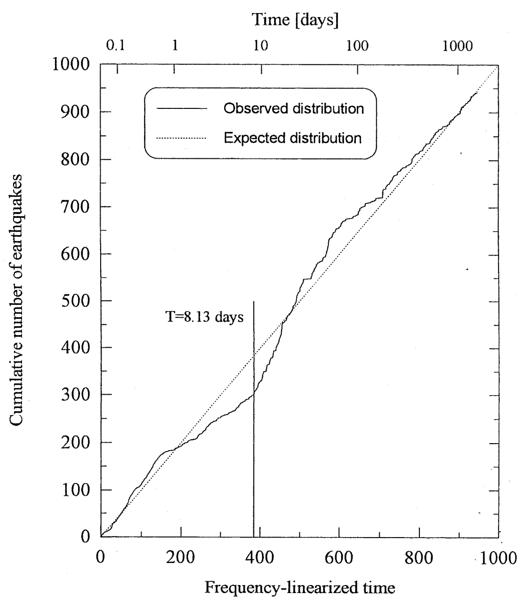

Fig. 4. Plot of the cumulative number of events *versus* the frequency-linearized time τ for the 1928 aftershock sequence (an ordinary aftershock sequence).

The temporal aftershock distributions of the 1986 Strazhitsa earthquakes are well modeled by the modified Omori formula (2.1), as can be seen in figs. 5a and 6. Figure 5a shows that a nearly linear trend of aftershock decay continues up to 170 days; thus the modified Omori formula (2.1) fits the observations up to 170 days after the main shock. About 170 days after the main shock the cumulative number of aftershocks increases rapidly with τ, showing a significant deviation from the prior trend. No large earthquake occurred in the region at that time. Furthermore, fig. 5b shows a significant misfit between observed and expected distributions for the 21.02.1986 sequence considering the time interval $T_a = 289$ days. Therefore this change in slope could be treated as a transition from aftershock activity to foreshock activity associated with the second 1986 Strazhitsa earthquake.

Consequently, we constructed models which take into account the effect of secondary aftershock activity, using eq. (2.5). The first two sequences (1903, 1904 Krupnik) are modeled by two secondary aftershock sequences, while the third is modeled by only one. We tested some models for secondary sequences, assuming: 1) the same p value as for the main aftershock sequence; 2) different p values. The estimated parameters, and the corresponding AIC (eq. (2.8)), are listed in table IV. The estimates of SD were obtained as the square roots of the diagonal elements of the inverse Hessian matrix (Ogata, 1983).

The AIC obtained for models with secondary aftershock sequences were much smaller than for those with ordinary aftershock sequences, for all three cases, as can be seen from tables III and IV. The best models for temporal aftershock distributions were combinations of the main and two secondary sequences with $p = p_1 = p_2$ for the 1903 sequence; two second-

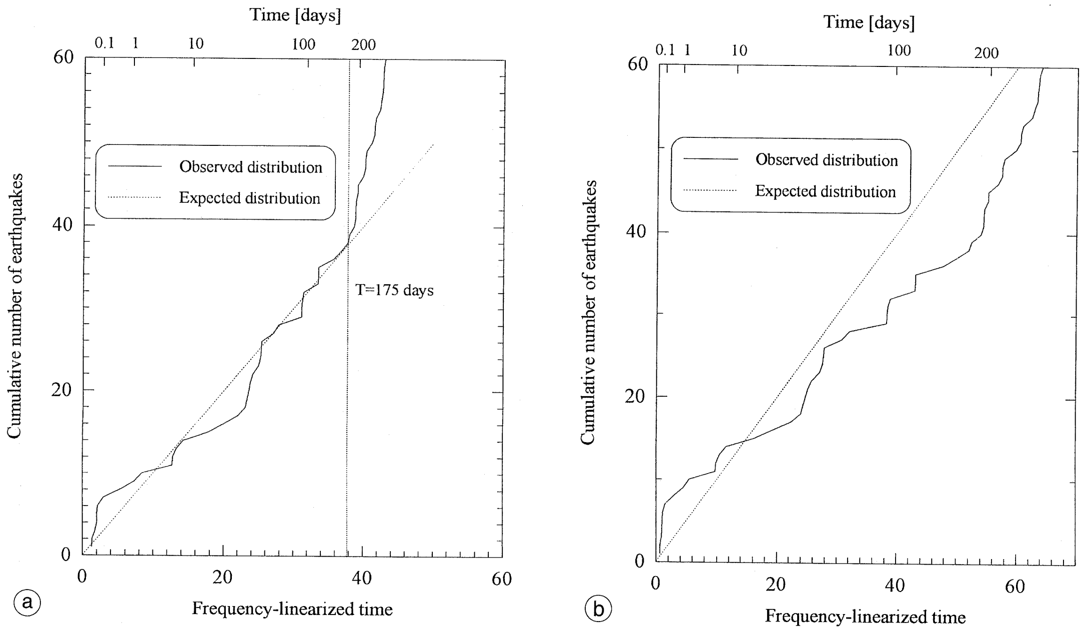

Fig. 5a,b. Plot of the cumulative number of events *versus* the frequency-linearized time τ for the 02.1986 aftershock sequence (an ordinary aftershock sequence): a) 151 days; b) 289 days.

Table IV. MLE's of the Omori formula parameters, and corresponding AIC, assuming main and secondary aftershock activity.

Earthquake	K SD	p SD	c SD	K_1 SD	p_1 SD	c_1 SD	K_2 SD	p_2 SD	c_2 SD	Max (ln$L$)	AIC
1903	8.62	1.18	0.03	15.49	–	0.52	8.40	–	0.33	177.48	– 340.96
$(p = p_1 = p_2)$	1.62	0.07	0.01	6.49		0.36	0.51		0.12		
1903	9.12	1.13	0.02	7.48	0.87	0.06	240.29	3.11	2.03	178.76	– 339.52
$(p \neq p_1 \neq p_2)$	1.73	0.15	0.02	2.92	0.11	0.14	2.94	0.40	0.45		
1904	109.91	1.16	0.11	19.62	–	0.21	90.39	–	4.88	1812.20	– 3610.40
$(p = p_1 = p_2)$	2.96	0.003	0.01	4.21		0.001	0.21		0.05		
1904	108.93	1.15	0.10	7805.5	3.37	4.49	11.14	0.69	0.004	1831.02	– 3644.04
$(p \neq p_1 \neq p_2)$	5.12	0.01	0.004	793.2	0.06	0.21	0.16	0.02	0.004		
1928	76.19	1.17	0.16	164.50	–	5.84	–	–	–	838.36	– 1666.72
$(p = p_1)$	4.82	0.01	0.01	16.05		0.001					
1928	86.10	1.29	0.22	140.59	1.10	5.21	–	–	–	839.02	– 1666.04
$(p \neq p_1)$	6.20	0.01	0.01	6.417	0.01	0.44					

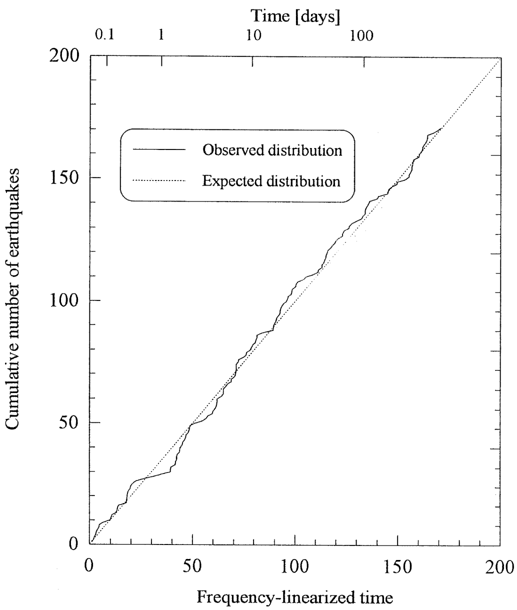

Fig. 6. Plot of the cumulative number of events *versus* the frequency-linearized time τ for the 12.1986 aftershock sequence (an ordinary aftershock sequence, 374 days).

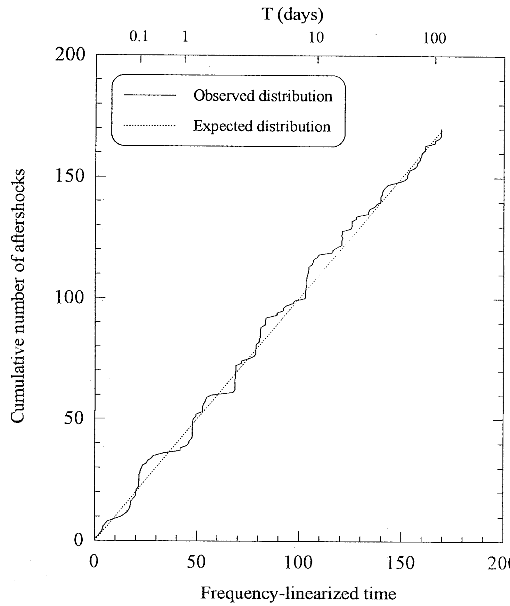

Fig. 7. Plot of the cumulative number of events *versus* the frequency-linearized time τ for the 1903 aftershock sequence (one ordinary and two secondary aftershock sequences with equal decay).

ary sequences with $p \neq p_1 \neq p_2$ for the sequence of the 1904 earthquake; and one secondary aftershock sequence with $p = p_1$ for the 1928 earthquake. Figures 7, 8 and 9 plot the cumulative number of events against the frequency-linearized time τ, using the parameters of the best model for each sequence. The three figures illustrate a good fit between observed and expected distributions without evident periods of decay and activation of the process.

The results in table IV support our suggestion on the existence of two types of secondary aftershock activity. Large values of the parameter c were obtained for «swarm» aftershock activity (table IV). These large values suggest a slow decay of secondary activity in certain (relatively large) time intervals, which could be caused by the occurrence of several secondary aftershock sequences in a short time interval.

Finally, the results in table IV concerning models with $p \neq p_1 \neq p_2$ for the 1903 and 1904 sequences show that p values for secondary aftershock sequences associated with the largest aftershocks were smaller than the corresponding p values of the main aftershock sequences, while p for «swarm» aftershock activity was significantly higher.

5. Discussion

During the last 33 years, more than 200 estimates of p value with a scatter from 0.6 to 2.5 and a median of 1.1 have been published for aftershock sequences in different parts of the world (Utsu *et al.*, 1995). The estimates of p value in the present study are in the lower part of this interval and somewhat lower than those obtained for aftershock sequences in Greece, *i.e.* p = 0.83-1.86 (Papazachos, 1975).

Our observations of the temporal distributions of aftershocks clearly demonstrate a sim-

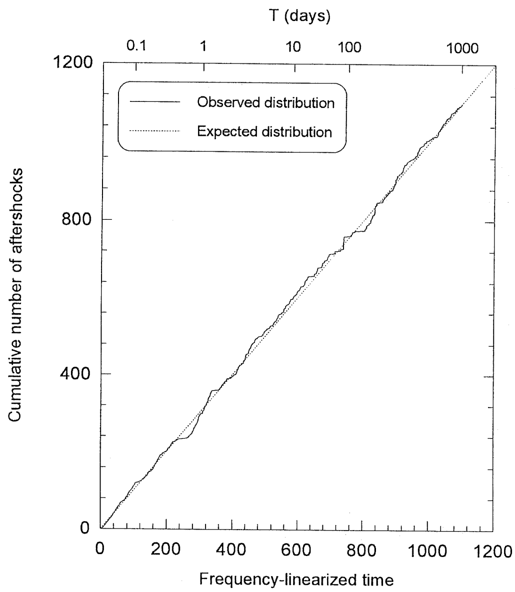

Fig. 8. Plot of the cumulative number of events *versus* the frequency-linearized time τ for the 1904 aftershock sequence (one ordinary and two secondary aftershock sequences with different decay).

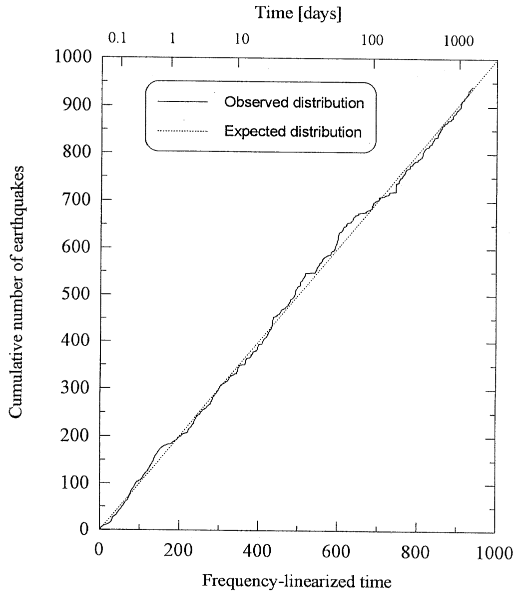

Fig. 9. Plot of the cumulative number of events *versus* the frequency-linearized time τ for the 1928 aftershock sequence (one ordinary and one secondary aftershock sequences with equal decay).

ilar behaviour in time for sequences which occurred close in space, *i.e.* the 1903 and 1904 sequences in Southwest Bulgaria, and the two 1986 sequences in North Bulgaria. The sequences in Southwest Bulgaria are characterized by well expressed secondary aftershock activity and relatively high values of p for the main sequence ($p \geq 1.13$). In contrast, those in Northern Bulgaria decay slowly ($p \leq 0.93$), without secondary aftershock sequences. The higher p values reported by Stanishkova and Slejko (1990) for the two 1986 aftershock sequences, *i.e.* 1.09 and 0.97, respectively, are not comparable with those obtained in the present study. Those authors did not use the same estimation method; moreover they assumed $c = 0$, and the time intervals and threshold magnitude are not stated. The analysis of the 1928 aftershock sequence, which occurred near the town of Plovdiv in the central part of South Bulgaria, shows only «swarm» secondary aftershock activity, and the decay of the main aftershock sequence is close to that of the Southwest Bulgaria sequences ($p = 1.17$ for the best model). Thus a regional variation of decay and type of aftershock activity is observed.

6. Conclusions

The main conclusions from our study of the temporal aftershock distribution in sequences of earthquakes in Bulgaria are as follows:

1) The maximum likelihood estimate of the parameters in the modified Omori formula and the selection of a statistical model based on AIC, show that aftershock sequences are best modeled by a combination of ordinary aftershock sequences with secondary aftershock activity for earthquakes in South Bulgaria. As for North Bulgaria, the aftershock sequences can be represented by the model with only ordinary aftershock activity.

2) An objectively defined transition from aftershock to foreshock activity was observed prior to the second 1986 Strazhitsa earthquake, using a frequency linearized time τ.

3) The temporal distributions of aftershocks also suggest the existence of two types of secondary aftershock activity. The first is related to a strong aftershock (usually the strongest in the sequence). The second – the «swarm» type – could be associated with a cluster of strong events, each with its own aftershock activity. This «swarm» type is characterized by a large value of parameter c.

4) A regional variation of the p value for the main aftershock activity was observed. The p values varied from 0.7 to 1.2, as those obtained for sequences in South Bulgaria are higher.

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