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Design an Integral Sliding Mode Controller for a Nonlinear System

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Abstract

The goal of this paper is to design a robust controller for controlling a pendulum system. The control of nonlinear systems is a common problem that is facing the researchers in control systems design. The Sliding Mode Controller (SMC) is the best solution for controlling a nonlinear system. The classical SMC consists from two phases. The first phase is the reaching phase and the second is the sliding phase. The SMC suffers from the chattering phenomenon which is considered as a severe problem and undesirable property. It is a zigzag motion along the switching surface. In this paper, the chattering is reduced by using a saturation function instead of sign function. In spite of SMC is a good method for controlling a nonlinear system but it still suffers from long settling time which is considered as undesired property. The Integral Sliding Mode controller (ISMC) can be used to reduce the settling time. Also, the ISMC is a good method for controlling a nonlinear systems. ISMC is simple, has a high performance and can be considered as an effective and powerful technique. In ISMC method, the reaching phaseis eliminated which considered a main part in designing classical SMC. The important property of the ISMC as well as the Classical Sliding Mode Controller (CSMC),is the ability to make the systems asymptotically stable. The pendulum system was used for testing the CSMC and ISMC. The results obtained from the simulation showed the advantages of using the ISMC when comparied with the CSMC. Finally, MATLAB software package was adopted in this paper.

***Keywords**: Chattering phenomenon, Classical sliding mode controller, Integral sliding mode controller, Switching surface.*

1. Introduction

In last two decades, the methods for controlling nonlinear systems take a much interest from many researchers and as a result many methods were developed [1]. One of them is the Sliding Mode Control system (SMC) which is consider as an effective and robust control method that is used successfully in wide variety of systems. The most important property in using SMC is the ability of this controller to make the system insensitive to external disturbance and parameters uncertainty [2]. In spite of the SMC robustness and its better performance, but it is sever from the problem of "chattering phenomenon ", which is considered as drawback property. To reduce this chattering phenomenon in the SMC, many methods were developed to overcome this problem. One of them is by using a boundary layer instead of signmum function in nonlinear part of controller [2]. Other researchers proposed to use a fuzzy logic system with a sliding mode controller to get a new structure called sliding mode fuzzy controller [3]. On the other hands, some researchers suggest to reduce the chattering by using a genetic algorithm [4]. Recently some researches, proposed to use the particle swarm optimization technique in order to reduce the drawbacks of the chattering phenomenon [5]. The advantage of using the SMC is the reduction in order by one from the original system equation [6]. The designed control law in SMC can drive the system state trajectory towards the manifold surface and stay in this surface for all future time until reaching the origin. Churn and We was first introduced the

Integral Sliding Mode (ISMC) [7], which is similar to the SMC since it is insensitive to external disturbance and parameters uncertainty [8]. The control law in the ISMC consists from two major parts, the first part is the nominal control which is responsible for the performance of the nominal system and the second part is the discontinuous control that is used to reject the external disturbance and parameters uncertainty [8]. In this paper the performance of pendulum system will be improved by using the ISMC and the results show high validity when using the proposed controller.

2. Classical Sliding Mode Controller (SMC)

In modern control systems, the SMC is a common method for designing a robust controller technique. This controller was extremely used with nonlinear system since 1950, and it is extended to use with large various types of applications such as electrical servo drives, pendulum, ETV and others. The differential equation that is used to govern the sliding mode control has order less by one than the order of original system. The main drawback of the SMC is the "chattering" which is phenomenon of oscillations having a finite frequency and amplitude along the sliding surface. The problem of chattering phenomenon can be solved by using many methods as mentioned above in section 1. Sliding Mode Controller consists of two major phases [6]:

A: Reaching phase: in this phase the state trajectories are oriented toward the switching surface S=0; hence, the sliding phase will be started at this instant as shown in Figure (1).

B: Sliding phase: in this phase the state trajectory is enforced to stay on the switching surface and to move along this surface until reaching the origin in finite time as shown in Figure (1).

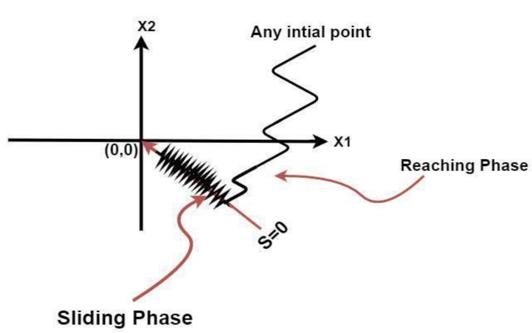

Fig. 1. The two phases of the sliding control [2].

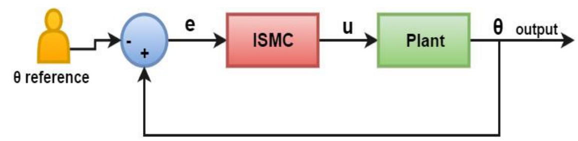

Fig. 2. The shape of sliding surface [2].

The control law is defined as:

$$u = u_{eq+} u_{dis} \quad \text{... (1)}$$

where, u_{eq} is the equivalent control part which required to oriented the system state trajectory toward switching surface ($s = 0$) and u_{dis} is the discontinuous control part to enforce the state trajectory to move along the switching surface towards the origin. The control u_{dis} is defined as below [10]:

$$u_{dis} = -k(x)\, sign(s) \quad \text{... (2)}$$

where, k is a constant with positive value.

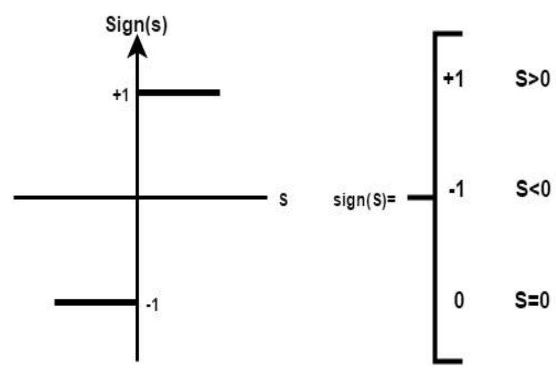

Fig. 3. The shape of a signum function.

By substituting equation (2) in (1), the control law can be rewritten as:

$$u = u_{eq} - k(x) sign(s) \quad \text{... (3)}$$

The sliding surface characterized as:

$$s = \lambda e + \dot{e} \;\; ; \;\; \lambda > 0 \quad \text{... (4)}$$

where, λ is a constant parameter with a positive value.

Let the error and its derivatives defined as:

$x_1 = e = \theta - \theta_f$ and $x_2 = \dot{e} = \dot{\theta}$

where θ_f is the final position that can be considered as the desired position. Then, equation (4) will be rewritten as below:

$$s = \lambda x_1 + x_2 \quad \text{... (5)}$$

for $\lambda = 1$, equation (5) will be as:

$$s = x_1 + x_2$$

the derivative of the sliding variable

$$\dot{s} = \dot{x}_1 + \dot{x}_2 \quad \text{... (6)}$$

The main goal is to keep the $s(x,t)$ close to switching surface in phase plane.
The general form of motion equation for any nonlinear system:

$$\dot{x} = f(x) + B(\mathrm{x})u + d(\mathrm{x},\mathrm{t}) \quad \text{... (7)}$$

To satisfy the condition ($s = 0$) that the right side of equation (6) equal zero by selecting the discontinuous gain $k(x)$ as follows [6]:

$$k(x) = ko + \left(\frac{\frac{\partial So}{\partial x}}{\frac{\partial So}{\partial x}} \frac{d(x,t)}{B_n(x)} \right) , ko > 0 \quad \dots (8)$$

In above control equation (3), the $sign(s)$ function caused a chattering phenomenon. This chattering phenomenon is undesirable property appearing along the sliding surface, the major reason that caused this phenomenon is the "$sign$ function" that is present in control equation (3). The Classical SMC is suffering from the chattering which is considered as a severe problem in SMC. The boundary layer function can be using to reduce the chattering. The $sat\ (s)$ function is used instead of $sign\ (s)$ function in control law. The $sat\ (s)$ function that shown in figure (4) can be described as below [6]:

$$sat(s/\varphi) = \begin{cases} +1 & (s/\varphi > 0) \\ s/\varphi & (-1 < s/\varphi < 1) \\ -1 & (s/\varphi < 0) \end{cases} \quad \dots (9)$$

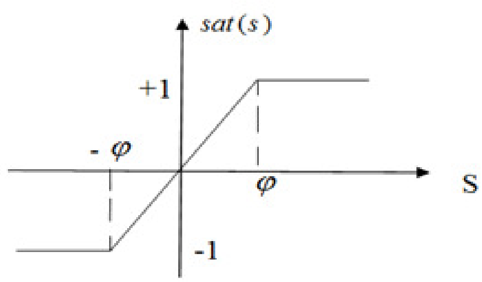

Fig. 4. The $sat\ (s)$ function [2].

The $sign(s)$ function in equation (3) is replaced by $sat\ (s)$ function and the control law can be written as below:

$$u = u_{eq} - k(x)sat \quad \text{... (10)}$$

3. Integral Sliding Mode Controller (ISMC)

Integral sliding mode control (ISMC) is a nonlinear robust controller, designed for controlling nonlinear systems [1]. The goal of the proposed ISMC is to eliminate the reaching phase by enforcing the state trajectory, which starts from any initial state, to be in sliding phase throughout the entire plant trajectory and to slide along the switching surface until reaching the origin. The difference between the ISMC approach and the classical sliding mode is that the order of motion equation in the ISMC is the same as the order of the original system, while in the classical sliding mode; the order is less by one from the original system [9]. The robustness of the system in the ISMC is guaranteed because in final trajectory the error and its derivatives reach zero value. Also in ISMC the system is insensitive to variations of system parameter and external disturbance. The main problem in this controller, as well as in classical SMC, is the chattering phenomenon in the control action; and to reduce this chattering may use some functions such as saturation, dead zone and inverse of tan instead of sign function which is usually used in classical sliding mode.
The complete system with the ISMC is shown in the following figure:

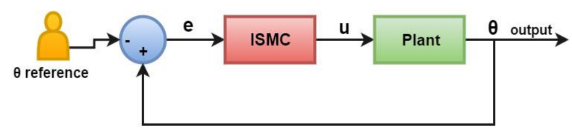

Fig. 5. The closed loop control system using ISMC.

The procedure of designing the ISMC with any nonlinear system can be described as below:

$$s(x) = s_o + z \quad ; \quad z(0) = -s_o(0) \quad \text{... (11)}$$

The $s(x)$ is sliding surface, z is the integral term and s_o defined as in classical sliding mode control as described in equation (5). The integral term $z(0)$ is determined based on the initial condition $s_o(0)$. The integral term z will be selected in order that the sliding variable $s(x)$ has a zero value and this make the system dynamic in the sliding mode from the initial instant of time.
The derivative of the sliding variable $s(x)$ is given as:

$$\dot{s} = \frac{\partial s_o}{\partial x}\dot{x} + \dot{z} \quad \text{... (12)}$$

The second step is to describe the control law of the ISMC as:

$$u = u_n + u_{dis} \quad \text{... (13)}$$

The nominal part of controller u_n is used to maintain the nominal system dynamics with reference characteristics, where u_{dis} is the second part of the controller that is used to reject the external disturbance and parameters uncertainty

As in [9] and by substuting equation (7) in equation (12)

$$\dot{s}=\frac{\partial s_o}{\partial x}\left[f(x)+B(x)u_n+B(x)u_{dis}+d(x,t)\right] +\dot{z} \quad \dots (14)$$

where the discontinuous controller is defined as

$$u_{dis} = -k(x)sign(s) \quad \dots (15)$$

where, $k(x)$ is the same as presented in equation (8) with positive value.

Therefore, equation (12) will be as bellow:

$$u = u_n - k(x)sign(s)$$

To satisfy the rejection of the external disturbance and variation of system parameters, the Integral term is assumed to be as:

$$\dot{z}=-\frac{\partial s_o}{\partial x}\left[f(x)+B(x)u_n\right] \quad \dots (16)$$

By substituting the equation (16) in (14), we will get the equation as:

$$\dot{s}=\frac{\partial s_o}{\partial x}\left[B(x)u_{dis}+d(x,t)\right] \quad \dots (17)$$

In the design of the ISMC, the sliding surface and control will be described as:

$$s(x) = s_o + z \ ; \quad z(0) = -s_o(0)$$
$$\dot{s}(x) = \dot{s}_o + \dot{z} \ ;$$
$$u = u_n - k(x)sign(s) \quad \dots (18)$$

Finally, when using the boundary layer, the equation of control law (18) rewritten as below:

$$u = u_n - k(x)sat \quad \dots (19)$$

4. Plant Description

Consider a second order of the pendulum system described by equation:

$$\ddot{\theta} = -a\sin\theta - b\dot{\theta} + c\big(T + d(t)\big) \quad \dots (20)$$

Where:

θ the angular position of the rod with vertical axis and it is measure in (*radian*) and it consider as the controlled variable (output),

$\dot{\theta}$ the angular velocity and it is measure in (*radian/ second*),

T the torque applied at the end of the pendulum and it measure in (*Newton. meter*) and it is considering as the control input.

and *d(t)* is the external disturbance applied to the system.

A common problem in real plant is the presence of an external disturbance and parameters uncertainty.

The nominal value of parameter *a=10, b=1 and c=10,*

The uncertainties values of above parameters are $\delta a = 0, \delta b = \mp 0.5, \delta c = \mp 5$.

Table 1,
The maximum and minimum pendulum parameters values)

Parameter value	Minimal value	Maximal Value
a	**10**	**10**
b	0.5	1.5
c	5	15

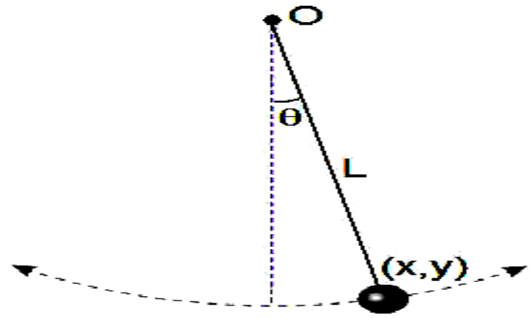

Fig. 6. The simple pendulum [4].

The error of the pendulum system can be described in the state equation as:

Let the error $x_1 = \theta - \theta_f$ and $x_2 = \dot{\theta}$

Where, θ_f is the desired position which is the equilibrium point. Equation above can be rewritten as:

$$\dot{x}_1 = \dot{x}_2 \quad \dots (21)$$

$\dot{x}_2 = -a * \text{si}$

Where,

$a = \delta a \mp a_n,$

$b = \delta b \mp b_n,$

$c = \delta c \mp ac_n$

$$\dot{s} = \dot{x}_1 + \dot{x}_2 \quad \dots (23)$$

By substituting equation (21), (22) and (2) in equation (23),

$k(x) = k_o +$

Case (A): Design the Classical sliding mode controller (CSMC) for pendulum system

The design of CSMC controller is written as in equation (3):

$$u = \frac{1}{c}\left[a\sin\big(x_1+\theta_f\big) + bx_2\right] - k(x)sign(s) \quad \dots (25)$$

when the boundary layer is used, the control law rewritten as described in equation (10):

$$u = \frac{1}{c}\left[a\,sin\big(x_1+\theta_f\big) + bx_2\right] - k(x)sign(s) \quad \dots (26)$$

Case (B): Design the Integral sliding mode controller (ISMC) for pendulum system

The Sliding variable is written according to equation (18) as:

$s(x) = s_o + z\,;\quad z(0) = -s_o(0)$ ... (27)

And

$s_o = x_1 + x_2$ ... (28)

And the error equation for the pendulum system was described as:

$\ddot{e} = -c_1 e - c_2\dot{e}\ \ , c_1 , c_2 > 0$ ... (29)

the values c_1 and c_2 are assigned depending on the required characteristics of plant dynamics.

In ISMC design, the derivative of integral term is described as below:

$\dot{z} = c_1 * e - c_2 * \dot{e} - x_2$ ... (30)

The nominal control is described as:

$u_n = \frac{1}{c}[($

Finally, the control law is written according to equation (18) as:

$u = \frac{1}{c}[a$

when the boundary layer is used, the above equation will be rewritten as:

$u = \frac{1}{c}[a$

$- k(x)sat$ ... (33)

5. The Simulation Results

Case (A): The classical sliding mode controller (CSMC) with a *sign* function

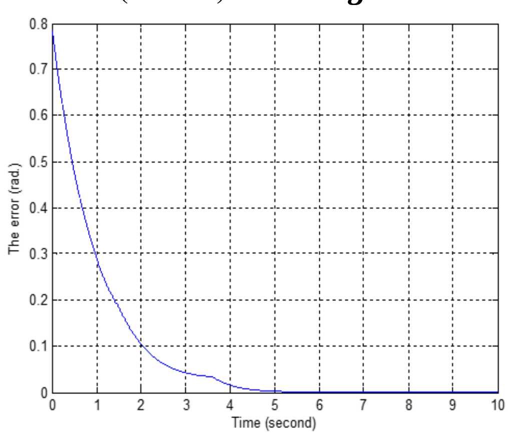

Fig. 7. The error x_1 vs. time of the CSMC.

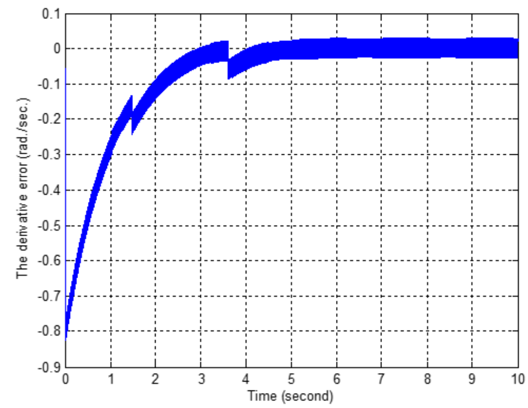

Fig. 8. The derivative error x_2 vs. time of the CSMC.

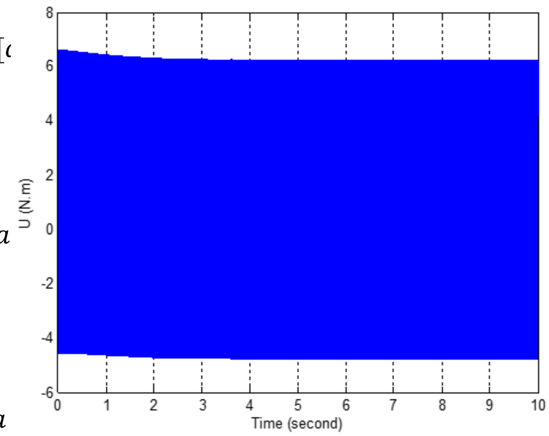

Fig. 9. The control action u vs. time of the CSMC.

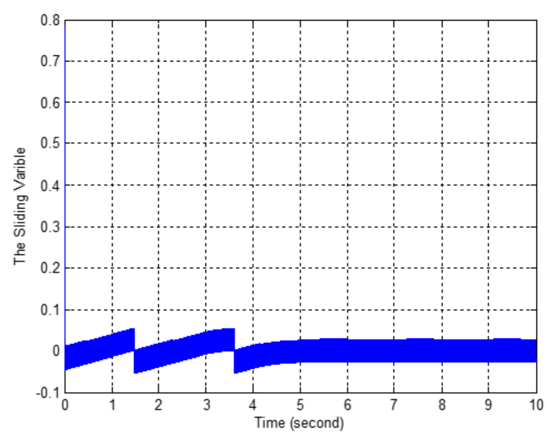

Fig. 10. The sliding variable S vs. time of the classical SMC .

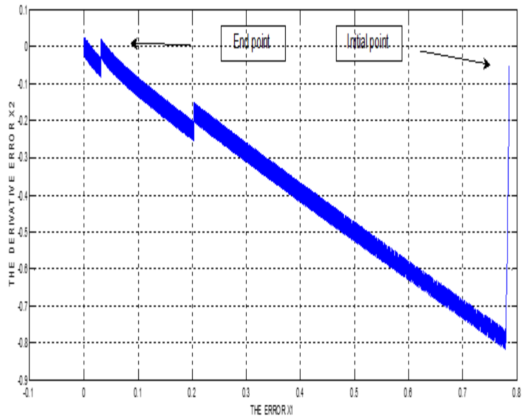

Fig. 11. The plot of the phase plane between x_1 and x_2 of the classical SMC.

Case (B): The classical sliding mode controller (CSMC) with boundary layer

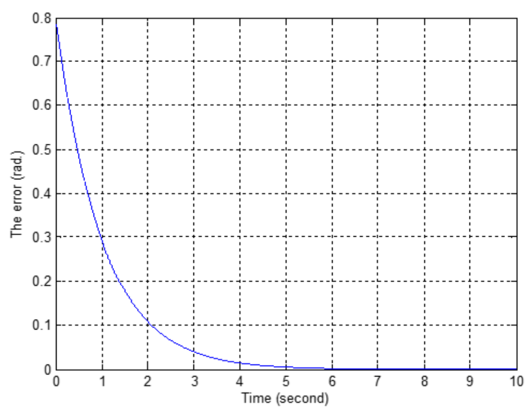

Fig. 12. The error x_1 vs. time of the CSMC.

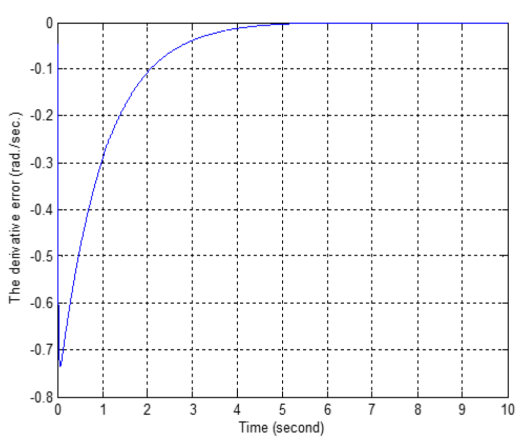

Fig. 13. The derivative error x_2 vs. time of the CSMC.

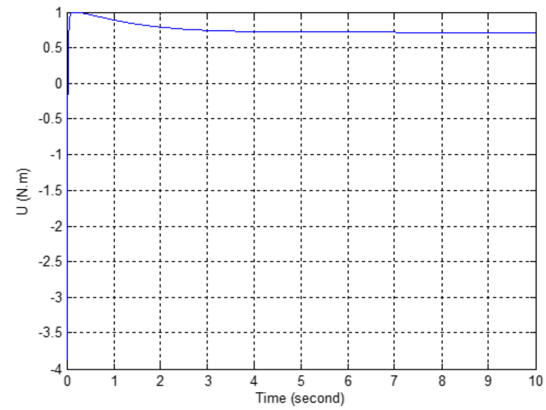

Fig. 14. The control action u vs. time of the CSMC.

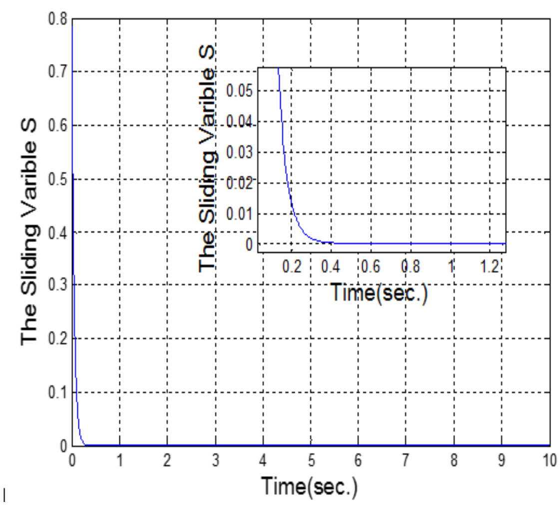

Fig. 15. The plot of sliding variable S vs. time of the CSMC.

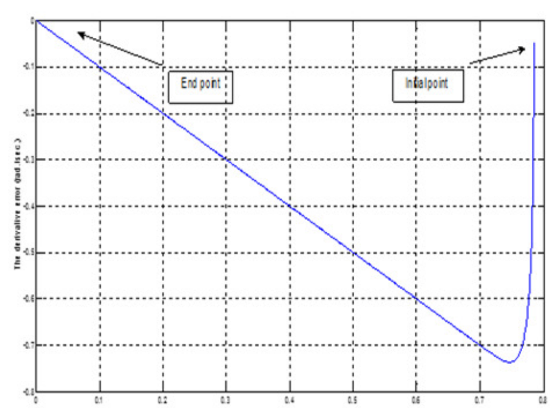

Fig. 16. The plot of phase plane between the error x_1 and the derivative error x_2 of the CSMC.

Case (C): The integral sliding mode controller with a $sign$ function

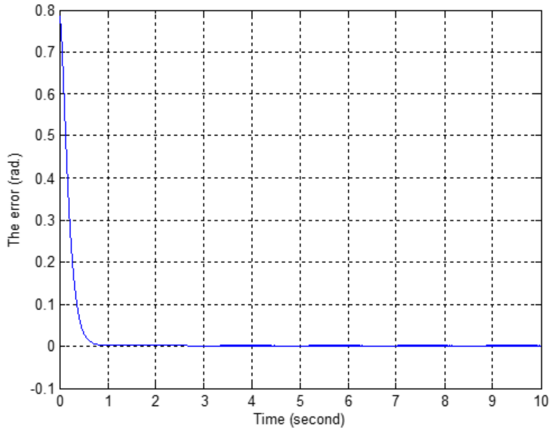

Fig. 17. The error x_1 vs. time of the ISMC.

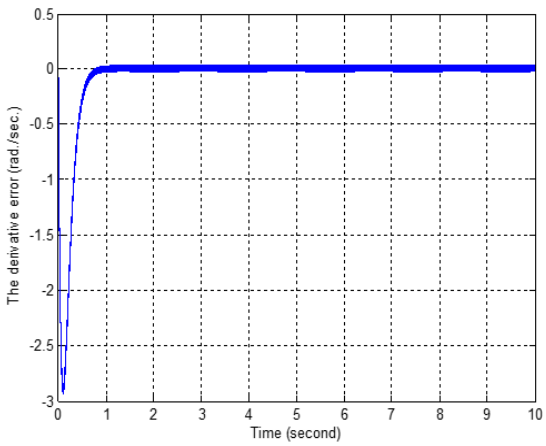

Fig. 18. The derivative error $x_2$vs. time of the ISMC.

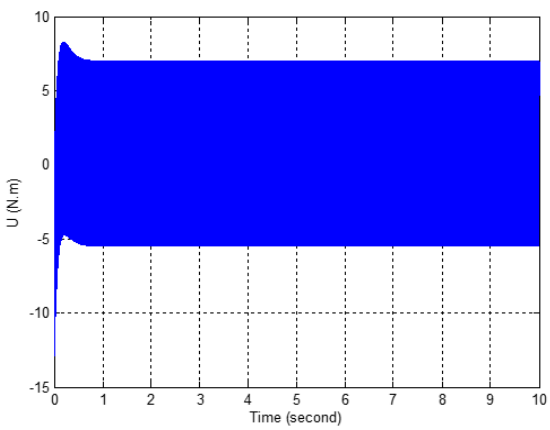

Fig. 19. The control action u vs. time of the ISMC.

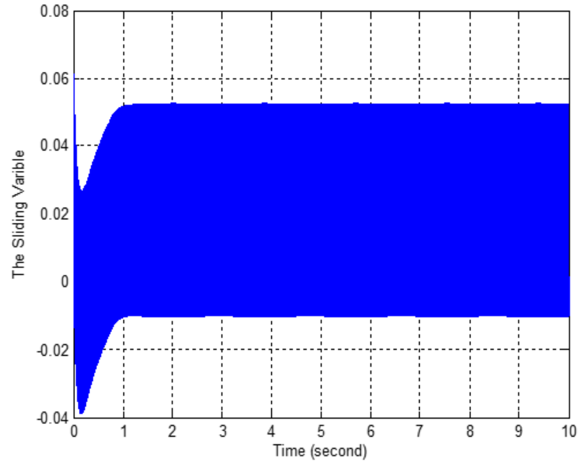

Fig. 20. The plot of sliding variable S vs. time of the ISMC.

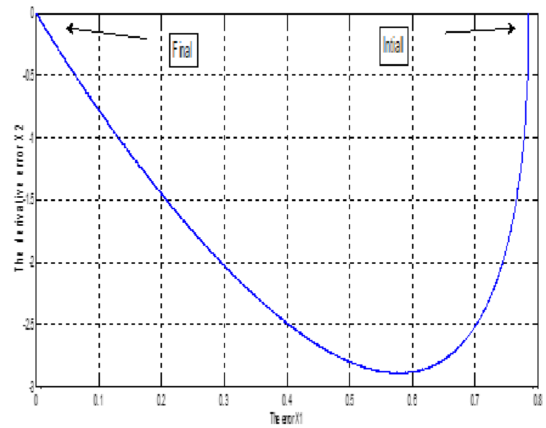

Fig. 21. The plot of phase plane between the error x_1 and the derivative error x_2 of the ISMC.

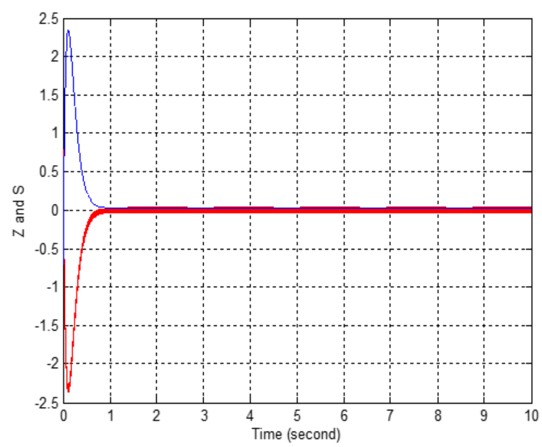

Fig. 22. The plot of the Sliding variable S and the Integral term Z vs. time of the ISMC

Case (D): The integral sliding mode controller with boundary layer

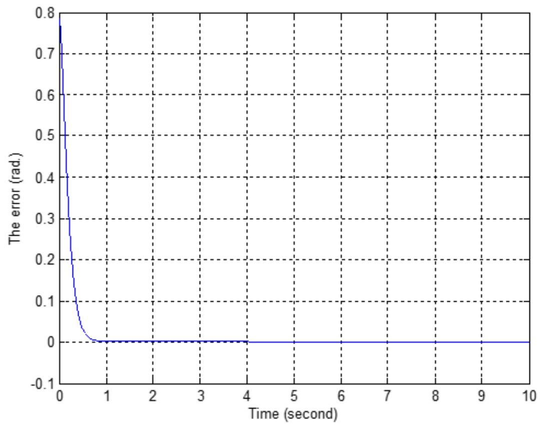

Fig. 23. The error x_1 vs. time of the ISMC.

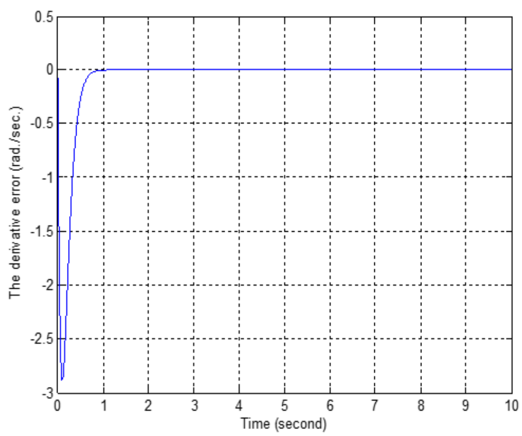

Fig. 24. The derivative error x_2 vs. time of the ISMC.

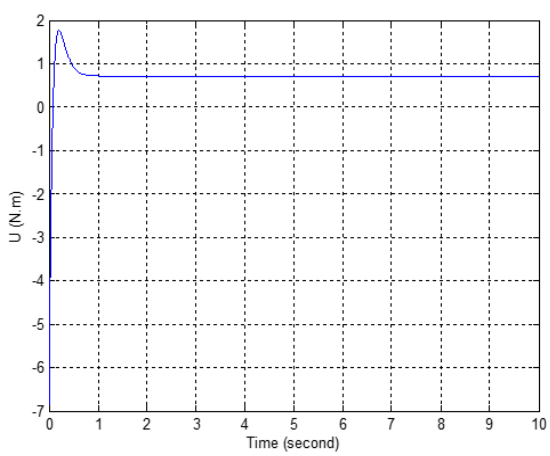

Fig. 25. The control action u vs. time of the ISMC.

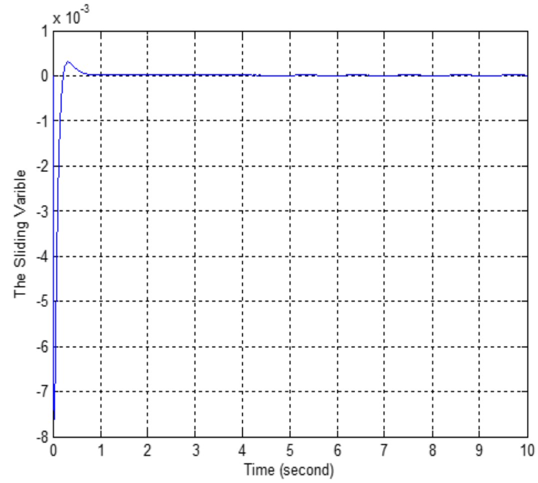

Fig. 26. The Sliding variable S vs. time of the ISMC.

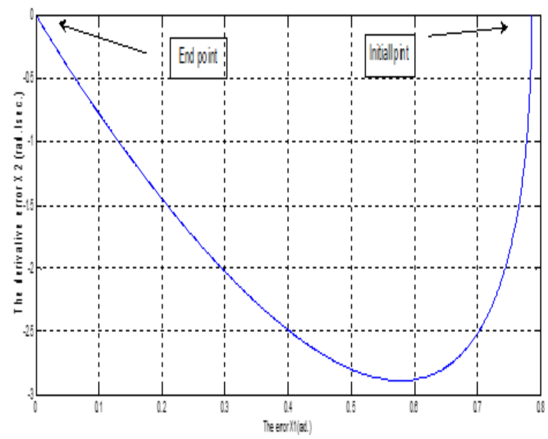

Fig. 27. The plot of phase plane between the error x_1 and the derivative error x_2 of the ISMC.

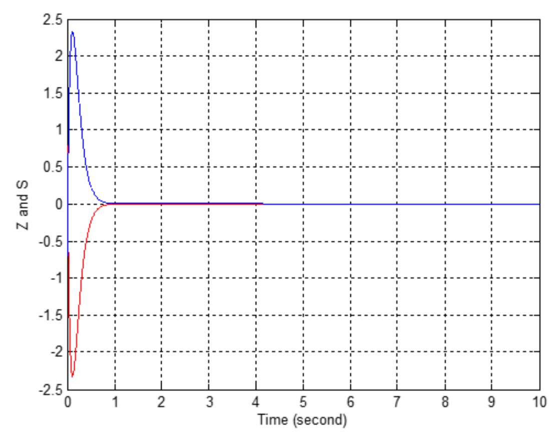

Fig. 28. The plot of the Sliding variable S and the Integral term Z vs. time of the ISMC.

6. Discussion

In this work, SMC and ISMC have been considered for controlling the position of the pendulum system. The results of SMC and ISMC have been included in this work to show the proprties of each controllers with the presence of the external disturbance and paramrters uncertinty. Each of the above controllers has the ablility to make the system asymptotically stable under the effect of external disturbance and the parameters uncertainty by making the error and the derivative of error equal to zero value as shown in figures (7), (8), (17) and (18). Both the SMC and ISMC are suffering from the chattering problem because of the effect of $sign(x)$ function as shown clearly in figures (9) and (19), this problem is solved by using the boundary layer as shown in figures (14) and (16). When using the $sign(x)$ function, the state trajectory in the CSMC hits the switching surface vertically as shown in figures (11) and (9), and this caused a chattering phenomenun, while in using the boundary layer the state trajectory hits the sliding surface in arc shape as shown in figures (16) and (14). The same thing was apear clearly when using the ISMC as shown in figures (21), (19), (27) and (25). The results show that the effect of the external disturbance and the parameters uncertainty of the dynamic system is cancelled by using the ISMC as shown in figures (25) and (27). In figure (27), the error (x_1) and the dervitive of error (x_2) reaches the origin in final trajectory, which means that its values equal to zero.

7. Conclusion

The most important improvement of using ISMC is the reducing of the settling time response of system comparing with the CSMC as shown in figures (12) and (23). In the CSMC the settling time is 6.5 sec as shown in figure (12), where in the ISMC the settling time is reduced to 0.8 sec as shown in figure (23). The ISMC consists of two parts, The first part is nominal control that is used to control the nominal system dynamics while the second part is the discontinous control which is used to reject the pertubaration term (the perterbation term consists of the external disturbance and parameters uncertainty).In using the ISMC, the pendulum system is is preseented by the nominal model from the from thr first instsnt . this nominal model is not effected by the perturbation term. The results shows that the CSMC and ISMC can be considered as a rubust controller, since it can give a good response even with the presence of disturbance and parameters uncertainty as shown in figures (14) and (25).
From figures (11), (16), (21) and (27), it is seen clearly that the CSMC and ISMC are able to make the system asymptotically stable.

8. References

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تصميم مسيطر تكاملي منزلق النمط للسيطرة على الأنظمة اللا خطية

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الخلاصة

الهدف من هذا البحث هو تصميم مسيطر متين وفعال للسيطرة على منظومة البندول. تعد السيطرة على المنظومات اللاخطية من المشكلات الشائعة التي تواجه الباحثين عند تصميم منظومات السيطرة. المسيطر ذي السطح الانزلاقي التقليدي هو احد الحلول للسيطرة على المنظومات اللا خطية . في المسيطر ذا السطح الانزلاقي التقليدي تتكون إشارة السيطرة من جزئين هما مرحله الوصول ومرحله الانزلاق . وهذا المسيطر يعاني من ظاهرة التذبذب التي هي أشارة متعرجه على طول سطح الانزلاق . في هذا البحث تم تقليل هذا التذبذب باستخدام دالة الإشباع (sat) بدل من داله الأشارة (sign).بالرغم من ان المسيطر ذي السطح الانزلاقي هو طريقه جيدة للسيطرة على الأنظمة اللاخطية لكنه يبقى يعاني من طول وقت الوصول والتي تعد خاصية غير مرغوب فيها. المسيطر ذا سطح الانزلاقي التكاملي يستطيع تقليل وقت الوصول. كذلك ان المسيطر ذي السطح الانزلاقي التكاملي يعتبر طريقه جيدة للسيطرة على الانظمة اللاخطية. المسيطر ذو السطح الانزلاقي التكاملي هو مسيطر بسيط ذو مواصفات عالية و يعتبر من التقنيات الفعالة والمفيدة . في المسيطر الانزلاقي التكاملي يتم الغاء مرحله الوصول والتي هي جزء أساس من المسيطر ذو السطح لانزلاقي التكاملي. من أهم خصائص المسيطر ذو السطح الانزلاقي التكاملي وكذلك المسيطر ذو السطح الانزلاقي التقليدي هو جعل المنظومة في حالة استقرار (أشارة الخطأ ومشتقة أشارة الخطأ قيمتهم صفر) . ومن اهم المميزات زمن الوصول الى الاستقرار (ts) اقل في المسيطر ذو السطح الانزلاقي التكاملي . في هذا البحث تم استخدام منظومة البندول لأختبار المسيطرات أعلاه وكذلك النتائج المستحصلة أظهرت فعالية المسيطرات وأخيرا" تم استخدام برنامج الماتلاب للبرمجة للحصول على نتائج.