ANNALS OF GEOPHYSICS, VOL. 45, N. 6, December 2002

Collapse modes of structures under strong motions of earthquake

Hiroshi Akiyama
Real Estate Science, School of Science and Technology, Nihon University, Kanda-Surugadai, Chiyoda-ku, Tokyo, Japan

Abstract
Under strong motion earthquakes, structures receive various types of damage. The most fatal damage is the loss of lateral strengths of a structure. The loss of lateral resistance causes a total collapse of the structure due to the $P-\delta$ effect associated with the lateral displacements and the gravity loading. To eliminate such a collapse mode, to introduce into the ordinary stiff structure a flexible element which remains elastic is very effective. The flexible-stiff mixed structure can behave preferably in many aspects under strong earthquakes.

Key words *collapse mode – damage concentration – $P-\delta$ effect – flexible-stiff mixed structure*

1. Objective

In the Hyogoken-Nanbu Earthquake, 1995, various types of failure modes and collapse modes were disclosed not only in reinforced concrete structures but also in steel structures. Among these modes of failure and collapse, the most fundamental and common cause for the total collapse of structure is the impairment of the strength to support the gravity loading due to the seismic loading.

In this paper, the process to the final collapse is described and the effective measure to eliminate the final cause of collapse is made clear. Mainly, the collapse modes of the multi-storied frames under shear deformations are discussed.

2. Loading effect

Under earthquakes, structures are subjected to a combined effect of gravity loading and seismic loading. Gravity loading has a definite effect expressed by the force proportional to the mass under the field of constant acceleration of g. Thus, a structure is characterized by a potential energy proportional to the height of the structure.

The ground motions under an earthquake are divided into horizontal and vertical. Since the framed structure is primarily designed against gravitational loading with a sufficiently large margin of safety, the loading effect due to the vertical seismic ground motion is of minor importance. Therefore, only the loading effect due to the horizontal ground motion is discussed in the paper.

The loading effect of a seismic ground motion can be grasped as an energy input. The total

Mailing address: Dr. Hiroshi Akiyama, Real Estate Science, School of Science and Technology, Nihon University, Kanda-Surugadai, Chiyoda-ku, Tokyo, Japan.

energy input into a structure by an earthquake is a stable amount influenced by the total mass and the fundamental natural period and is scarcely influenced by the other structural parameters such as the strength distribution, the mass distribution and the stiffness distribution (Akiyama, 1985).

The balance of energy is described as follows:

$$W_r = W_e + W_p = E \tag{2.1}$$

where W_r is the resistance of a structure in energy absorption; W_e is the elastic vibrational energy in a structure; W_p is the cumulative inelastic strain energy in a structure, and E is the total energy input by an earthquake. $W_e + W_p$ forms the absorbed energy by a structure.

Under the horizontal ground motion, the structure develops horizontal displacements. Horizontal displacements of structure and the gravity loading form an additional effect of so-called «$P-\delta$ effect». The $P-\delta$ effect corresponds to the release of potentional energy in the gravitational acceleration field, and causes the reduction of the horizontal resistance of the structure in strength. Considering the $P-\delta$ effect, the total resistance of a structure against a horizontal ground excitation in terms of energy, ${}_tW_r$ is described as

$$ {}_tW_r = W_e + W_p - W_{P\delta} \tag{2.2}$$

where $W_{P\delta}$ is the release of potential energy by the $P-\delta$ effect.

The total energy input can be expressed by more understandable quantities such as the equivalent velocity and the equivalent height in the gravitational field defined as follows:

$$E = \frac{MV_E^2}{2} \qquad \left(V_E = \sqrt{\frac{2E}{M}} \right) \tag{2.3}$$

$$E = Mgh_{eq} \qquad \left(h_{eq} = \frac{E}{Mg} = \frac{V_E^2}{2g} \right) \tag{2.4}$$

where M is the total mass of the structure; V_E is the equivalent velocity, and h_{eq} is the equivalent height in the gravitational field.

Even under the strongest level earthquakes which occurred in Japan, it is very rare for V_E to exceed 300 cm/s. h_{eq} for V_E = 300 cm/s becomes 46 cm.

When one story of a multi-story frame fails to support the gravitational force, the story collapses and the potential energy stored in the upper stories beyond the collapsed story is released. The equivalent height of the released energy is the story height, H. Taking an ordinary value of H to be 400 cm, the seismic energy input which corresponds to h_{eq} = 46 cm is realized to be far modest compared to the released potential energy in the event of story collapse.

3. Collapse modes under earthquakes

Collapse of a structure starts at the weakest point of the structure. In cases influenced by the $P-\delta$ effect, the energy balance shown by eq. (2.1) is modified as

$$W_r = W_e + r_k W_{pk} = E + W_{p\delta} \tag{3.1}$$

where W_r is the resistance of a structure in inelastic energy absorption; W_{pk} is the resistance of the weakest story in inelastic energy absorption; k is the number of the story which yields the minimum value of r_k, and $r_k = W_p/W_{pk}$ is the damage concentration factor.

The damage concentration factor takes unity when the sheer damage concentration takes place in the k th story, generally ranging

$$1.0 \le r_k \le {}_e r_k \tag{3.2}$$

where ${}_e r_k$ is the energy distribution when the structure remains elastic.

The value of r_k is influenced by the structural type and the strength distribution. In weak-column type, r_k becomes smaller compared to the case of weak-beam type. The strength distribution is normalized by dividing with the strength distribution in the elastic case. r_k tends to take the smallest value at the weakest story in the normalized strength distribution. Collapse occurs when W_r can not reach $E + W_{p\delta}$. The energy absorption capacity W_{pk} is determined by the mechanical properties of structure including material properties.

In the reinforced concrete structures, an adequate reinforcement and yielding of reinforcing steel bars ensure the energy absorption capacity. Crush of concrete and buckling of reinforcing bars limit the capacity. In particular, shear failure accompanied by crush of concrete drastically impedes the development of inelastic deformation.

In steel structures, a high energy absorption capacity can be expected as far as the inherently ductile material properties are adequately developed. Various modes of buckling impair the energy absorption capacity. Another important mode of failure is brittle fracture of steel members which was widely disclosed in the Hyogoken-Nanbu Earthquake, 1995 and the Northridge Earthquake, 1994. As a result of the intensive researches (Nakashima *et al.*, 1998; Akiyama, 2000; Roeder, 2000), the effective measures to eliminate the brittle fracture were made clear in the following directions.

1) To exclude the material which indicates low toughness against brittle structure.

2) To apply minute structural details to reduce stress concentration.

The final cause for collapse is the $P-\delta$ effect. The $P-\delta$ effect appears as a release of energy by the amount of $W_{p\delta}$ in eq. (3.1). And also, the $P-\delta$ effect prompts the damage concentration in the multi-story frames, making r_k smaller. In fig. 1, typical examples of story collapse found in the Hyogoken-Nanbu (Kobe) earthquake (1995) are shown. These buildings are reinforced concrete frames.

Fig. 1. Story collapse of reinforced concrete frames in Kobe earthquake.

In the weakest intermediate story, the energy absorption capacity of columns expired. After the energy balance was lost the potential energy was released, resulting in the story collapse. Stories other than the collapsed story were equipped with sufficient strength to resist the impulsive forces associated with the release of potential energy, and thus, escaped collapse. When stories other than the initially collapsed story cannot resist the impulsive forces, a successive collapse of other stories can take place, leading to the total release of potential energy, and the total collapse of the entire frame.

4. $P-\delta$ effect

Referring to fig. 2, the $P-\delta$ effect in the relationship between the story shear force and the story drift in the multi-storied structure is expressed by a negative spring effect as shown in the next equation

$$\Delta Q_{P\delta} = -\frac{W\delta}{H} = k_{P\delta}\delta \tag{4.1}$$

where W is the total weight which rests on the story; $\Delta Q_{P\delta}$ is the decrease of shear force resistance due to the $P-\delta$ effect; δ is the story drift; H is the height of story; and $k_{P\delta} = -W/H$ is the spring constant characterized by the $P-\delta$ effect.

The work done by this additional shear force, $\Delta Q_{P\delta}$ until the maximum deformation δ_m is reached, is written as

$$\Delta W = \frac{W\delta_m^2}{2H} \tag{4.2}$$

where ΔW is the work done by the $P-\delta$ effect.

Under the maximum story deformation of δ_m, the weight which rests on the story moves downward by Δ as shown in fig. 1. The downward displacement Δ is written as

$$\Delta = \frac{\delta_m^2}{2H} \; . \tag{4.3}$$

The work done by the $P-\delta$ effect is just equal to the product of Δ and W.

The influence of the $P-\delta$ effect on the seismic response of structures can be most explicitly stated in terms of the energy input. Through numerical analyses, it was found that the increase in energy input due to the $P-\delta$ effect can be related to $k_{P\delta}$ as follows (Akiyama, 1984):

$$W_{P\delta} = \frac{10|k_{P\delta 1}|E}{k_1} \tag{4.4}$$

where k_1 is the spring constant of the first story in the elastic range, and $k_{P\delta 1}$ is $k_{P\delta}$ in the first story.

Thus, observing the first story, eq. (3.1) is rewritten as

$$W_e + r_1 W_{p1} = E\left(1 + \frac{10|k_{P\delta 1}|}{k_1}\right) . \tag{4.5}$$

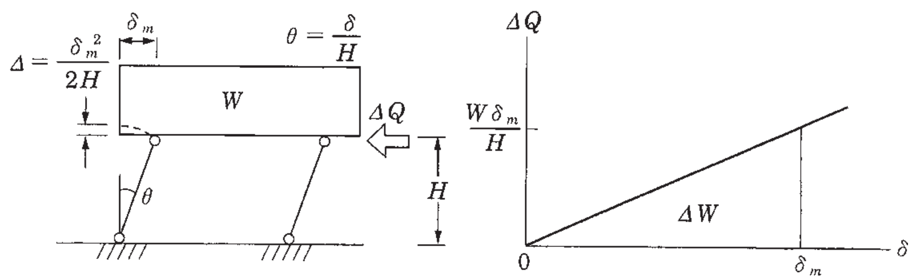

Fig. 2. $P-\delta$ effect.

When the story shear force resistance is totally nullified due to the combined action of structural collapse modes and the $P-\delta$ effect, the static equilibrium of forces in the vertical direction is broken. Then, the weakest story takes a path to the total collapse under the release of potential energy in the gravitational acceleration field. The released potential energy can make stories other than the weakest story successively collapse, thus inviting a progressive process to the total collapse of the structure.

5. Cancel of gravity loading effect

If it is possible to eliminate the $P-\delta$ effect, the major cause of collapse of structure can be removed. The most effective measure to eliminate the $P-\delta$ effect is to equip a structure with an elastic element which can cancel the negative spring effect shown by eq. (4.1). In the multistoried structure, the elastic element is required to satisfy the following conditions:

$$ {}_f k_0 \geq |k_{P\delta}| = \frac{W}{H} \qquad \delta \leq \delta_m \tag{5.1}$$

where ${}_f k_0$ is the original elastic spring constant of the frame; and δ_m is the predicted maximum story drift under earthquakes.

Then, the substantial elastic spring constant, k_f after cancelling the $P-\delta$ effect becomes as

$$k_f = {}_f k_0 + k_{P\delta} \,. \tag{5.2}$$

The condition given by eq. (5.1) is same as

$$k_f \geq 0 \,. \tag{5.3}$$

The structure must also be equipped with the energy absorbing element to resist earthquakes. The energy absorbing element must be equipped with high elastic rigidity and large inelastic deformation capacity. The elastic element is characterized by the relatively smaller elastic rigidity and can be categorized by «the flexible element». The energy absorbing element with relatively high elastic rigidity can be categorized by «the stiff element». The flexible element is designed under the condition of supporting the gravity loading and keeping elasticity within the range of the maximum drift anticipated under the seismic loading. Such a compound structure consisting of the flexible element and stiff element is recognized to be the flexible-stiff mixed structure (Akiyama, 1998). Structural performances in the flexible-stiff mixed structure are enhanced as the elasticity in the flexible element is increased. The intensity of elasticity of the flexible element is measured by r_q defined by the following equation:

$$r_q = \frac{{}_f Q_m}{{}_s Q_Y} = \frac{k_f \delta_m}{{}_s Q_Y} \tag{5.4}$$

where r_q is the elasticity intensity in the flexible-stiff mixed structure; ${}_s Q_Y$ is the yield strength of the stiff element; ${}_f Q_m$ is the maximum shear force in the flexible element, and prefix f and s identify quantities of the flexible and stiff elements.

The shear force-story deformation relationship of the flexible-stiff mixed structure is shown in fig. 3a-c. The shear force-story deformation relationship of the stiff element is assumed to be the elastic-perfectly plastic type.

Under the elastic-perfectly plastic $Q-\delta$ relationship as shown in fig. 4, the energy absorption due to plastic deformation is related to the cumulative plastic deformation as follows:

$$\begin{gathered} W_P = Q_Y \delta_P = \eta Q_Y \delta_Y \\ \delta_P = \delta_P^+ + \delta_P^- \end{gathered} \tag{5.5}$$

where Q_Y is the yield shear force; δ_P is the total cumulative plastic deformation; δ_P^+, δ_P^- are cumulative plastic deformations in positive and negative loading domains, and $\eta = \delta_P / \delta_Y$ is the cumulative plastic deformation ratio.

On the other hand, the maximum deformation in the story, δ_m is another important damage index. The non-dimensionallized maximum deformation is defined as

$$\mu_m = \frac{\delta_m - \delta_Y}{\delta_Y} \tag{5.6}$$

where μ_m is the plastic deformation ratio.

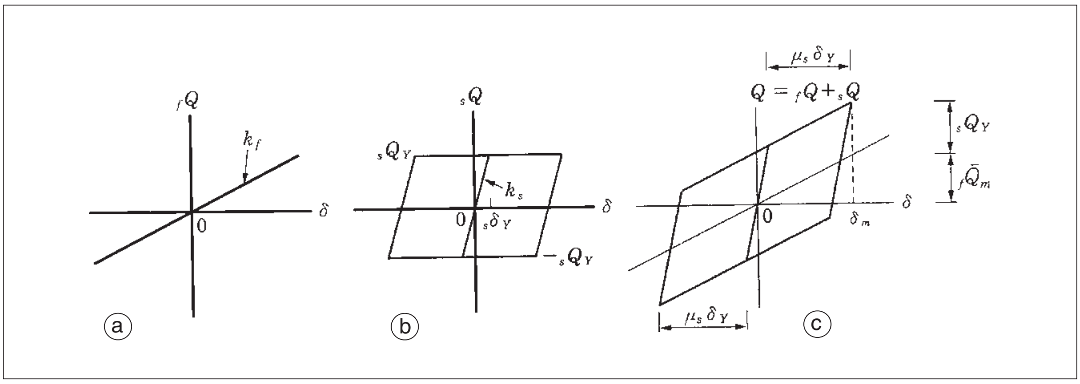

Fig. 3a-c. a) Flexible element; b) stiff element; c) mixed system.

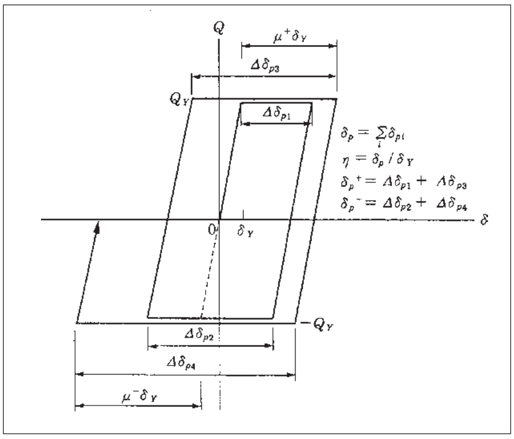

Fig. 4. Elastic-perfectly plastic type of $Q-\delta$ relationship.

The efficiency in plastic energy absorption can be measured by η/μ_m. The hysteretic loop which corresponds to η/μ_m is shown in fig. 5. When the flexible element does not exist, η/μ_m was found to be in the following range:

$$1.0 \le \frac{\eta}{\mu_m} \le 4.0. \qquad (5.7)$$

In this case, r_q can be negative due to the P–δ effect. Then, the deformation tends to develop one-sidedly. The smallest value of η/μ_m can be easily realized in the case where the P–δ effect becomes significantly large.

As r_q increases, η/μ_m increases and the following values are practicable in the design of the flexible-stiff mixed structure:

$$\frac{\eta}{\mu_m} = 8.0 \text{ to } 12.0\ . \qquad (5.8)$$

In fig. 5, n_{eq} denotes the number of hysteretic loop with the plastic deformation amplitude of $\mu_m\delta_Y$.

The advantage of the flexible-stiff mixed structure is found in cancellation of the $P-\delta$ effect, unification of damage distribution and minimization of drift under a certain amount of energy absorption. These preferable performances are secured as far as the following condition is satisfied:

$$r_q \ge 1.0\ . \qquad (5.9)$$

The $P-\delta$ effect is intensified as the number of story increases. Therefore, the importance of introducing the flexible-stiff mixed structure becomes clear in high-rise buildings. The efficiency of the flexible-stiff mixed structures is expressed by the largeness of μ_m and η. In order to make μ_m large, two methods are effective as shown in eq. (5.6):

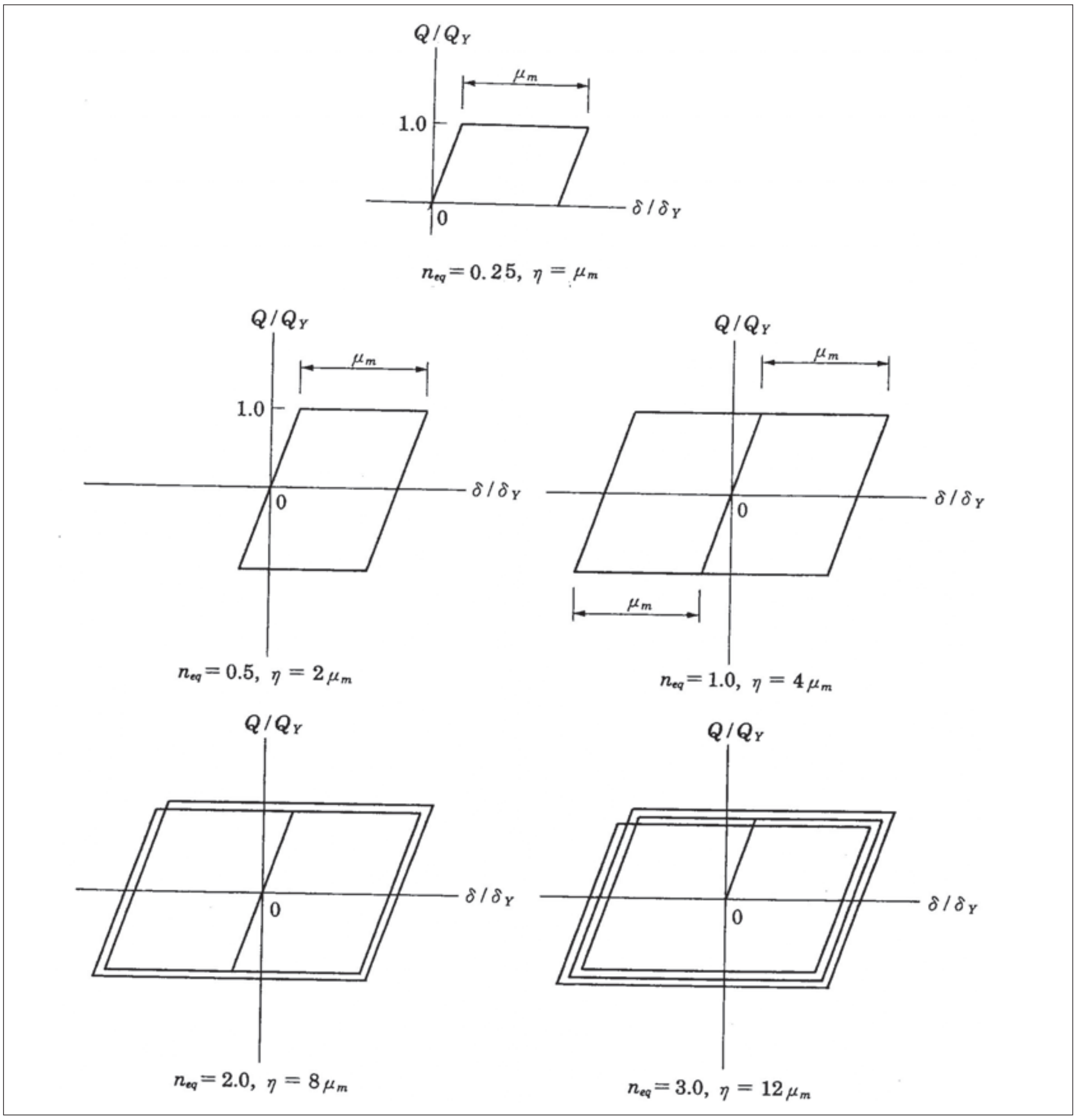

Fig. 5. Equivalent hysteretic loop associated with η/μ_m .

1) To maximize δ_m (maximum elastic deformation).

2) To minimize ${}_s\delta_Y$ (elastic limit deformation of the stiff element).

In particular, to minimize ${}_s\delta_Y$ is a promising approach, resulting also in the reduction of δ_m. Now in Japan, high-rise buildings with high performance are being pursued in a structural type which consists of main structural skeletons remaining elastic and additionally equipped energy absorbing elements with small elastic limit deformations.

6. Conclusions

Structures can withstand earthquakes by absorbing the total energy input, as far as the release of the potential energy in the gravitational acceleration field is prevented. Since the total energy input exerted by an earthquake is a very stable amount, the damage concentration occurs inevitably and accelerates the collapse of structures. There can be various modes of collapse inherent to individual structures. When the structural collapse modes are combined with the $P-\delta$ effect, the structure loses its lateral strength and the total collapse can occur.

The essential measure to eliminate the total collapse is taken by cancelling the $P-\delta$ effect by introducing the elastic element. The generalized structural form equipped with the elastic element is identified to be the flexible-stiff mixed structure and can develop higher performances under earthquakes.

REFERENCES

AKIYAMA, H. (1984): $P-\delta$ effect of energy absorption capacity of steel framed structures subjected to earthquakes, *Trans. A.I.J.*, No. 340, 11-16.

AKIYAMA, H. (1985): *Earthquake-Resistant Limit-State Design for Buildings* (University of Tokyo Press), 1-372.

AKIYAMA, H. (1998): A prospect for future earthquake resistant design, *Eng. Struct.*, **20** (4-6), 447-451.

AKIYAMA, H. (2000): Evaluation of fractural mode of failure in steel structures following Kobe lessons, *J. Constr. Steel Res.*, **55**, 211-227.

NAKASHIMA, M., K. SUITA, K. MORISAKO and Y. MARUOKA (1998): Tests of welded beam-column subassemblies, *J. Struct. Eng.*, **124** (11), 1245-1252.

ROEDER, C.W. (2000): SAC program to assure ductile connection performance, behaviour of steel structures in seismic areas, in *Proceedings of the Third International Conference STESSA 2000*, 659-666.