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On the role of the *J-E* constitutive relationship in applied geoelectromagnetism

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Abstract
In current applications of the Induced Polarization (IP) method, the Debye and Cole-Cole models are used to study relaxation and dispersion properties of rocks, though it is believed that this type of modelisation is confused and vague, because of the lack of a background physical description. In this paper, we show that the Debye model can physically be deduced as a consequence of the electrodynamic behaviour of a mixture of bound and unbound charged particles immersed in an external electric field. We also clarify that the Cole-Cole model is a synthetic model, which can physically be explained as a continuous distribution of Debye terms.

Key words *rock electrodynamics – induced polarization – impedivity models*

1. Introduction

Electrical relaxation and dispersion in rocks are observed in the Time-Domain (TD) and Frequency-Domain (FD), respectively, using any standard Induced Polarization (IP) device. These effects are often investigated in mining and environmental exploration.

Figure 1a-c shows schematically the IP response of a polarizable medium, prospected, *e.g.*, by a quadrupolar electrode device (fig. 1a). In the TD, a voltage transient is detected across the two receiving electrodes, after both the onset and the shutdown of a current step excitation through the two emitting electrodes, as depicted in fig. 1b. A remarkable IP feature is that at the onset of the excitation current, the voltage response suddenly jumps up from zero to a finite value and then gradually rises towards a stable value. Accordingly, at the shutdown of the current, the voltage suddenly falls down to a finite non-null value and then gradually vanishes. In the FD, one observes a voltage-to-current ratio which is a complex function of frequency, showing amplitude and phase spectra as in fig. 1c. Worth noting is the high-frequency asymptote of the amplitude curve, which is a horizontal straight-line generally placed at a finite non-vanishing level, lower than that corresponding to the low-frequency asymptote.

Despite such different behaviour, relaxation in TD and dispersion in FD represent the same physical phenomenon and the relative responses are connected to each other via Fourier Transform (FT). Thus, IP observations are the evidence of a complex form of conduction of electricity in rocks, which cannot be synthesized by Ohm's law. In fact, Ohm's law states that the current density is linearly related to the electrical field by a factor σ, known as the conductivity, which is assumed independent of time t in the TD, or frequency ω in the FD.

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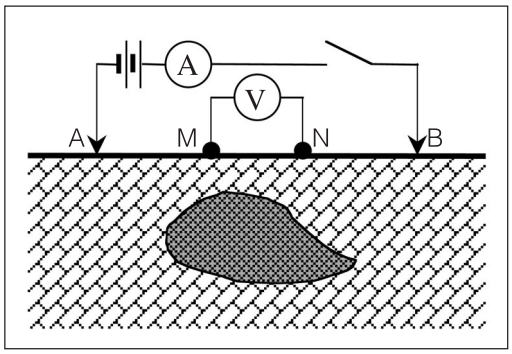
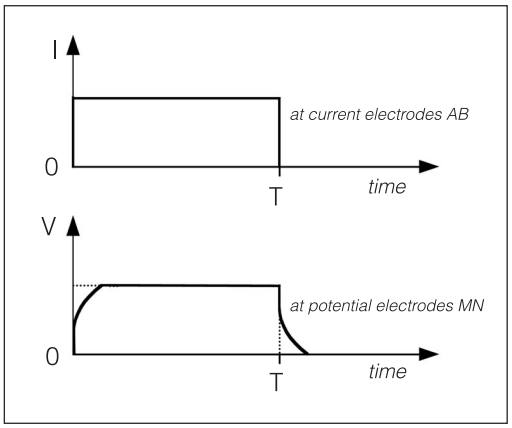


(a)



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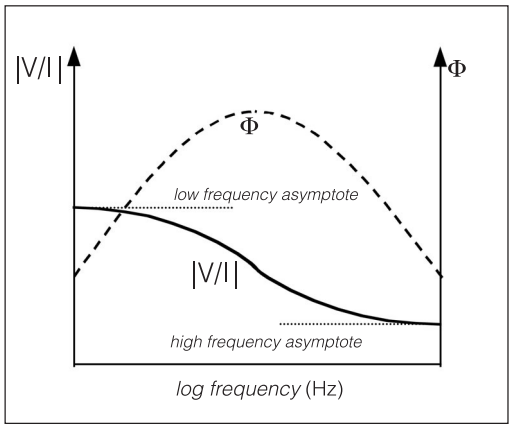

(c)

Fig. 1a-c. The Induced Polarization phenomenology observed with a quadrupolar electrode array (a) in the time-domain (b) and in the frequency-domain (c).

Rigorously speaking, Ohm's law can be assumed valid in rocks only when a Direct Current (DC) regime is utilized, as in the DC resistivity method. With variable currents, as in many natural or artificial source TD or FD electromagnetic (EM) methods, Ohm's law can be adopted provided that relaxation and dispersion effects are negligible, but in general this condition cannot be established beforehand. Therefore, in order to deal with the influence of IP in all EM methods, a generalized current density-to-electrical field relationship, including Ohm's law as particular case, is needed.

In earth materials, many mechanisms have been developed to explain IP effects (Marshall and Madden, 1959; Nilsson, 1971; Zonge and Wynn, 1975; Wong, 1979; Klein *et al.*, 1984; Olhoeft, 1985; Wyller *et al.*, 1992). Comprehensive treatments can be found in the review books by Wait (1959), Bertin and Loeb (1976), Sumner (1976) and Fink *et al.* (1990).

Of the many empirical laws proposed so far, the most utilised in geophysics are the Debye (1928) TD and FD pair (see also Wait, 1959; Patella and Di Maio, 1989)

$$V^D(t) = V_0 \sum_{n=1}^{N} a_n \exp\left(-t / t_n\right) \qquad (1.1a)$$

$$r^D(w) = r_0 \left[1 - \sum_{n=1}^{N} \frac{iwb_n}{1 + iwt_n}\right] \qquad (1.1b)$$

where $\alpha_n \geq 0$, $\tau_n \geq 0$, $\beta_n \geq 0$, $\Sigma\alpha_n = m$ with $0 \leq m \leq 1$, $\beta_n = \alpha_n \tau_n$, and the Cole and Cole (1941) TD and FD pair (see also Pelton *et al.*, 1978)

$$V^{CC}(t) = mV_0 \sum_{n=0}^{\infty} \frac{(-1)^n (t / t)^{nc}}{\Gamma(1 + nc)} \qquad (1.2a)$$

$$r^{CC}(w) = r_0 \left[1 - m \frac{(iwt)^c}{1 + (iwt)^c}\right] \qquad (1.2b)$$

with $0 \leq c \leq 1$, $\tau \geq 0$.

In both pairs of formulas, t and ω are the time and frequency, respectively, $V(t)$ is the voltage during the TD relaxation phase, V_0 is the steady

voltage at the end of the TD charging phase, m is the chargeability (Seigel, 1959), $\rho(\omega)$ is the electric field-to-current density ratio in the FD, called impedivity (Patella, 1993), and ρ_0 is the zero-frequency impedivity, *i.e.* the DC resistivity. Finally, $\Gamma(.)$ is Euler's function, i is the imaginary unit, and α_n, τ_n, β_n and c, τ, m are assumed as heuristic parameters required to adapt the formulas to experimental data. It must be mentioned that the Cole-Cole model includes, as particular cases, Warburg (1899) and Madden and Cantwell (1967) models, for $c = 0.5$ and $c = 0.25$, respectively.

Eqs. (1.1a) and (1.2a) can be derived from (1.1b) and (1.2b), respectively, using the relationship

$$\frac{V(t)}{V_0} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{\rho_0 - \rho(\omega)}{\rho_0} e^{i\omega t}\frac{d\omega}{i\omega}. \quad (1.3)$$

For a background knowledge of the analytical properties of the Debye and Cole-Cole formulas, reference is made to Shuey and Johnson (1973), Patella and Ciminale (1979), Pelton *et al.* (1983, 1984) and Caputo (1993). However, Wait (1982) points out that this characterization of IP is rather confused and vague, because fundamentally non-physical descriptions are employed. To try to overcome this conceptual drawback, in this paper a mechanical approach is suggested to derive a generalised current density-to-electrical field relationship, allowing the physical properties of Debye and Cole-Cole descriptions to be fully investigated.

2. The J-E constitutive relationship in presence of polarization

We consider Ampère-Maxwell equation in the TD written as (Stratton, 1941)

$$\nabla \times \boldsymbol{h}(t) = \boldsymbol{j}'(t) + \partial \boldsymbol{d}'(t)/\partial t \quad (2.1)$$

where $\boldsymbol{h}$ is the magnetic field, and $\boldsymbol{d}'$ and $\boldsymbol{j}'$ are the bound charges displacement field and the unbound charges current density, respectively.

Postulating that $\boldsymbol{j}'$ and $\boldsymbol{e}$, and $\boldsymbol{d}'$ and $\boldsymbol{e}$ are related to each other by a convolutional form typical of a linear, causal and time-invariant system, eq. (2.1) can be expanded as

$$\nabla \times \boldsymbol{h}(t) = \int_0^\infty \tilde{\sigma}'(t')\boldsymbol{e}(t-t')dt' + \frac{\partial}{\partial t}\int_0^\infty \tilde{\varepsilon}'(t')\boldsymbol{e}(t-t')dt' \quad (2.2)$$

where the causal functions $\tilde{\sigma}'(t')$ and $\tilde{\varepsilon}'(t')$ represent the electrical conductivity and dielectric permeability impulse responses, respectively.

Applying the derivation rule of a convolution integral and putting

$$\tilde{\sigma}(t') = \left[\tilde{\sigma}'(t') + \partial\tilde{\varepsilon}'(t')/\partial t'\right] \quad (2.3)$$

Ampère-Maxwell equation takes the compact form

$$\nabla \times \boldsymbol{h}(t) = \int_0^\infty \tilde{\sigma}(t')\boldsymbol{e}(t-t')dt'. \quad (2.4)$$

The right-hand integral in eq. (2.4) defines the total current density vector $\boldsymbol{j}(t)$, say

$$\boldsymbol{j}(t) = \int_0^\infty \tilde{\sigma}(t')\boldsymbol{e}(t-t')dt'. \quad (2.5)$$

As a consequence of eq. (2.3), the causal function $\tilde{\sigma}(t)$, which is called the admittivity impulse response, includes the electrical conductivity $\tilde{\sigma}'(t)$ and dielectric permeability $\tilde{\varepsilon}'(t)$ impulse responses.

In the FD, putting with $\boldsymbol{J}(\omega)$, $\boldsymbol{E}(\omega)$, $\sigma(\omega)$, $\sigma'(\omega)$ and $\varepsilon'(\omega)$ the FT of $\boldsymbol{j}(t)$, $\boldsymbol{e}(t)$, $\tilde{\sigma}(t)$, $\tilde{\sigma}'(t)$ and $\tilde{\varepsilon}'(t)$, respectively, eq. (2.3) and eq. (2.5) are transformed, respectively, into

$$\sigma(\omega) = \sigma'(\omega) + i\omega\varepsilon'(\omega) \quad (2.6)$$

and

$$\boldsymbol{J}(\omega) = \sigma(\omega)\boldsymbol{E}(\omega). \quad (2.7)$$

Equation (2.7) can equivalently be put in the form

$$\boldsymbol{E}(\omega) = \rho(\omega)\boldsymbol{J}(\omega) \quad (2.8)$$

where $\rho(\omega) = 1/\sigma(\omega)$. The frequency-dependent functions $\rho(\omega)$ and $\sigma(\omega)$ are called here *impedivity* and *admittivity*, respectively, and represent a useful extension of the classical parameters of the resistivity ρ and conductivity σ, respectively.

Using now the identity

$$\tilde{\sigma}'(t') = \frac{\partial}{\partial t'} \int_{-\infty}^{t'} \tilde{\sigma}'(\tau)\, d\tau \tag{2.9}$$

equation (2.2) can also be written as

$$\nabla \times \boldsymbol{h}(t) = \frac{\partial}{\partial t} \int_0^{\infty} \left[\int_{-\infty}^{t'} \tilde{\sigma}'(\tau) d\tau + \tilde{\varepsilon}'(t') \right] \boldsymbol{e}(t - t')\, dt'. \tag{2.10}$$

Putting

$$\tilde{\varepsilon}(t') = \int_{-\infty}^{t'} \tilde{\sigma}'(\tau) d\tau + \tilde{\varepsilon}'(t') \tag{2.11}$$

Ampère-Maxwell eq. (2.1) finally takes also the compact form

$$\nabla \times \boldsymbol{h}(t) = \frac{\partial}{\partial t} \int_0^{\infty} \tilde{\varepsilon}(t') \boldsymbol{e}(t - t') dt' \tag{2.12}$$

whose right-hand integral defines now the total displacement field vector $\boldsymbol{d}(t)$, say

$$\boldsymbol{d}(t) = \int_0^{\infty} \tilde{\varepsilon}(t') \boldsymbol{e}(t - t') dt'. \tag{2.13}$$

According to eq. (2.11), the causal function $\tilde{\varepsilon}(t)$, which is called permittivity impulse response, also includes the electrical conductivity and dielectric permeability impulse responses $\tilde{\sigma}'(t)$ and $\tilde{\varepsilon}'(t)$.

In the FD, using the FT, eq. (2.11) and eq. (2.13) are transformed, respectively, into

$$\varepsilon(\omega) = \pi\sigma'(0)\delta(\omega) + \frac{\sigma'(\omega)}{i\omega} + \varepsilon'(\omega) \tag{2.14}$$

and

$$\boldsymbol{D}(\omega) = \varepsilon(\omega)\boldsymbol{E}(\omega) \tag{2.15}$$

where $\delta(\omega)$ is the Dirac delta function and $\boldsymbol{D}(\omega)$ is the FT of $\boldsymbol{d}(t)$.

Equation (2.15) can equivalently be put in the form

$$\boldsymbol{E}(\omega) = \eta(\omega)\boldsymbol{D}(\omega) \tag{2.16}$$

where $\eta(\omega) = 1/\varepsilon(\omega)$. The frequency-dependent parameters $\eta(\omega)$ and $\varepsilon(\omega)$ are here called *preventivity* and *permittivity*, respectively, and represent an extension of the classical concept of dielectric permeability ε.

Equation (2.13) is fully equivalent to eq. (2.5): in fact, comparing eq. (2.3) with eq. (2.11), we readily derive the transformation formulas

$$\tilde{\sigma}(t) = \partial\tilde{\varepsilon}(t) / \partial t \tag{2.17}$$

$$\tilde{\varepsilon}(t) = \int_{-\infty}^{t} \tilde{\sigma}(t)\, d\tau \tag{2.18}$$

and finally, comparing eq. (2.4) with eq. (2.12) and eq. (2.5) with eq. (2.13), we readily get

$$\boldsymbol{j}(t) = \partial\boldsymbol{d}(t) / \partial t. \tag{2.19}$$

Concluding, we can state that for linear, causal and time-invariant natural systems, electrical current density and displacement field are equivalent descriptive vectors of the electromagnetic properties of matter. The bulk response can indifferently be synthesized either by the admittivity or the permittivity parameters, which are linked together with simple transformation formulas. By this formulation, a distinction between unbound and bound charge carriers, in relation to the long- or short-distance capacity of movement under the influence of an external force, becomes unessential.

In order to make Ampère-Maxwell equation applicable in practice we must define the physical model of the admittivity (permittivity) TD and FD functions, for which we need to study at first the electrodynamic behaviour of a charged particle. To this aim, it is worth explaining beforehand how we shall combine the bulk electrical properties of a geophysical

structure and the microscopic electrodynamic properties of a compound of charged particles. Indeed, in all equations so far developed, we have not indicated the spatial dependence of the constitutive parameters and field vectors, tacitly admitting that the medium is a continuum. This is a useful, practical assumption at the scale of the geophysical field and laboratory experiments. However, as will be discussed later, the physical derivation of the analytical expression of the constitutive parameters requires a zooming at the smallest scale of the charge carriers, for which discontinuous properties must be introduced. Thus, the medium will be considered as an aggregate of elementary cells with unitary volume, each cell being sufficiently small to assimilate the medium to a continuum and sufficiently large to incorporate the full diversity of a charged mixture.

3. The electrodynamic equation of a charged particle

Let us consider a charged particle confined in a medium immersed in an external electric field $\boldsymbol{e}(t)$. The trajectory $\boldsymbol{r}(t)$ of the particle can be determined by solving the following general mechanical differential equation

$$\sum_{\lambda=0}^{2} m_\lambda \frac{d^\lambda \boldsymbol{r}(t)}{dt^\lambda} = q\boldsymbol{e}(t) \tag{3.1}$$

where q and m_2 are the charge and mass of the particle, m_1 is a friction-like coefficient accounting for dissipative effects due to collisions and interactions with other charged particles, and m_0 is an elastic-like coefficient accounting for recall effects on bound charges. Obviously, $m_\lambda \geq 0$ for any λ.

In the FD eq. (3.1) transforms to

$$\sum_{\lambda=0}^{2} m_\lambda (i\omega)^\lambda \boldsymbol{R}(\omega) = q\boldsymbol{E}(\omega) \tag{3.2}$$

where $\boldsymbol{R}(\omega)$ is the FT of $\boldsymbol{r}(t)$. From eq. (3.2) we readily obtain the solution for $\boldsymbol{R}(\omega)$ as

$$\boldsymbol{R}(\omega) = \frac{q}{\sum_{\lambda=0}^{2} m_\lambda (i\omega)^\lambda} \boldsymbol{E}(\omega). \tag{3.3}$$

Supposing that the system has $K > 0$ equal charged particles per unit of volume, by definition it is (Stratton, 1941; Parkhomenko, 1967)

$$\boldsymbol{J}(\omega) = Kqi\omega\boldsymbol{R}(\omega) \tag{3.4}$$

which, using eq. (3.4), becomes

$$\boldsymbol{J}(\omega) = \frac{i\omega Kq^2}{\sum_{\lambda=0}^{2} m_\lambda (i\omega)^\lambda} \boldsymbol{E}(\omega). \tag{3.5}$$

Comparing eq. (2.7) with eq. (3.5), we get

$$\sigma(\omega) = \frac{i\omega Kq^2}{\sum_{\lambda=0}^{2} m_\lambda (i\omega)^\lambda}. \tag{3.6}$$

Though limited to only one species of charge carrier, eq. (3.6) is a useful admittivity starting model. It describes a circuit-like cell with a resistance-capacitance-inductance series combination. In fact, where it is $\omega m_2 << m_1$ and $m_0 << \omega m_1$, eq. (3.6) reduces to

$$\sigma(\omega) = \frac{Kq^2}{m_1} \tag{3.7}$$

which describes a purely resistive cell, as assumed in DC geoelectrical methods. Where, instead, it is $\omega m_1 << m_0$ and $\omega^2 m_2 << m_0$ eq. (3.6) reduces to

$$\sigma(\omega) = \frac{i\omega Kq^2}{m_0} \tag{3.8}$$

describing a purely capacitive cell. Finally, if

$m_1 << \omega m_2$ and $m_0 << \omega^2 m_2$ eq. (3.6) reduces to

$$\sigma(\omega) = \frac{Kq^2}{i\omega m_2} \tag{3.9}$$

which represents the admittivity of a purely inductive cell.

From eq. (3.6) more complex cases can also be discussed. If unbound charges ($m_0 = 0$) are considered, the admittivity model results to be

$$\sigma(\omega) = \frac{Kq^2}{m_1 + i\omega m_2} \tag{3.10}$$

which fits the behaviour of a circuit-like cell consisting of a resistance and inductance in series. This model is assumed to explain dispersion effects in metals (Stratton, 1941).

For bound charges ($m_0 \sum 0$) with negligible inertia ($m_2 \approx 0$) the admittivity reduces to

$$\sigma(\omega) = \frac{i\omega Kq^2}{m_0 + i\omega m_1} \tag{3.11}$$

which characterises a circuit-like cell made of a resistance and capacitance in series.

Finally, for bound charges ($m_0 \neq 0$) with vanishing friction ($m_1 \approx 0$) the admittivity is

$$\sigma(\omega) = \frac{i\omega Kq^2}{m_0 - \omega^2 m_2} \tag{3.12}$$

which characterises a circuit-like cell made of an inductance and capacitance in series.

4. The Debye impedivity model

Let us consider now a system with two different species of charge carriers and put with K_j, q_j and $m_{\lambda j}$ the number per unit of volume, the electrical charge and the passive coefficients, respectively, of the carriers of the j-th species (j = 1,2). Such a pair of ionic species can, *e.g.*, be the result of ionic dissociation of a salt dissolved in pore water.

Since the total current density in the FD is now defined as (Parkhomenko, 1967)

$$\mathbf{J}(\omega) = i\omega \sum_{j=1}^{2} K_j q_j \mathbf{R}_j(\omega) \tag{4.1}$$

we readily obtain the following analytical model of admittivity

$$\sigma(\omega) = \sum_{j=1}^{2} \frac{i\omega K_j q_j^2}{m_{0j} + i\omega m_{1j} - \omega^2 m_{2j}} \tag{4.2}$$

For our purpose, the most interesting case to investigate is when one species ($j = 1$) consists of unbound charges and the other species ($j = 2$) of bound charges, both showing negligible inertia. This simplified physical set can already explain, at least qualitatively, non-resonant IP phenomena in rocks (Keller and Frischknecht, 1966). In fact, we have a two-branch parallel circuit-like cell, with a branch consisting of a single resistance and the other branch of a resistance-capacitance series combination, *i.e.*

$$\sigma(\omega) = \frac{K_1 q_1^2}{m_{11}} + \frac{i\omega K_2 q_2^2}{m_{02} + i\omega m_{12}}. \tag{4.3}$$

Putting

$$a = \frac{K_1 q_1^2}{m_{11}} \tag{4.4}$$

$$\tau = \frac{K_1 q_1^2 m_{12} + K_2 q_2^2 m_{11}}{K_1 q_1^2 m_{02}} \tag{4.5}$$

$$b = \frac{K_2 q_2^2 m_{11}}{K_1 q_1^2 m_{02}} \tag{4.6}$$

with $a, \tau, b \geq 0$ and $\tau \geq b$, from eq. (4.3) we obtain the admittivity function as

$$\sigma(\omega) = \frac{a(1 + i\omega\tau)}{1 + i\omega(\tau - b)} \tag{4.7}$$

and the impedivity function as

$$\rho(\omega) = \frac{1 + i\omega(\tau - b)}{a(1 + i\omega\tau)}. \tag{4.8}$$

We have thus got admittivity and impedivity functions qualitatively reflecting the IP frequency spectral evolution from $\sigma_0 = \lim_{\omega \to 0} \sigma(\omega) = a$, or $\rho_0 = \lim_{\omega \to 0} \rho(\omega) = 1/a$, to $\sigma_\infty = \lim_{\omega \to \infty} \sigma(\omega) = a\tau/(\tau - b) \Delta a$, or $\rho_\infty = \lim_{\omega \to \infty} \rho(\omega) = (\tau - b)/a\tau \leq 1/a$.

Generalising, we can now consider within each elementary cell either a parallel or a series combination of N two-branch circuits, or a network of both combinations. For a parallel combination, the admittivity becomes

$$\sigma(\omega) = \sum_{n=1}^{N} a_n \frac{1 + i\omega\tau_n}{1 + i\omega(\tau_n - b_n)} \qquad (4.9)$$

whereas for a series combination, the impedivity becomes

$$\rho(\omega) = \sum_{n=1}^{N} \frac{1 + i\omega(\tau_n - b_n)}{a_n(1 + i\omega\tau_n)} \qquad (4.10)$$

which takes the same form as in eq. (1.1b), putting

$$\rho_0 = \lim_{\omega\to 0} \rho(\omega) = \sum_{n=1}^{N} (1/a_n) \qquad (4.11)$$

$$\beta_n = b_n / a_n \rho_0. \qquad (4.12)$$

5. The Cole-Cole impedivity model

Shuey and Johnson (1973) showed that the analysis of the decay spectrum allows the properties of an impedivity model to be more easily deduced and understood.

The decay spectrum $A(k)$ is defined as

$$A(k) = \lim_{\gamma\to 0} \frac{\rho(ik - \gamma) - \rho(ik + \gamma)}{2\pi i} \qquad (5.1)$$

where $\rho(ik \mp \gamma)$ is any given impedivity function, where ω is replaced by $ik \mp \gamma$ with γ an arbitrarily small positive number.

Using the Debye impedivity model given in eq. (1.1b), after some simple steps we get the Debye decay spectrum $A^D(k)$ as (Patella and Di Maio, 1989)

$$A^D(k) = \rho_0 \sum_{n=1}^{N} \frac{\beta_n}{\tau_n^2} \lim_{\gamma\to 0} \frac{\gamma}{\pi\left[(k - 1/\tau_n)^2 + \gamma^2\right]}. \qquad (5.2)$$

The limit in the right-hand side of eq. (5.2) defines the Dirac delta function $\delta(k\text{-}1/\tau_n)$. Hence, we obtain at last the very important result

$$A^D(k) = \rho_0 \sum_{n=1}^{N} \frac{\beta_n}{\tau_n^2} \delta(k - 1/\tau_n). \qquad (5.3)$$

Now, considering the definition of the δ-function (Papoulis, 1962) applied to a generic decay spectrum $A(k)$

$$A(k) = \int_0^\infty A(\varphi)\delta(k - \varphi)d\varphi \qquad (5.4)$$

it follows that $A(k)$ can always be considered as a continuous distribution of impulses with varying time constants, and thereof approximable to a sum as in eq. (5.3) with any desired accuracy. In fact, discretising eq. (5.4) using the trapezoidal rule for expressing the area under the curve and putting $\varphi = 1/\tau$, we obtain the approximation

$$A(k) \approx \sum_{n=1}^{N} A_n \delta(k - 1/\tau_n) \qquad (5.5)$$

which conforms to eq. (5.3) putting $A_n = \rho_0 \beta_n / \tau_n^2$.

This conclusion allows Cole-Cole, Warburg and Madden-Cantwell models to be considered as a continuous superposition of Debye terms with varying time constants distributed around $k = 1/\tau$. Therefore, τ in eq. (1.2b) takes the additional significance of main relaxation time constant, when the Cole-Cole model is considered as a sum of Debye terms (Pelton *et al.*, 1983), *i.e.* as

$$\rho^{CC}(\omega) \approx \rho_0 \left(1 - \sum_{n=1}^{N} \frac{i\omega\beta_n}{1 + i\omega\tau_n}\right). \qquad (5.6)$$

6. Conclusions

We have demonstrated that the Debye impedivity model for rocks can physically be deduced as a consequence of the electrodynamic behaviour of a mixture of bound and unbound charged particles immersed in an external electrical field. We have also clarified that the Cole-Cole impedivity model is a synthetic

model, which can physically be explained as a continuous distribution of Debye terms.

Though being the Debye and derived Cole-Cole impedivity models related to the physical parameters describing the motion of ionic species in rock capillaries under the influence of an external electric field, no possibility exists to distinguish each ionic species in a compound by means of only IP meaurements. Indeed, admitting that one may be able to single out the N exponential decay terms in the TD voltage transient or the N complex fractions in the FD impedivity spectrum and obtain the set of α_n, τ_n or β_n, τ_n IP parameters, by no way the mechanical and electric parameters of bound and unbound charged particles can be determined singly.

This conclusion contrasts the current belief that using IP it should be possible to distinguish ionic particles and to estimate their abundances, as, *e.g.*, in environmental applications for monitoring contaminants in sediments. In this application field, only vague information on the possible existence of contaminants seems to be deducible from IP experiments. Weller and Börner (1996), *e.g.*, have shown a reliable correlation between the imaginary part of admittivity and porespace internal surface in sediments, and concluded that the observed high sensitivity of the imaginary component to changes at the internal surface may be used as an indicator for contaminations.

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