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# Cam Dynamic Synthesis 

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#### Abstract

The paper presents an original method to make the geometric synthesis of the rotary cam and translated tappet with roll. Classical method uses to the geometric synthesis and the reduced tappet velocity, and in this mode the geometric classic method become a geometric and kinematic synthesis method. The new geometric synthesis method uses just the geometric parameters (without velocities), but one utilizes and a condition to realize at the tapped the velocities predicted by the tapped movement laws imposed by the cam profile. Then, it makes the dynamic analyze for the imposed cam profile, and one modify the cam profile geometric parameters to determine a good dynamic response (functionality). In this mode it realizes the dynamic synthesis of the cam, and we obtain a normal functionality.


Keywords: Geometric synthesis, Dynamic synthesis, Rotary cam, Translated tappet with roll.

## 1. Introduction

In conditions which started to magnetic motors, oil fuel is decreasing, energy which was obtained by burning oil is replaced with nuclear energy, hydropower, solar energy, wind, and other types of unconventional energy, in the conditions in which electric motors have been instead of internal combustion in public transport, but more recently they have entered in the cars world (Honda has produced a vehicle that uses a compact electric motor and electricity consumed by the battery is restored by a system that uses an electric generator with hydrogen combustion in cells, so we have a car that burns hydrogen, but has an electric motor), which is the role and prospects which have internal combustion engines type Otto or Diesel?

Internal combustion engines in four-stroke (Otto, Diesel) are robust, dynamic, compact, powerful, reliable, economic, autonomous, independent and will be increasingly clean.

Let's look at just remember that any electric motor that destroy ozone in the atmosphere needed our planet by sparks emitted by collecting brushes. Immediate consequence is that if we only use electric motors in all sectors, we'll have problems with higher ozone shield that protects our planet and without which no life could exist on Earth.

Magnetic motors (combined with the electromagnetic) are just in the beginning, but they offer us a good perspective, especially in the aeronautics industry.

Probably at the beginning they will not be used to act as a direct transmission, but will generate electricity that will fill the battery that will actually feed the engine (probably an electric motor).

The Otto engines or those with internal combustion in general, will have to adapt to hydrogen fuel.

It is composed of the basic (hydrogen) can extract industrially, practically from any item (or
combination) through nuclear, chemical, photonic by radiation, by burning, etc... (Most easily hydrogen can be extracted from water by breaking up into constituent elements, hydrogen and oxygen; by burning hydrogen one obtains water again that restores a circuit in nature, with no losses and no pollution).

Hydrogen must be stored in reservoirs cell (a honeycomb) for there is no danger of explosion; the best would be if we could breaking up water directly on the vehicle, in which case the reservoir would feed water (and there were announced some successful).

As a backup, there are trees that can donate a fuel oil, which could be planted on the extended zone, or directly in the consumer court. With many years ago, Professor Melvin Calvin, (Berkeley University), discovered that "Euphora" tree, a rare species, contained in its trunk a liquid that has the same characteristics as raw oil. The same professor discovered on the territory of Brazil, a tree which contains in its trunk a fuel with properties similar to diesel.

During a journey in Brazil, the natives driven him (Professor Calvin) to a tree called by them "Copa-Iba".

At the time of boring the tree trunk, from it to begin flow a gold liquid, which was used as indigenous raw material base for the preparation of perfumes or, in concentrated form, as a balm. Nobody see that it is a pure fuel that can be used directly by diesel engines.

Calvin said that after he poured the liquid extracted from the tree trunk directly into the tank of his car (equipped with a diesel), engine functioned irreproachable.

In Brazil the tree is fairly widespread. It could be adapted in other areas of the world, planted in the forests, and the courts of people.

From a jagged tree is filled about half of the tank; one covers the slash and it is not open until after six months; it means that having 12 trees in a courtyard, a man can fill monthly a tank with the new natural diesel fuel.

In some countries (USA, Brazil, Germany) producing alcohol or vegetable oils, for their use as fuel.

In the future, aircraft will use ion engines, magnetic, laser or various micro particles accelerated. Now, and the life of the jet engine begin to end.

Even in these conditions internal combustion engines will be maintained in land vehicles (at least), for power, reliability and especially their dynamics. Thermal engine efficiency is still low and, about one third of the engine power is lost
just by the distribution mechanism. Mechanical efficiency of cam mechanisms was about 4-8\%. In the past 20 years, managed to increase to about $14-18 \%$, and now is the time to pick it up again at up to $60 \%$. This is the main objective of this paper.

## 2. Presenting a Dynamic Model, with one Grade of Freedom, with Variable Internal Amortization

### 2.1. Determining the Amortization Coefficient of the Mechanism

Starting with the kinematical schema of the classical valve gear mechanism (see the Figure 1), one creates the translating dynamic model, with a single degree of freedom (with a single mass), with variable internal amortization (see the Figure $2)$, having the motion equation (1) [3, 13].

The formula (1) is just a Newton equation, where the sum of forces on a single element is 0 .

$$
\begin{equation*}
M \cdot \ddot{x}=K \cdot(y-x)-k \cdot x-c \cdot \dot{x}-F_{0} \tag{1}
\end{equation*}
$$



Fig. 1. The Kinematical schema of the classical valve gear mechanism.

The Newton equation (1) can be written in the form (2).

$$
\begin{equation*}
M \cdot \ddot{x}+c \cdot \dot{x}=K \cdot(y-x)-\left(F_{0}+k \cdot x\right) \tag{2}
\end{equation*}
$$



Fig. 2. Dynamic Model with a Single Liberty, with variable internal amortization.

The differential equation, Lagrange, can be written in the form (3).
$M \cdot \ddot{x}+\frac{1}{2} \cdot \frac{d M}{d t} \cdot \dot{x}=F_{m}-F_{r}$
Comparing the two equations, ( 2 and 3 ), one identifies the coefficients and one obtains the resistant force (4), the motor force (5) and the coefficient of internal amortization (6). One can see that the internal amortization coefficient, $c$, is a variable.
$F_{r}=F_{0}+k \cdot x=k \cdot x_{0}+k \cdot x=k \cdot\left(x_{0}+x\right)$
$F_{m}=K \cdot(y-x)=K \cdot(s-x)$
$c=\frac{1}{2} \cdot \frac{d M}{d t}$
One places the variable coefficient, c , (see the relation 6), in the Newton equation (form 1 or 2 ) and obtains the equation (7).
$M \cdot \ddot{x}+\frac{1}{2} \cdot \frac{d M}{d t} \cdot \dot{x}+(K+k) \cdot x=K \cdot y-F_{0} .$.
The reduced mass can be written in the form (8), (the reduced mass of the system, reduced at the valve).
$M=m_{5}+\left(m_{2}+m_{3}\right) \cdot\left(\frac{\dot{y}_{2}}{\dot{x}}\right)^{2}+$
$+J_{1} \cdot\left(\frac{\omega_{1}}{\dot{x}}\right)^{2}+J_{4} \cdot\left(\frac{\omega_{4}}{\dot{x}}\right)^{2}$
With the following notations:
$\mathrm{m}_{2}=$ the mass of the tappet (of the valve lifter); $\mathrm{m}_{3}$ $=$ the mass of the valve push rod; $\mathrm{m}_{5}=$ the valve mass; $\mathrm{J}_{1}=$ the inertia mechanical moment of the
cam; $\mathrm{J}_{4}=$ the inertia mechanical moment of the valve rocker; $\dot{y}_{2}=$ the tappet velocity, or the second movement-low, imposed by the cam's profile; $\dot{x}=$ the real (dynamic) valve velocity.

If one notes with $i=i_{25}$, the ratio of transmission tappet-valve, given from the valve rocker, the theoretically velocity of the valve, $\dot{y}$, (the tappet velocity reduced at the valve), takes the form (9), where the ratio of transmission, $i$, is given from the formula (10).
$\dot{y} \equiv \dot{y}_{5}=\frac{\dot{y}_{2}}{i}$
$i=\frac{C C_{0}}{C_{0} D}$
One can write the following relations (11-16), where $y$ ' is the reduced velocity forced at the tappet by the cam's profile. With the relations (10, $13,14,16$ ) the reduced mass (8), can be written in the forms (17-19).

$$
\begin{align*}
& \dot{x}=\omega_{1} \cdot x^{\prime}  \tag{11}\\
& \ddot{x}=\omega_{1}^{2} \cdot x^{\prime \prime}  \tag{12}\\
& \dot{y}_{2}=\omega_{1} \cdot y_{2}^{\prime}=\omega_{1} \cdot i \cdot y^{\prime}  \tag{13}\\
& \frac{\omega_{1}}{\dot{x}}=\frac{\omega_{1}}{\omega_{1} \cdot x^{\prime}}=\frac{1}{x^{\prime}}  \tag{14}\\
& \omega_{4}=\frac{\dot{y}_{2}}{C C_{0}}=\frac{\omega_{1} \cdot y_{2}^{\prime}}{C C_{0}}=\frac{\omega_{1} \cdot y^{\prime} \cdot i}{C C_{0}}=  \tag{15}\\
& =\frac{\omega_{1} \cdot y^{\prime}}{C C_{0}} \cdot \frac{C C_{0}}{C_{0} D}=\frac{\omega_{1} \cdot y^{\prime}}{C_{0} D} \\
& \frac{\omega_{4}}{\dot{x}}=\frac{\omega_{1} \cdot y^{\prime}}{C_{0} D \cdot \omega_{1} \cdot x^{\prime}}=\frac{1}{C_{0} D} \cdot \frac{y^{\prime}}{x^{\prime}}  \tag{16}\\
& M=m_{5}+\left(m_{2}+m_{3}\right) \cdot\left(\frac{i \cdot y^{\prime}}{x^{\prime}}\right)^{2}+ \\
& +J_{1} \cdot\left(\frac{1}{x^{\prime}}\right)^{2}+J_{4} \cdot\left(\frac{1}{C_{0} D} \cdot \frac{y^{\prime}}{x^{\prime}}\right)^{2}  \tag{17}\\
& M=m_{5}+\left[i^{2} \cdot\left(m_{2}+m_{3}\right)+\right. \\
& \left.+\frac{J_{4}}{\left(C_{0} D\right)^{2}}\right] \cdot\left(\frac{y^{\prime}}{x^{\prime}}\right)^{2}+J_{1} \cdot\left(\frac{1}{x^{\prime}}\right)^{2}  \tag{18}\\
& M=m_{5}+m * \cdot\left(\frac{y^{\prime}}{x^{\prime}}\right)^{2}+J_{1} \cdot\left(\frac{1}{x^{\prime}}\right)^{2} \tag{19}
\end{align*}
$$

It derives $\mathrm{dM} / \mathrm{d} \varphi$ and obtains the relations (2022).

$$
\begin{align*}
& \frac{d\left[\left(\frac{y^{\prime}}{x^{\prime}}\right)^{2}\right]}{d \varphi}=\frac{2 \cdot y^{\prime}}{x^{\prime}} \cdot \frac{\left(y^{\prime \prime} \cdot x^{\prime}-x^{\prime \prime} \cdot y^{\prime}\right)}{x^{\prime 2}}=  \tag{20}\\
& =\frac{2 \cdot y^{\prime}}{x^{\prime 2}} \cdot\left(y^{\prime \prime}-x^{\prime} \cdot \cdot \frac{y^{\prime}}{x^{\prime}}\right)=2 \cdot\left(\frac{y^{\prime}}{x^{\prime}}\right)^{2} \cdot\left(\frac{y^{\prime \prime}}{y^{\prime}}-\frac{x^{\prime \prime}}{x^{\prime}}\right) \\
& \frac{d\left[\left(\frac{1}{x^{\prime}}\right)^{2}\right]}{d \varphi}=\frac{2}{x^{\prime}} \cdot \frac{-x^{\prime \prime}}{x^{\prime 2}}=-2 \cdot \frac{x^{\prime \prime}}{x^{\prime 3}}  \tag{21}\\
& \frac{d M}{d \varphi}=2 \cdot m * \cdot\left(\frac{y^{\prime}}{x^{\prime}}\right)^{2} \cdot\left(\frac{y^{\prime \prime}}{y^{\prime}}-\frac{x^{\prime \prime}}{x^{\prime}}\right)-2 \cdot J_{1} \cdot \frac{x^{\prime \prime}}{x^{\prime 3}} \tag{22}
\end{align*}
$$

The relation (6) can be written in form (23) and with relation (22), it's taking the forms (24-25).

$$
\begin{equation*}
c=\frac{\omega}{2} \cdot \frac{d M}{d \varphi} \tag{23}
\end{equation*}
$$

$c=\omega \cdot\left\{\left[i^{2} \cdot\left(m_{2}+m_{3}\right)+\frac{J_{4}}{\left(C_{0} D\right)^{2}}\right]\right.$.
$\left.\cdot\left(\frac{y^{\prime}}{x^{\prime}}\right)^{2} \cdot\left(\frac{y^{\prime \prime}}{y^{\prime}}-\frac{x^{\prime \prime}}{x^{\prime}}\right)-J_{1} \cdot \frac{x^{\prime \prime}}{x^{\prime 3}}\right\}$
$c=\omega \cdot\left[m * \cdot\left(\frac{y^{\prime}}{x^{\prime}}\right)^{2} \cdot\left(\frac{y^{\prime \prime}}{y^{\prime}}-\frac{x^{\prime \prime}}{x^{\prime}}\right)-J_{1} \cdot \frac{x^{\prime \prime}}{x^{\prime 3}}\right]$
With the notation (26):

$$
\begin{equation*}
m^{*}=i^{2} \cdot\left(m_{2}+m_{3}\right)+\frac{J_{4}}{\left(C_{0} D\right)^{2}} \tag{26}
\end{equation*}
$$

### 2.2. Determining the Movement Equations

With the relations $(19,12,25,11)$ the equation (2) take the forms (27, 28, 29, 30 and 31) [13]:
$M \cdot \omega^{2} \cdot x^{\prime \prime}+c \cdot \omega \cdot x^{\prime}+(K+k) \cdot x=K \cdot y-F_{0}$
$\omega^{2} \cdot x^{\prime \prime} \cdot m_{5}+\omega^{2} \cdot m^{*} \cdot\left(\frac{y^{\prime}}{x^{\prime}}\right)^{2} \cdot x^{\prime \prime}+$
$+J_{1}\left(\frac{1}{x^{\prime}}\right)^{2} x^{\prime \prime} \omega^{2}+\omega^{2} \cdot x^{\prime} \cdot m^{*}\left(\frac{y^{\prime}}{x^{\prime}}\right)^{2} \cdot\left(\frac{y^{\prime \prime}}{y^{\prime}}-\frac{x^{\prime \prime}}{x^{\prime}}\right)-$
$-x^{\prime} \cdot \omega^{2} \cdot J_{1} \cdot \frac{x^{\prime \prime}}{x^{\prime 3}}+(K+k) \cdot x=K \cdot y-F_{0}$

$$
\begin{align*}
& \omega^{2} \cdot m_{5} \cdot x^{\prime \prime}+\omega^{2} \cdot m^{*} x^{\prime} \cdot\left(\frac{y^{\prime}}{x^{\prime}}\right)^{2}-\omega^{2} m^{*} \cdot\left(\frac{y^{\prime}}{x^{\prime}}\right)^{2} \cdot x^{\prime \prime}+ \\
& +\omega^{2} \cdot m^{*} \cdot y^{\prime \prime} \cdot \frac{y^{\prime}}{x^{\prime}}+(K+k) \cdot x=K \cdot y-F_{0} \tag{29}
\end{align*}
$$

$\omega^{2} m_{5} x^{\prime \prime}+(K+k) x+\omega^{2} m^{*} y^{\prime \prime} \cdot \frac{y^{\prime}}{x^{\prime}}=K y-F_{0}$
$\omega^{2} \cdot\left(m_{5} \cdot x^{\prime \prime}+m^{*} \cdot y^{\prime \prime} \cdot \frac{y^{\prime}}{x^{\prime \prime}}\right)+(K+k) x=K \cdot y-F_{0}$

The exact equation (31) can be approximated at the form (32) with $x^{\prime} \cong y^{\prime}$.

$$
\begin{equation*}
\omega^{2} \cdot\left(m_{5} \cdot x^{\prime \prime}+m^{*} \cdot y^{\prime \prime}\right)+(K+k) \cdot x=K \cdot y-F_{0} \tag{32}
\end{equation*}
$$

With the following notations: $y=s, y^{\prime}=s$, $y^{\prime \prime}=s^{\prime \prime}, y^{\prime \prime}=s^{\prime \prime} \prime$, the equation (32) takes the approximate form (33) and the complete equation (31) takes the exact form (34).

$$
\begin{equation*}
\omega^{2} \cdot\left(m_{5} \cdot x^{\prime \prime}+m^{*} \cdot s^{\prime \prime}\right)+(K+k) \cdot x=K \cdot s-F_{0} \tag{33}
\end{equation*}
$$

$\omega^{2} \cdot\left(m_{5} \cdot x^{\prime \prime}+m^{*} \cdot s^{\prime \prime} \cdot \frac{s^{\prime}}{x^{\prime}}\right)+(K+k) \cdot x=K \cdot s-F_{0}$

## Solving the Differential Equation by Direct Integration and Obtaining the Mother Equation

One integrates the equation (31) directly. One prepares the equation (31) for the integration. First, one writes (31) in form (35) [13].
$-(K+k) \cdot x+K \cdot y-k \cdot x_{0}-$
$-m_{S}^{*} \cdot \omega^{2} \cdot x^{I I}=\frac{m_{T}^{*} \cdot \omega^{2} \cdot y^{I I} \cdot y^{I}}{x^{I}}$
The equation (35), can be amplified by $x^{\prime}$ and one obtains the relation (36).
$-(K+k) \cdot x \cdot x^{I}+K \cdot y \cdot x^{I}-k \cdot x_{0} \cdot x^{I}-$
$-m_{S}^{*} \cdot \omega^{2} \cdot x^{I} \cdot x^{I I}=m_{T}^{*} \cdot \omega^{2} \cdot y^{I} \cdot y^{I I}$

Now, one replaces the term K.y.x' with $K \cdot y \cdot \frac{K}{K+k} \cdot y^{I}$, (taken in calculation the statically assumption, $\mathrm{F}_{\mathrm{m}}=\mathrm{F}_{\mathrm{r}}$ ) and one obtains the form (37).
$-(K+k) \cdot x \cdot x^{I}+\frac{K^{2}}{K+k} \cdot y \cdot y^{I}-k \cdot x_{0} \cdot x^{I}-$ $-m_{S}^{*} \cdot \omega^{2} \cdot x^{I} \cdot x^{I I}=m_{T}^{*} \cdot \omega^{2} \cdot y^{I} \cdot y^{I I}$

One integrates directly the equation (37) and one obtains the mother equation (38).
$-(K+k) \cdot \frac{x^{2}}{2}+\frac{K^{2}}{K+k} \cdot \frac{y^{2}}{2}-k \cdot x_{0} \cdot x-$
$-m_{S}^{*} \cdot \omega^{2} \cdot \frac{x^{\prime 2}}{2}=m_{T}^{*} \cdot \omega^{2} \cdot \frac{y^{\prime 2}}{2}+C$

With the initial condition, at the $\varphi=0, y=y^{\prime}=0$ and $x=x=0$, one obtains for the constant of integration, $C$ the value 0 . In this case the equation (38), takes the form (39).
$-(K+k) \cdot \frac{x^{2}}{2}+\frac{K^{2}}{K+k} \cdot \frac{y^{2}}{2}-k \cdot x_{0} \cdot x-$
$-m_{S}^{*} \cdot \omega^{2} \cdot \frac{x^{\prime 2}}{2}=m_{T}^{*} \cdot \omega^{2} \cdot \frac{y^{\prime 2}}{2}$
The equation (39) can be put in the form (40), if one divides it with the $-\frac{K+k}{2}$.

$$
\begin{align*}
& x^{2}+2 \cdot \frac{k \cdot x_{0}}{K+k} \cdot x+\frac{m_{S}^{*} \cdot \omega^{2}}{K+k} \cdot x^{\prime 2}+ \\
& +\frac{m_{T}^{*} \cdot \omega^{2}}{K+k} y^{\prime 2}-\frac{K^{2}}{(K+k)^{2}} \cdot y^{2}=0 \tag{40}
\end{align*}
$$

The mother equation (40), take the form (41), if one notes: $x^{\prime}=\frac{K}{K+k} \cdot y^{\prime}, \quad$ (the static assumption, $\mathrm{F}_{\mathrm{m}}=\mathrm{F}_{\mathrm{r}}$ ).

$$
\begin{align*}
& x^{2}+2 \cdot \frac{k \cdot x_{0}}{K+k} \cdot x-\frac{K^{2}}{(K+k)^{2}} \cdot y^{2}+ \\
& +\frac{\frac{K^{2}}{(K+k)^{2}} \cdot m_{S}^{*}+m_{T}^{*}}{(K+k)} \cdot \omega^{2} \cdot y^{\prime 2}=0 \tag{41}
\end{align*}
$$

Solving the Mother Equation (41) Directly

The equation (41) is a two degree equation in x ; One determines directly, $\Delta(42-43)$ and $\mathrm{X}_{1,2}$ (44) [13].
$\Delta=\frac{\left(k \cdot x_{0}\right)^{2}+(K \cdot s)^{2}}{(K+k)^{2}}-\frac{m_{S}^{*} \frac{K^{2}}{(K+k)^{2}}+m_{T}^{*}}{(K+k)} y^{\prime 2} \omega^{2}$
$\Delta=\frac{\left(k x_{0}\right)^{2}+(K s)^{2}}{(K+k)^{2}}-\frac{m_{S}^{*} \frac{K^{2}}{(K+k)^{2}}+m_{T}^{*}}{(K+k)}\left(D s^{\prime}\right)^{2} \omega^{2}$
$X_{1,2}=-\frac{k \cdot x_{0}}{K+k} \pm \sqrt{\Delta}$
Physically, just the positive solution is valid (see the relation 45).

$$
\begin{equation*}
X=\sqrt{\Delta}-\frac{k \cdot x_{0}}{K+k} \tag{45}
\end{equation*}
$$

## Solving the Mother Equation (41) with Finished Differences

One can solve the mother equation (41) using the finished differences [13]. One notes:

$$
\begin{equation*}
X=s+\Delta X \tag{46}
\end{equation*}
$$

With the notation (46) placed in the mother equation (41), it obtains the equation (47).

$$
\begin{align*}
& s^{2}+(\Delta X)^{2}+2 \Delta X s+2 \frac{k \cdot x_{0}}{K+k} s+2 \frac{k \cdot x_{0}}{K+k} \Delta X- \\
& -\frac{K^{2}}{(K+k)^{2}} s^{2}+\frac{\frac{K^{2}}{(K+k)^{2}} m_{S}^{*}+m_{T}^{*}}{(K+k)} \omega^{2} \cdot y^{\prime 2}=0 \tag{47}
\end{align*}
$$

The equation (47) is a two degree equation in $\Delta X$, which can be solved directly with $\Delta(49)$ and $\Delta X_{1,2},(50)$, or transformed in a single degree equation in $\Delta X$, with $(\Delta X)^{2} \cong 0$, solved by the relation (48).
$\Delta X=(-1)$.
$\frac{\left(k^{2}+2 k K\right) s^{2}+2 k x_{0}(K+k) s+\left[\frac{K^{2}}{K+k} m_{S}^{*}+(K+k) m_{T}^{*}\right] \omega^{2}\left(D s^{\prime}\right)^{2}}{2 \cdot\left(s+\frac{k \cdot x_{0}}{K+k}\right) \cdot(K+k)^{2}}$
$\Delta=\frac{K^{2} s^{2}+k^{2} x_{0}^{2}-\left[\frac{K^{2}}{K+k} m_{S}^{*}+(K+k) m_{T}^{*}\right] \omega^{2}\left(D s^{\prime}\right)^{2}}{(K+k)^{2}}$

$$
\begin{equation*}
\Delta X=\sqrt{\Delta}-\left(s+\frac{k \cdot x_{0}}{K+k}\right) \tag{50}
\end{equation*}
$$

### 2.3. Mechanism with Rotary Cam and Translated Tappet with Roll

First, one presents an original method to determine the efficiency at the mechanism with rotary cam and translated follower with roll [5].

With this occasion it presents and the forces and the velocities as well (Figure 3).

The pressure angle $\delta$ (Figure 3), is determined by relations (1.5-1.6).

We can write the next forces, speeds and powers (1.13-1.18).
$\mathrm{F}_{\mathrm{m}}\left(\mathrm{v}_{\mathrm{m}}\right)$ is perpendicular to the vector $\mathrm{r}_{\mathrm{A}}$ at A .
$F_{m}$ is divided into $F_{a}$ (the sliding force) and $F_{n}$ (the normal force). $\mathrm{F}_{\mathrm{n}}$ is divided too, into $\mathrm{F}_{\mathrm{i}}$ (the bending force) and $F_{u}$ (the useful force). The momentary dynamic efficiency can be obtained from relation (1.18).


Fig. 3. Forces and velocities at the cam with translated follower with roll.

The written relations are the following.

$$
\begin{align*}
& \mathrm{r}_{\mathrm{B}}^{2}=\mathrm{e}^{2}+\left(\mathrm{s}_{0}+\mathrm{s}\right)^{2}  \tag{1.1}\\
& r_{B}=\sqrt{r_{B}^{2}}  \tag{1.2}\\
& \cos \alpha_{B} \equiv \sin \tau=\frac{e}{r_{B}}  \tag{1.3}\\
& \sin \alpha_{B} \equiv \cos \tau=\frac{s_{0}+s}{r_{B}}  \tag{1.4}\\
& \cos \delta=\frac{s_{0}+s}{\sqrt{\left(s_{0}+s\right)^{2}+\left(s^{\prime}-e\right)^{2}}} \tag{1.5}
\end{align*}
$$

$\sin \delta=\frac{s^{\prime}-e}{\sqrt{\left(s_{0}+s\right)^{2}+\left(s^{\prime}-e\right)^{2}}}$
$\cos (\delta+\tau)=\cos \delta \cdot \cos \tau-\sin \delta \cdot \sin \tau$

$$
\left\{\begin{array}{l}
r_{A}^{2}=\left(e+r_{b} \cdot \sin \delta\right)^{2}+\left(s_{0}+s-r_{b} \cdot \cos \delta\right)^{2}  \tag{1.7}\\
r_{A}^{2}=r_{B}^{2}+r_{b}^{2}-2 \cdot r_{b} \cdot r_{B} \cdot \cos (\delta+\tau)
\end{array}\right.
$$

$$
\left\{\begin{array}{l}
\cos \alpha_{A}=\frac{e \cdot \sqrt{\left(s_{0}+s\right)^{2}+\left(s^{\prime}-e\right)^{2}}+r_{b} \cdot\left(s^{\prime}-e\right)}{r_{A} \cdot \sqrt{\left(s_{0}+s\right)^{2}+\left(s^{\prime}-e\right)^{2}}}  \tag{1.8}\\
\cos \alpha_{A}=\frac{e+r_{b} \cdot \sin \delta}{r_{A}}
\end{array}\right.
$$

$$
\left\{\begin{array}{l}
\sin \alpha_{A}=\frac{\left(s_{0}+s\right) \cdot\left[\sqrt{\left(s_{0}+s\right)^{2}+\left(s^{\prime}-e\right)^{2}}-r_{b}\right]}{r_{A} \cdot \sqrt{\left(s_{0}+s\right)^{2}+\left(s^{\prime}-e\right)^{2}}}  \tag{1.9}\\
\sin \alpha_{A}=\frac{s_{0}+s-r_{b} \cdot \cos \delta}{r_{A}}
\end{array}\right.
$$

$$
\left\{\begin{array}{l}
\cos \left(\alpha_{A}-\delta\right)=\frac{\left(s_{0}+s\right) \cdot s^{\prime}}{r_{A} \cdot \sqrt{\left(s_{0}+s\right)^{2}+\left(s^{\prime}-e\right)^{2}}}  \tag{1.10}\\
\cos \left(\alpha_{A}-\delta\right)=\frac{s^{\prime}}{r_{A}} \cdot \cos \delta
\end{array}\right.
$$

$\cos \left(\alpha_{A}-\delta\right) \cdot \cos \delta=\frac{s^{\prime}}{r_{A}} \cdot \cos ^{2} \delta$
$\left\{\begin{array}{l}v_{a}=v_{m} \cdot \sin \left(\alpha_{A}-\delta\right) \\ F_{a}=F_{m} \cdot \sin \left(\alpha_{A}-\delta\right)\end{array}\right.$
$\left\{\begin{array}{l}v_{n}=v_{m} \cdot \cos \left(\alpha_{A}-\delta\right) \\ F_{n}=F_{m} \cdot \cos \left(\alpha_{A}-\delta\right)\end{array}\right.$
$\left\{\begin{array}{l}v_{i}=v_{n} \cdot \sin \delta \\ F_{i}=F_{n} \cdot \sin \delta\end{array}\right.$
$\left\{\begin{array}{l}v_{2}=v_{n} \cdot \cos \delta=v_{m} \cdot \cos \left(\alpha_{A}-\delta\right) \cdot \cos \delta \\ F_{u}=F_{n} \cdot \cos \delta=F_{m} \cdot \cos \left(\alpha_{A}-\delta\right) \cdot \cos \delta\end{array}\right.$
$\left\{\begin{array}{l}P_{u}=F_{u} \cdot v_{2}=F_{m} \cdot v_{m} \cdot \cos ^{2}\left(\alpha_{A}-\delta\right) \cdot \cos ^{2} \delta \\ P_{c}=F_{m} \cdot v_{m}\end{array}\right.$
$\left\{\begin{array}{l}\eta_{i}=\frac{P_{u}}{P_{c}}=\frac{F_{m} \cdot v_{m} \cdot \cos ^{2}\left(\alpha_{A}-\delta\right) \cdot \cos ^{2} \delta}{F_{m} \cdot v_{m}} \\ \eta_{i}=\left[\cos \left(\alpha_{A}-\delta\right) \cdot \cos \delta\right]^{2}=\left[\frac{s^{\prime}}{r_{A}} \cdot \cos ^{2} \delta\right]^{2} \\ \eta_{i}=\frac{s^{\prime 2}}{r_{A}^{2}} \cdot \cos ^{4} \delta\end{array}\right.$
$r_{B_{0}}=r_{0}+r_{b}$
$s_{0}=\sqrt{r_{B_{0}}^{2}-e^{2}}$
$\cos \alpha_{0}=\frac{e}{r_{B_{0}}}$
$\sin \alpha_{0}=\frac{s_{0}}{r_{B_{0}}}$

### 2.4. The Relations to Design the Profile

Now one determines the profile of the cam (relations 1.23-1.28).
$\gamma=\alpha_{A}-\alpha_{0}$
$\cos \gamma=\cos \alpha_{A} \cdot \cos \alpha_{0}+\sin \alpha_{A} \cdot \sin \alpha_{0}$
$\sin \gamma=\sin \alpha_{A} \cdot \cos \alpha_{0}-\cos \alpha_{A} \cdot \sin \alpha_{0} \ldots(1.25)$
$\theta_{A}=\varphi-\gamma$
$\cos \theta_{A}=\cos \varphi \cdot \cos \gamma+\sin \varphi \cdot \sin \gamma$
$\sin \theta_{A}=\sin \varphi \cdot \cos \gamma-\sin \gamma \cdot \cos \varphi$

### 2.5. The Exact Kinematics of B Module

From the triangle OCB (fig. 3) the length $\mathrm{r}_{\mathrm{B}}$ (OB) and the complementary angles $\alpha_{\mathrm{B}}$ (COB) and $\tau(\mathrm{CBO})$ are determined by the relation (1.11.4).

From the general triangle OAB , where one knows $\mathrm{OB}, \mathrm{AB}$, and the angle between them, B (ABO, which is the sum of $\tau$ with $\delta$ ), the length OA and the angle $\mu$ (AOB) can be determined with the relations (1.7-1.8, 1.29-1.31):
$\cos \mu=\frac{r_{A}^{2}+r_{B}^{2}-r_{b}^{2}}{2 \cdot r_{A} \cdot r_{B}}$
$\sin (\delta+\tau)=\sin \delta \cdot \cos \tau+\sin \tau \cdot \cos \delta$
$\sin \mu=\frac{r_{b}}{r_{A}} \cdot \sin (\delta+\tau)$
With $\alpha_{B}$ and $\mu$ we can deduce now $\alpha_{A}$ and $\dot{\alpha}_{A}$ (the relations 1.32-1.33):
$\alpha_{A}=\alpha_{B}-\mu$
$\dot{\alpha}_{A}=\dot{\alpha}_{B}-\dot{\mu}$
From (1.3) one obtains $\dot{\alpha}_{B}$ (1.37), (see 1.341.37) where $\dot{r}_{B}$ (1.36) can be deduced from (1.1). Then, (1.38) will be obtained from (1.29):
$-\sin \alpha_{B} \cdot \dot{\alpha}_{B}=-\frac{e \cdot \dot{r}_{B}}{r_{B}^{2}}$
$\dot{\alpha}_{B}=\frac{e \cdot r_{B} \cdot \dot{r}_{B}}{\left(s_{0}+s\right) \cdot r_{B}^{2}}$
$\left\{\begin{array}{l}2 \cdot r_{B} \cdot \dot{r}_{B}=2 \cdot\left(s_{0}+s\right) \cdot \dot{s} \\ r_{B} \cdot \dot{r}_{B}=\left(s_{0}+s\right) \cdot \dot{s}\end{array}\right.$
$\dot{\alpha}_{B}=\frac{e \cdot\left(s_{0}+s\right) \cdot \dot{s}}{\left(s_{0}+s\right) \cdot r_{B}^{2}}=\frac{e \cdot \dot{s}}{r_{B}^{2}}$
$\left\{\begin{array}{l}2 \cdot \dot{r}_{A} \cdot r_{B} \cdot \cos \mu+2 \cdot r_{A} \cdot \dot{r}_{B} \cdot \cos \mu- \\ -2 \cdot r_{A} \cdot r_{B} \cdot \sin \mu \cdot \dot{\mu}=2 \cdot r_{A} \cdot \dot{r}_{A}+2 \cdot r_{B} \cdot \dot{r}_{B}\end{array}\right.$

From (1.38) one writes $\dot{\mu}$ (1.43), but it is necessary to obtain first $\dot{r}_{A}$ (1.39) from expression (1.8):

$$
\left\{\begin{array}{l}
2 \cdot r_{A} \cdot \dot{r}_{A}=2 \cdot r_{B} \cdot \dot{r}_{B}-2 \cdot r_{b} \cdot \dot{r}_{B} \cdot \cos (\delta+\tau)  \tag{1.39}\\
+2 \cdot r_{b} \cdot r_{B} \cdot \sin (\delta+\tau) \cdot(\dot{\delta}+\dot{\tau})
\end{array}\right.
$$

To solve (1.39) we need the derivatives $\dot{\delta}$ (1.40 and 1.41) and $\dot{\tau}$ (1.42).

$$
\begin{equation*}
\delta^{\prime}=\frac{s^{\prime} \cdot\left(s_{0}+e\right)-s^{\prime} \cdot\left(s^{\prime}-e\right)}{\left(s_{0}+s\right)^{2}+\left(s^{\prime}-e\right)^{2}} \tag{1.40}
\end{equation*}
$$

$\dot{\delta}=\delta^{\prime} \cdot \omega$
$\dot{\tau}=-\dot{\alpha}_{B}=-\frac{e \cdot \dot{s}}{r_{B}^{2}}$
Now we can determine $\dot{\mu}(1.43), \dot{\alpha}_{A}$ (1.33) and $\dot{\theta}_{A}$ (1.44):

$$
\begin{equation*}
\dot{\mu}=\frac{\dot{r}_{A} r_{B} \cos \mu+r_{A} \dot{r}_{B} \cos \mu-r_{A} \dot{r}_{A}-r_{B} \dot{r}_{B}}{r_{A} \cdot r_{B} \cdot \sin \mu} \tag{1.43}
\end{equation*}
$$

$\dot{\theta}_{A}=\dot{\varphi}-\dot{\gamma}=\omega-\dot{\alpha}_{A}$

We write $\cos \alpha_{A}$ and $\sin \alpha_{A}$ (1.9-1.10):
Further, we can obtain the expression $\cos \left(\alpha_{A^{-}}\right.$ $\delta)(1.11)$, and $\cos \left(\alpha_{A}-\delta\right) \cdot \cos \delta$ (1.12).

Finally the forces and the velocities are deduced as follows (1.13-1.16):

### 2.6. Determining the Efficiency of the Module B

With the relationships (1.17-1.18) we can determine the powers and the momentary mechanical efficiency [14].

## Determining the (Dynamic) Transmission Function D, for the Module B

The follower's velocity (1.16) can be written into the form (1.45).

$$
\left\{\begin{array}{l}
v_{2}=v_{n} \cdot \cos \delta=v_{m} \cdot \cos \left(\alpha_{A}-\delta\right) \cdot \cos \delta=  \tag{1.45}\\
=v_{m} \cdot \frac{s^{\prime}}{r_{A}} \cdot \cos ^{2} \delta=r_{A} \cdot \dot{\theta}_{A} \cdot \frac{s^{\prime}}{r_{A}} \cdot \cos ^{2} \delta= \\
=\dot{\theta}_{A} \cdot s^{\prime} \cdot \cos ^{2} \delta=\theta_{A}^{I} \cdot \omega \cdot s^{\prime} \cdot \cos ^{2} \delta
\end{array}\right.
$$

With the relationships (1.45) and (1.46) we determine the transmission function (the dynamic modulus), D (1.47):

$$
\begin{equation*}
v_{2}=s^{\prime} \cdot D \cdot \omega \tag{1.46}
\end{equation*}
$$

$D=\theta_{A}^{I} \cdot \cos ^{2} \delta$
Expression $\cos ^{2} \delta$ is known (1.48):

$$
\begin{equation*}
\cos ^{2} \delta=\frac{\left(s_{0}+s\right)^{2}}{\left(s_{0}+s\right)^{2}+\left(s^{\prime}-e\right)^{2}} \tag{1.48}
\end{equation*}
$$

The expression of the $\theta^{\prime}{ }_{A}(1.49)$ is more difficult.

$$
\left\{\begin{array}{l}
\theta_{A}^{I}=\left[\left(s_{0}+s\right)^{2}+e^{2}-e \cdot s^{\prime}-r_{b} .\right.  \tag{1.49}\\
\left.\cdot \sqrt{\left(s_{0}+s\right)^{2}+\left(s^{\prime}-e\right)^{2}}\right] \cdot\left\{\left[\left(s_{0}+s\right)^{2}+\right.\right. \\
\left.+\left(s^{\prime}-e\right)^{2}\right] \cdot \sqrt{\left(s_{0}+s\right)^{2}+\left(s^{\prime}-e\right)^{2}}+ \\
+r_{b} \cdot\left[s^{\prime} \cdot \cdot\left(s_{0}+s\right)-s^{\prime} \cdot\left(s^{\prime}-e\right)-\left(s_{0}+s\right)^{2}-\right. \\
\left.\left.-\left(s^{\prime}-e\right)^{2}\right]\right\} /\left[\left(s_{0}+s\right)^{2}+\left(s^{\prime}-e\right)^{2}\right] / \\
/\left\{\left[\left(s_{0}+s\right)^{2}+e^{2}+r_{b}^{2}\right]\right. \\
\cdot \sqrt{\left(s_{0}+s\right)^{2}+\left(s^{\prime}-e\right)^{2}}- \\
\left.-2 \cdot r_{b} \cdot\left[\left(s_{0}+s\right)^{2}+e^{2}-e \cdot s^{\prime}\right]\right\}
\end{array}\right.
$$

We will determine $\mu$ by its expressions (1.501.51):
$\cos \mu=$

$$
\left\{\begin{array}{l}
=\frac{\left[\left(s_{0}+s\right)^{2}+e^{2}\right] \cdot \sqrt{\left(s_{0}+s\right)^{2}+\left(s^{\prime}-e\right)^{2}}}{r_{A} \cdot r_{B} \cdot \sqrt{\left(s_{0}+s\right)^{2}+\left(s^{\prime}-e\right)^{2}}}-  \tag{1.50}\\
-\frac{r_{b} \cdot\left[\left(s_{0}+s\right)^{2}+e^{2}-e \cdot s^{\prime}\right]}{r_{A} \cdot r_{B} \cdot \sqrt{\left(s_{0}+s\right)^{2}+\left(s^{\prime}-e\right)^{2}}}
\end{array}\right.
$$

$\sin \mu=\frac{r_{b} \cdot\left(s_{0}+s\right) \cdot s^{\prime}}{r_{A} \cdot r_{B} \cdot \sqrt{\left(s_{0}+s\right)^{2}+\left(s^{\prime}-e\right)^{2}}}$

## The Dynamics of Distribution Mechanisms with Translated Follower with Roll

For the dynamics of the Module $B$ the relationships (49-50) are used in the forms (1.521.54 ), where D is determined from (1.47).
$\Delta X=-\frac{\frac{k^{2}+2 k K}{(K+k)^{2}} \cdot s^{2}+\frac{2 k x_{0}}{K+k} \cdot s+\frac{\left[\frac{K^{2}}{(K+k)^{2}} \cdot m_{S}^{*}+m_{T}^{*}\right] \cdot \omega^{2}}{K+k} \cdot y^{\prime 2}}{2 \cdot\left[s+\frac{k x_{0}}{K+k}\right]}$
$\Delta X=-\frac{\frac{k^{2}+2 k K}{(K+k)^{2}} \cdot s^{2}+\frac{2 k x_{0}}{K+k} \cdot s+\frac{\left[\frac{K^{2}}{(K+k)^{2}} \cdot m_{S}^{*}+m_{T}^{*}\right] \cdot \omega^{2}}{K+k} \cdot\left(D \cdot s^{\prime}\right)^{2}}{2 \cdot\left[s+\frac{k x_{0}}{K+k}\right]}$
$X=s+\Delta X$

### 2.7. The Dynamic Analysis of the Module B

It presents now the dynamics of the module $B$ for some known movement laws.

We begin with the classical law SIN (see the diagram in Figure 4); A speed rotation $\mathrm{n}=5500$ [rot/min], for a maxim theoretical displacement of the valve $\mathrm{h}=6[\mathrm{~mm}]$ is used.

The phase angle is $\varphi_{u}=\varphi_{c}=65$ [degree]; the ray of the basic circle is $\mathrm{r}_{0}=13$ [mm].

For the ray of the roll the value $\mathrm{r}_{\mathrm{b}}=13[\mathrm{~mm}]$ has been adopted.


Fig. 4. The dynamic analysis of the module $B$. The law SIN, $\mathrm{n}=550 \mathrm{rpm}, \varphi_{\mathrm{u}}=65^{\circ}, \mathrm{r}_{0}=13[\mathrm{~mm}], \mathrm{r}_{\mathrm{b}}=13$ [mm], $h_{T}=6$ [mm], $e=0$ [ mm$], k=30[N / m m]$, and $\mathrm{x}_{0}=20$ [mm].


Fig. 5. The profile SIN at the module B. $\mathbf{n}=5500 \mathrm{rpm}$ $\varphi_{u}=65^{0}, r_{0}=13[\mathrm{~mm}], r_{b}=13[\mathrm{~mm}], h_{T}=6[\mathrm{~mm}]$.

The dynamics are better than for the classical module C. For a phase angle of just 65 degrees the accelerations have the same values as for the classical module C for a relaxed phase $\left(75^{0}-80^{\circ}\right)$.

In Figure 5 we can see the cam's profile. It uses the profile sin, a rotation speed $\mathrm{n}=5500 \mathrm{rpm}$, and $\varphi_{\mathrm{u}}=65^{\circ}, \mathrm{r}_{0}=13[\mathrm{~mm}], \mathrm{r}_{\mathrm{b}}=13[\mathrm{~mm}], \mathrm{h}_{\mathrm{T}}=6[\mathrm{~mm}]$.

The law COS can be seen in the Figures 6 and 7.

In the Figure 6 is presented the dynamic analyze of the profile cos, and its profile design can be seen in the Figure 7.

The principal parameters are:
Law COS, $\mathrm{n}=5500 \mathrm{rpm}, \varphi_{\mathrm{u}}=65^{\circ}, \mathrm{r}_{0}=13$ [mm], $r_{b}=6[\mathrm{~mm}], h_{T}=6[\mathrm{~mm}], \eta=10.5 \%$.


Fig. 6. The dynamic analysis of the module B. Law COS, $n=5500 \mathrm{rpm}, \varphi_{u}=65^{0}, r_{0}=13[\mathrm{~mm}], r_{b}=6[\mathrm{~mm}]$, $h_{T}=6[\mathrm{~mm}], \eta=10.5 \%$.


Fig. 7. The profile $\operatorname{COS}$ at the module $\mathbf{B}, \mathbf{n}=5500$ $\mathrm{rpm}, \varphi_{u}=65^{0}, r_{0}=13[\mathrm{~mm}], r_{b}=6[\mathrm{~mm}], h_{T}=6[\mathrm{~mm}]$.


Fig. 8. The dynamic analysis. Law C4P1-0, $n=5500$ $\mathrm{rpm}, \varphi_{u}=80^{0}, r_{0}=13[\mathrm{~mm}], r_{b}=6[\mathrm{~mm}], h_{T}=6[\mathrm{~mm}]$.

In the figure 8 the law C 4 P , created by the author, is analyzed dynamic. The vibrations are diminished, the noises are limited, the effective displacement of the valve is increased, $\mathrm{s}_{\max }=5.37$ [mm].


Fig. 9. The profile C4P of the module B.

The efficiency has a good value $\eta=8.6 \%$. In the Figure 9 the profile of C4P law is presented.

It starts at the law C4P with $\mathrm{n}=5500$ [rpm], but for this law the rotation velocity can increase to high values of 30000-40000 [rpm] (see the Figure 10).


Fig. 10. The dynamic analysis of the module B. Law C4P1-5, $n=40000 \mathrm{rpm}$.

### 2.8. The New Cam Synthesis

The rotary cam with translated follower with roll (Figure 3 or 13), is synthesized dynamic by the new next relationships.

It has exchanged the rotation sense from the Figure 1 to Figure 11, from the clockwise to the counter-clockwise.

First, one determines the mass moment of inertia (mechanical) of the mechanism, reduced to the element of rotation, ie cam (basically using kinetic energy conservation, system 2.1).

$$
\left\{\begin{array}{l}
J_{c a m a}=\frac{1}{2} \cdot M_{c} \cdot R^{2} \\
R^{2} \equiv r_{A}^{2}=x_{A}^{2}+y_{A}^{2}=e^{2}+r_{b}^{2} \cdot \sin ^{2} \delta+2 \cdot e \cdot r_{b} \cdot \sin \delta+ \\
+\left(s_{0}+s\right)^{2}+r_{b}^{2} \cdot \cos ^{2} \delta-2 \cdot r_{b} \cdot\left(s_{0}+s\right) \cdot \cos \delta \\
r_{A}^{2}=e^{2}+r_{b}^{2}+\left(s_{0}+s\right)^{2}+2 \cdot r_{b} \cdot\left[e \cdot \sin \delta-\left(s_{0}+s\right) \cdot \cos \delta\right] \\
r_{A}^{2}=e^{2}+r_{b}^{2}+\left(s_{0}+s\right)^{2}+2 \cdot r_{b} \cdot e \cdot \frac{s^{\prime}-e}{\sqrt{\left(s_{0}+s\right)^{2}+\left(s^{\prime}-e\right)^{2}}}- \\
-2 \cdot r_{b} \cdot\left(s_{0}+s\right) \cdot \frac{\left(s_{0}+s\right)}{\sqrt{\left(s_{0}+s\right)^{2}+\left(s^{\prime}-e\right)^{2}}} \\
r_{A}^{2}=e^{2}+r_{b}^{2}+\left(s_{0}+s\right)^{2}-\frac{2 \cdot r_{b} \cdot\left(s_{0}+s\right)^{2}}{\sqrt{\left(s_{0}+s\right)^{2}+\left(s^{\prime}-e\right)^{2}}}+ \\
+\frac{2 \cdot r_{b} \cdot e \cdot\left(s^{\prime}-e\right)}{\sqrt{\left(s_{0}+s\right)^{2}+\left(s^{\prime}-e\right)^{2}}} \\
J_{m}^{*}=\frac{1}{2} \cdot M_{c} \cdot\left(r_{0}^{2}+r_{b}^{2}+r_{0} \cdot r_{b}\right)+\frac{1}{4} \cdot M_{c} \cdot s_{0} \cdot h+\frac{1}{16} \cdot M_{c} \cdot h^{2}+ \\
+\frac{1}{2} \cdot M_{c} \cdot r_{b} \cdot \frac{\pi \cdot \frac{\pi \cdot h}{2 \cdot \varphi_{0}}-e^{2}-\left(s_{0}+\frac{h}{2}\right)^{2}}{\sqrt{\left(s_{0}+\frac{h}{2}\right)^{2}+\left(\frac{\pi \cdot h}{2 \cdot \varphi_{0}}-e\right)^{2}}+\frac{m_{T} \cdot \pi^{2} \cdot h^{2}}{8 \cdot \varphi_{0}^{2}}} \\
J^{*}=\frac{1}{2} \cdot M_{c} \cdot\left(2 \cdot r_{b}^{2}+r_{0}^{2}+2 \cdot r_{0} \cdot r_{b}\right)+M_{c} \cdot s_{0} \cdot s+\frac{1}{2} \cdot M_{c} \cdot s^{2}+ \\
+M_{c} \cdot r_{b} \cdot \frac{e \cdot s^{\prime}-e^{2}-\left(s_{0}+s\right)^{2}}{\sqrt{\left(s_{0}+s\right)^{2}+\left(s^{\prime}-e\right)^{2}}+m_{T} \cdot s^{\prime 2}} \tag{2.1}
\end{array}\right.
$$

We considered the law of motion of the tappet classic version already used the cosine law (both ascending and descending).

The angular velocity is a function of the cam position ( $\varphi$ ) but also its rotation speed (2.2). Where $\omega_{\mathrm{m}}$ is the nominal angular velocity of cam and express at the distribution mechanisms based on the motor shaft speed (2.3).

$$
\begin{equation*}
\omega^{2}=\frac{J_{m}^{*}}{J^{*}} \cdot \omega_{m}^{2} \tag{2.2}
\end{equation*}
$$

$$
\begin{align*}
& \omega_{m}=2 \cdot \pi \cdot v_{c}=2 \cdot \pi \cdot \frac{n_{c}}{60}= \\
& =\frac{2 \cdot \pi}{60} \cdot \frac{n_{\text {motor }}}{2}=\frac{\pi \cdot n}{60} \tag{2.3}
\end{align*}
$$

We start the simulation with a classical law of motion, namely the cosine law. To climb cosine law system is expressed by the relationships (2.4).

$$
\left\{\begin{array}{l}
s=\frac{h}{2}-\frac{h}{2} \cdot \cos \left(\pi \cdot \frac{\varphi}{\varphi_{u}}\right) \\
s^{\prime} \equiv v_{r}=\frac{\pi \cdot h}{2 \cdot \varphi_{u}} \cdot \sin \left(\pi \cdot \frac{\varphi}{\varphi_{u}}\right) \\
s^{\prime \prime} \equiv a_{r}=\frac{\pi^{2} \cdot h}{2 \cdot \varphi_{u}^{2}} \cdot \cos \left(\pi \cdot \frac{\varphi}{\varphi_{u}}\right)  \tag{2.4}\\
s^{\prime \prime \prime} \equiv \alpha_{r}=-\frac{\pi^{3} \cdot h}{2 \cdot \varphi_{u}^{3}} \cdot \sin \left(\pi \cdot \frac{\varphi}{\varphi_{u}}\right)
\end{array}\right.
$$

Where $\varphi$ takes values from 0 to $\varphi_{u}$.
$\mathrm{J}_{\text {max }}$ occurs for $\varphi=\varphi_{\mathrm{u}} / 2$.
With the relation (2.5) is expressed the first derivative of the reduced mechanical moment of inertia. It is necessary to determine the angular acceleration (2.6).

$$
\begin{align*}
& J^{* \prime}=M_{c} \cdot s_{0} \cdot s^{\prime}+M_{c} \cdot s \cdot s^{\prime}+2 \cdot m_{T} \cdot s^{\prime} \cdot s^{\prime \prime}+ \\
& +M_{c} \cdot r_{b} \cdot \frac{\left[e \cdot s^{\prime \prime}-2 \cdot\left(s_{0}+s\right) \cdot s^{\prime}\right] \cdot\left[\left(s_{0}+s\right)^{2}+\left(s^{\prime}-e\right)^{2}\right]}{\left[\left(s_{0}+s\right)^{2}+\left(s^{\prime}-e\right)^{2}\right]^{3 / 2}} \\
& -M_{c} \cdot r_{b} \cdot \frac{\left[e \cdot s^{\prime}-e^{2}-\left(s_{0}+s\right)^{2}\right] \cdot\left[\left(s_{0}+s\right) \cdot s^{\prime}+\left(s^{\prime}-e\right) \cdot s^{\prime \prime}\right]}{\left[\left(s_{0}+s\right)^{2}+\left(s^{\prime}-e\right)^{2}\right]^{3 / 2}} \tag{2.5}
\end{align*}
$$

Differentiating the formula (2.2), against time, is obtained the angular acceleration expression (2.6).
$\varepsilon=-\frac{\omega^{2}}{2} \cdot \frac{J^{* \prime}}{J^{*}}$
Relations (2.2) and (2.6) a general nature and are basically two original equations of motion, crucial for mechanical mechanisms.

For a rotary cam and translated tappet with roll mechanism (without valve), dynamic movement tappet is expressed by equation (2.7), which is an
original dynamic equation deduced for the distribution mechanism (50) and now by canceling the valve mass, will customize and reaching form below (2.7).
$x=s-$
$-\frac{(K+k) \cdot m_{T} \cdot \omega^{2} \cdot s^{\prime 2}+\left(k^{2}+2 k \cdot K\right) \cdot s^{2}+2 k \cdot x_{0} \cdot(K+k) \cdot s}{2 \cdot(K+k)^{2} \cdot\left(s+\frac{k \cdot x_{0}}{K+k}\right)}$

Where x is the dynamic movement of the pusher, while $s$ is its normal, kinematics movement. K is the spring constant of the system, and k is the spring constant of the tappet spring.

It note, with $\mathrm{x}_{0}$ the tappet spring preload, with $\mathrm{m}_{\mathrm{T}}$ the mass of the tappet, with $\omega$ the angular rotation speed of the cam (or camshaft), where s' is the first derivative in function of $\varphi$ of the tappet movement, s. Differentiating twice successively, the expression (2.7) in the angle $\varphi$, we obtain a reduced tappet speed (equation 2.8), and reduced tappet acceleration (2.9).

$$
\left\{\begin{array}{l}
N=(K+k) \cdot m_{T} \cdot \omega^{2} \cdot s^{\prime 2}+\left(k^{2}+2 k \cdot K\right) \cdot s^{2}+2 k \cdot x_{0} \cdot(K+k) \cdot s \\
M=\left[(K+k) m_{T} \omega^{2} \cdot 2 s^{\prime} s^{\prime \prime}+\left(k^{2}+2 k K\right) \cdot 2 s s^{\prime}+2 k x_{0}(K+k) \cdot s^{\prime}\right] . \\
\cdot\left(s+\frac{k x_{0}}{K+k}\right)-N \cdot s^{\prime} \\
x^{\prime}=s^{\prime}-\frac{M}{2 \cdot(K+k)^{2} \cdot\left(s+\frac{k x_{0}}{K+k}\right)^{2}} \tag{2.8}
\end{array}\right.
$$

$$
N=(K+k) \cdot m_{T} \cdot \omega^{2} \cdot s^{\prime 2}+\left(k^{2}+2 k \cdot K\right) \cdot s^{2}+2 k \cdot x_{0} \cdot(K+k) \cdot s
$$

$$
\begin{aligned}
& M=\left[(K+k) m_{T} \omega^{2} \cdot 2 s^{\prime} s^{\prime \prime}+\left(k^{2}+2 k K\right) \cdot 2 s s^{\prime}+2 k x_{0}(K+k) \cdot s^{\prime}\right] \\
& \cdot\left(s+\frac{k x_{0}}{K+k}\right)-N \cdot s^{\prime}
\end{aligned}
$$

$$
O=(K+k) \cdot m_{T} \cdot \omega^{2} \cdot 2 \cdot\left(s^{\prime \prime 2}+s^{\prime} \cdot s^{\prime \prime \prime}\right)+
$$

$$
+\left(k^{2}+2 \cdot k \cdot K\right) \cdot 2 \cdot\left(s^{\prime 2}+s \cdot s^{\prime \prime}\right)+2 \cdot k \cdot x_{0} \cdot(K+k) \cdot s^{\prime \prime}
$$

$$
\begin{equation*}
x^{\prime \prime}=s^{\prime \prime}-\frac{\left[O \cdot\left(s+\frac{k x_{0}}{K+k}\right)-N \cdot s^{\prime \prime}\right] \cdot\left(s+\frac{k x_{0}}{K+k}\right)-M \cdot 2 \cdot s^{\prime}}{2 \cdot(K+k)^{2} \cdot\left(s+\frac{k x_{0}}{K+k}\right)^{3}} \tag{2.9}
\end{equation*}
$$

Further the acceleration of the tappet can be determined directly real (dynamic) using the relation (2.10).

$$
\begin{equation*}
\ddot{x}=x^{\prime} \cdot \omega^{2}+x^{\prime} \cdot \varepsilon \tag{2.10}
\end{equation*}
$$

## 3. New Dynamic Synthesis

Give the following parameters:
$\mathrm{r}_{0}=0.013[\mathrm{~m}] ; \mathrm{r}_{\mathrm{b}}=0.005[\mathrm{~m}] ; \mathrm{h}=0.008[\mathrm{~m}] ; \mathrm{e}=0.01$ $[\mathrm{m}] ; \mathrm{x}_{0}=0.03[\mathrm{~m}] ; \varphi_{\mathrm{u}}=\pi / 2 ; \varphi_{\mathrm{c}}=\pi / 2 ; \mathrm{K}=5000000$ $[\mathrm{N} / \mathrm{m}] ; \mathrm{k}=20000[\mathrm{~N} / \mathrm{m}] ; \mathrm{m}_{\mathrm{T}}=0.1 \quad[\mathrm{~kg}] ; \quad \mathrm{M}_{\mathrm{c}}=0.2$ $[\mathrm{kg}] ; \mathrm{n}_{\text {motor }}=5500[\mathrm{rot} / \mathrm{min}]$.

To sum up dynamically based on a computer program, you can vary the input data until the corresponding acceleration is obtained (see Figure 11). It then summarizes the corresponding cam profile (Figure 12) using the relations (2.11).


Fig. 11. Dynamic diagram to the rotary cam with translated follower with roll.



Fig. 12. The cam profile to the rotary cam with translated follower with roll; $\mathrm{r}_{\mathrm{b}}=\mathbf{0 . 0 0 3}[\mathrm{m}] ; \mathrm{e}=\mathbf{0 . 0 0 3}$ [m]; h=0.006 [m]; $\mathbf{r}_{0}=\mathbf{0 . 0 1 3 ~ [ m ] ; ~} \varphi_{0}=\pi / 2[\mathrm{rad}] ;$.

### 3.1. The New Geometry of the Rotary Cam and the Translated Follower with Roll

Now, we shall see the geometry of a rotary cam with translated follower with roll (Figure 13). The cam rotation sense is positive (trigonometric).

We can make the geometrical synthesis of the cam profile with the help of the cinematics of the mechanism. One uses as well the reduced speed, $\mathrm{s}^{\prime} . \mathrm{OA}=\mathrm{r}=\mathrm{r}_{\mathrm{A}} ; \mathrm{r}^{2}=\mathrm{r}_{\mathrm{A}}{ }^{2}$

It establishes a system fixed Cartesian, $\mathrm{xOy}=$ $\mathrm{x}_{\mathrm{f}} \mathrm{Oy}_{\mathrm{f}}$, and a mobil Cartesian system, $\mathrm{xOy}=$ $\mathrm{x}_{\mathrm{m}} \mathrm{Oy}_{\mathrm{m}}$ fixed with the cam.

From the lower position 0, the tappet, pushed by cam, uplifts to a general position, when the cam rotates with the $\varphi$ angle. The contact point A, go from $A_{i}^{0}$ to $A^{0}$ (on the cam), and to $A$ (on the tappet). The position angle of the point A from the tappet is $\theta_{\mathrm{f}}$, and from the cam is $\theta_{\mathrm{m}}$. We can determine the coordinates of the point A from the tappet (2.12), and from the cam (2.13).

$$
\left\{\begin{array}{l}
x_{T} \equiv x_{A}^{f}=-e+r_{b} \cdot \sin \delta=r_{A} \cdot \cos \theta_{f}=r \cdot \cos \theta_{f}  \tag{2.12}\\
y_{T} \equiv y_{A}^{f}=s_{0}+s-r_{b} \cdot \cos \delta=r_{A} \cdot \sin \theta_{f}=r \cdot \sin \theta_{f}
\end{array}\right.
$$

$$
\left\{\begin{array}{l}
x_{c} \equiv x_{A}^{m}=r_{A} \cdot \cos \theta_{m}=r \cdot \cos \left(\theta_{f}-\varphi\right)=r \cos \theta_{f} \cos \varphi+r \sin \theta_{f} \sin \varphi= \\
=x_{T} \cos \varphi+y_{T} \sin \varphi=\left(-e+r_{b} \cdot \sin \delta\right) \cdot \cos \varphi+\left(s_{0}+s-r_{b} \cdot \cos \delta\right) \cdot \sin \varphi \\
y_{c} \equiv y_{A}^{m}=r_{A} \cdot \sin \theta_{m}=r \cdot \sin \left(\theta_{f}-\varphi\right)=r \sin \theta_{f} \cos \varphi-r \sin \varphi \cos \theta_{f}= \\
=y_{T} \cos \varphi-x_{T} \sin \varphi=\left(s_{0}+s-r_{b} \cdot \cos \delta\right) \cdot \cos \varphi-\left(-e+r_{b} \cdot \sin \delta\right) \cdot \sin \varphi \tag{2.13}
\end{array}\right.
$$



Fig. 13. The geometry of the rotary cam with translated follower with roll.

One uses and the next relationships (2.14), where the pressure angle $\delta$ was obtained with the classic Antonescu P. method [2].

$$
\left\{\begin{array}{l}
s_{0}=\sqrt{\left(r_{0}+r_{b}\right)^{2}-e^{2}}  \tag{2.14}\\
\cos \delta=\frac{s_{0}+s}{\sqrt{\left(s_{0}+s\right)^{2}+\left(s^{\prime}-e\right)^{2}}} \\
\sin \delta=\frac{s^{\prime}-e}{\sqrt{\left(s_{0}+s\right)^{2}+\left(s^{\prime}-e\right)^{2}}}
\end{array}\right.
$$

### 3.2. Determining the Forces, the Velocities and the Efficiency

The driving force $F_{m}$, perpendicular on $r$ in $A$, is divided in two components: $\mathrm{F}_{\mathrm{n}}$, the normal force, and $\mathrm{F}_{\mathrm{a}}$, a force of slipping. Fn is divided, as well, in two components: $\mathrm{F}_{\mathrm{T}}$ is the transmitted (the utile) force, and $F_{R}$ is a radial force which bend the tappet (see 2.15, and the Figure 14).

$$
\left\{\begin{array} { l } 
{ \{ \begin{array} { l } 
{ F _ { n } = F _ { m } \cdot \operatorname { c o s } \alpha } \\
{ v _ { n } = v _ { m } \cdot \operatorname { c o s } \alpha }
\end{array} \quad \{ \begin{array} { l } 
{ F _ { T } = F _ { n } \cdot \operatorname { c o s } \delta = F _ { m } \cdot \operatorname { c o s } \alpha \cdot \operatorname { c o s } \delta } \\
{ v _ { T } = v _ { n } \cdot \operatorname { c o s } \delta = v _ { m } \cdot \operatorname { c o s } \alpha \cdot \operatorname { c o s } \delta }
\end{array} } \\
{ \mu _ { i } = \frac { P _ { u } } { P _ { c } } = \frac { F _ { T } \cdot v _ { T } } { F _ { m } \cdot v _ { m } } = \frac { F _ { m } \cdot \operatorname { c o s } \alpha \cdot \operatorname { c o s } \delta \cdot v _ { m } \cdot \operatorname { c o s } \alpha \cdot \operatorname { c o s } \delta } { F _ { m } \cdot v _ { m } } = } \\
{ = ( \operatorname { c o s } \alpha \cdot \operatorname { c o s } \delta ) ^ { 2 } = \operatorname { c o s } ^ { 2 } \alpha \cdot \operatorname { c o s } ^ { 2 } \delta }
\end{array} \left\{\begin{array}{l}
\beta=\pi-A ; \alpha=\frac{\pi}{2}-\beta=A-\frac{\pi}{2} ; \\
\cos A=\frac{r_{b}^{2}+r^{2}-r_{B}^{2}}{2 \cdot r_{b} \cdot r} ; \quad r_{B}=\sqrt{e^{2}+\left(s_{0}+s\right)^{2} ;} \\
r=\sqrt{x_{A f}^{2}+y_{A f}^{2}}=\sqrt{\left(-e+r_{b} \cdot \sin \delta\right)^{2}+\left(s_{0}+s-r_{b} \cdot \cos \delta\right)^{2}} \Rightarrow \\
\Rightarrow r \equiv r_{A}=\sqrt{e^{2}+r_{b}^{2}+\left(s_{0}+s\right)^{2}-2 \cdot r_{b} \cdot\left[e \cdot \sin \delta+\left(s_{0}+s\right) \cdot \cos \delta\right]}
\end{array}\right.\right.
$$



Fig. 14. Forces and velocities of the rotary cam with translated follower with roll.

### 3.3. New Geometro-Kinematics Synthesis

For a good work one proposes to make a new geometric and kinematics synthesis of the cam profile, using some new relationships for the pressure angle delta (2.16). The new synthesis relations already presented ( 2.12 and 2.13 ) will use delta pressure angle, deduced now with new relationship (2.16).

$$
\left\{\begin{array}{l}
x_{T} \equiv x_{A}^{f}=-e+r_{b} \cdot \sin \delta=r_{A} \cdot \cos \theta_{f}=r \cdot \cos \theta_{f}  \tag{2.12}\\
y_{T} \equiv y_{A}^{f}=s_{0}+s-r_{b} \cdot \cos \delta=r_{A} \cdot \sin \theta_{f}=r \cdot \sin \theta_{f}
\end{array}\right.
$$

$x_{c} \equiv x_{A}^{m}=r_{A} \cdot \cos \theta_{m}=r \cdot \cos \left(\theta_{f}-\varphi\right)=r \cos \theta_{f} \cos \varphi+r \sin \theta_{f} \sin \varphi=$
$=x_{T} \cos \varphi+y_{T} \sin \varphi=\left(-e+r_{b} \cdot \sin \delta\right) \cdot \cos \varphi+\left(s_{0}+s-r_{b} \cdot \cos \delta\right) \cdot \sin \varphi$
$y_{c} \equiv y_{A}^{m}=r_{A} \cdot \sin \theta_{m}=r \cdot \sin \left(\theta_{f}-\varphi\right)=r \sin \theta_{f} \cos \varphi-r \sin \varphi \cos \theta_{f}=$
$=y_{T} \cos \varphi-x_{T} \sin \varphi=\left(s_{0}+s-r_{b} \cdot \cos \delta\right) \cdot \cos \varphi-\left(-e+r_{b} \cdot \sin \delta\right) \cdot \sin \varphi$

One uses and the next relationships (where the pressure angle $\delta$ was obtained with the new method):


The new profile can be seen in the Figure 15.


Fig. 15. The new cam profile to the rotary cam with translated follower with roll; $\mathrm{r}_{\mathrm{b}}=\mathbf{0 . 0 0 3}[\mathrm{m}] ; \mathrm{e}=\mathbf{0 . 0 0 3}$ $[\mathrm{m}] ; \mathrm{h}=\mathbf{0 . 0 0 6}[\mathrm{m}] ; \mathrm{r}_{0}=\mathbf{0 . 0 1 3}[\mathrm{m}] ; \varphi_{0}=\pi / 2[\mathrm{rad}] ;$

### 3.4. Demonstration (Explication)

The original relationships (2.16) have been deduced by the expressions (2.17).

Classical method uses to the geometric synthesis and the reduced tappet velocity, and in this mode the geometric classic method become a geometric and kinematic synthesis method. The new geometric synthesis method uses just the geometric parameters (without velocities), but one utilizes and a condition to realize at the tapped the velocities predicted by the tapped movement laws imposed by the cam profile.

$$
\begin{align*}
& \left.\left\{\begin{array}{l}
\dot{s}=r_{A} \cdot \omega \cdot \cos \alpha \cdot \cos \delta \Rightarrow s^{\prime}=r_{A} \cdot \cos \alpha \cdot \cos \delta \\
\alpha+\beta=\frac{\pi}{2} ; \quad A+\beta=\pi ; \quad \alpha=\frac{\pi}{2}-\beta=A-\frac{\pi}{2} \Rightarrow \\
\Rightarrow \cos \alpha=\cos \left(A-\frac{\pi}{2}\right)=\cos \left(\frac{\pi}{2}-A\right)=\sin A
\end{array}\right\} \Rightarrow \begin{array}{l}
\frac{s^{\prime}}{r_{A} \cdot \cos \delta}=\sin A \\
\sin A=\frac{r_{B}}{r_{A}} \cdot \sin (\delta-B)
\end{array}\right\} \Rightarrow \\
& \left.\Rightarrow \begin{array}{l}
\frac{s^{\prime}}{r_{A} \cdot \cos \delta}=\frac{r_{B}}{r_{A}} \cdot \sin (\delta-B) \Rightarrow \frac{s^{\prime}}{r_{B} \cdot \cos \delta}=\sin (\delta-B) \\
\sin (\delta-B)=\sin \delta \cos B-\sin B \cos \delta=\frac{s_{0}+s}{r_{B}} \sin \delta-\frac{e}{r_{B}} \cos \delta
\end{array}\right\} \Rightarrow \\
& \Rightarrow \frac{s^{\prime}}{r_{B} \cdot \cos \delta}=\frac{\left(s_{0}+s\right) \cdot \sin \delta-e \cdot \cos \delta}{r_{B}} \Rightarrow s^{\prime}=\left(s_{0}+s\right) \cdot \sin \delta \cdot \cos \delta-e \cdot \cos ^{2} \delta \Rightarrow \\
& \Rightarrow\left(s_{0}+s\right) \cdot \cos \delta \cdot \sqrt{1-\cos ^{2} \delta}=s^{\prime}+e \cdot \cos ^{2} \delta \Rightarrow \\
& \Rightarrow\left(s_{0}+s\right)^{2} \cdot \cos ^{2} \delta-\left(s_{0}+s\right)^{2} \cdot \cos ^{4} \delta=s^{\prime 2}+e^{2} \cdot \cos ^{4} \delta+2 \cdot e \cdot s^{\prime} \cdot \cos ^{2} \delta \Rightarrow \\
& \Rightarrow\left[\left(s_{0}+s\right)^{2}+e^{2}\right] \cdot \cos ^{4} \delta-\left[\left(s_{0}+s\right)^{2}-2 \cdot e \cdot s^{\prime}\right] \cdot \cos ^{2} \delta+s^{\prime 2}=0 \Rightarrow \\
& \Rightarrow \cos ^{2} \delta=\frac{\left(s_{0}+s\right)^{2}-2 e s^{\prime} \pm \sqrt{\left[\left(s_{0}+s\right)^{2}-2 \cdot e \cdot s^{\prime}\right]^{2}-4 s^{\prime 2}\left[\left(s_{0}+s\right)^{2}+e^{2}\right]}}{2 \cdot\left[\left(s_{0}+s\right)^{2}+e^{2}\right]} \Rightarrow \\
& \Rightarrow \cos \delta=\sqrt{\frac{\left(s_{0}+s\right)^{2}+\left(s_{0}+s\right) \cdot \sqrt{\left(s_{0}+s\right)^{2}-4 \cdot s^{\prime 2}-4 \cdot e \cdot s^{\prime}}-2 \cdot e \cdot s^{\prime}}{2 \cdot\left[\left(s_{0}+s\right)^{2}+e^{2}\right]}} \tag{2.17}
\end{align*}
$$

Then, it makes the dynamic analyze for the imposed cam profile, and one modify the cam profile geometric parameters to determine a good dynamic response (functionality). In this mode it realizes the dynamic synthesis of the cam, and we obtain a normal functionality. The synthesis was made using the natural geometro-kinematics parameters (of cam mechanism). It follows the proper functioning dynamics. We will optimize and the couple cam-pusher efficiency. Forces, velocities and accelerations are also limited.

### 3.5. Increasing the mechanical efficiency at the Rotary Cam and Translated Follower with Roll

The used law is the classical law (2.4), cosine law.

The synthesis of the cam profile can be made with the relationships (3.1) when the cam rotates clockwise and with the expressions from the
system (3.2) when the cam rotates counterclockwise (trigonometric).
$\left\{\begin{array}{l}x_{C}=\left(-e-r_{b} \cdot \sin \delta\right) \cdot \cos \varphi-\left[\left(s_{0}+s\right)-r_{b} \cdot \cos \delta\right] \cdot \sin \varphi \\ y_{C}=\left[\left(s_{0}+s\right)-r_{b} \cdot \cos \delta\right] \cdot \cos \varphi+\left(-e-r_{b} \cdot \sin \delta\right) \cdot \sin \varphi\end{array}\right.$
$\left\{\begin{array}{l}x_{c}=\left(-e+r_{b} \cdot \sin \delta\right) \cdot \cos \varphi+\left(s_{0}+s-r_{b} \cdot \cos \delta\right) \cdot \sin \varphi \\ y_{c}=\left(s_{0}+s-r_{b} \cdot \cos \delta\right) \cdot \cos \varphi-\left(-e+r_{b} \cdot \sin \delta\right) \cdot \sin \varphi\end{array}\right.$

The $r_{0}$ (the radius of the base circle of the cam) is 0.013 [ m$]$. The h (the maximum displacement of the tappet) is $0.020[\mathrm{~m}]$. The angle of lift, $\square_{\mathrm{u}}$ is $\pi / 3$ [rad]. The radius of the tappet roll is $r_{b}=0.002$ [m]. The misalignment is $\mathrm{e}=0$ [m]. The cosine profile can be seen in the fig. 16.


Fig. 16. The cosine profile at the cam with translated follower with roll; $r_{0}=13[\mathrm{~mm}]$, $h=20[\mathrm{~mm}], \varphi_{u}=\pi / 3[\mathrm{rad}], \mathrm{r}_{\mathrm{b}}=2[\mathrm{~mm}], \mathrm{e}=0[\mathrm{~mm}]$.

The obtained mechanical yield (obtained by integrating the instantaneous efficiency throughout the climb and descent) is 0.39 or $\eta=39 \%$. The dynamic diagram can be seen in the fig. 17 (the dynamic setting are partial normal). Valve spring preload 9 cm no longer poses today. Instead, achieve a long arc very hard $(\mathrm{k}=500000[\mathrm{~N} / \mathrm{m}])$, require special technological knowledge.


Fig. 17. The dynamic diagram at the cosine profile at the cam with translated follower with roll; $\quad \mathrm{r}_{0}=13[\mathrm{~mm}] ; \quad \mathrm{h}=20[\mathrm{~mm}] ; \quad \varphi_{\mathrm{u}}=\pi / 3[\mathrm{rad}]$; $\mathrm{r}_{\mathrm{b}}=2[\mathrm{~mm}] ; \mathrm{e}=0[\mathrm{~mm}] ; \mathrm{n}=5500[\mathrm{rpm}] ; \mathrm{x}_{0}=9[\mathrm{~cm}]$; $\mathrm{k}=500[\mathrm{kN} / \mathrm{m}]$

It tries increase the yield [8-9, 15-16]; angle of climb is halved $\square_{\mathrm{u}}=\pi / 6[\mathrm{rad}]$ (see the profile in the Fig. 18).

The $\mathrm{r}_{0}$ (the radius of the base circle of the cam) is $0.015[\mathrm{~m}]$. The h (the maximum displacement of the tappet) is $0.010[\mathrm{~m}]$. The angle of lift, $\square_{\mathrm{u}}$ is $\pi / 6$ [rad]. The radius of the tappet roll is $r_{b}=0.002$ [m]. The misalignment is $\mathrm{e}=0$ [m]. The cosine profile can be seen in the Fig. 18.


Fig. 18. The cosine profile at the cam with translated follower with roll; $r_{0}=15[\mathrm{~mm}]$, $\mathrm{h}=10[\mathrm{~mm}], \varphi_{\mathrm{u}}=\pi / 6[\mathrm{rad}], \mathrm{r}_{\mathrm{b}}=2[\mathrm{~mm}], \mathrm{e}=0[\mathrm{~mm}]$.

The obtained mechanical yield (obtained by integrating the instantaneous efficiency throughout the climb and descent) is 0.428 or $\eta=43 \%$. The dynamic diagram can be seen in the fig. 19 (the dynamic setting are not normal). Valve spring preload 20 cm no longer poses today. Instead, achieve a long arc very-very hard $(\mathrm{k}=1500000[\mathrm{~N} / \mathrm{m}])$, require special technological knowledge.


Fig. 19. The dynamic diagram at the cosine profile at the cam with translated follower with roll; $\mathrm{r}_{0}=15[\mathrm{~mm}] ; \mathrm{h}=10[\mathrm{~mm}] ; \square_{\mathrm{u}}=\pi / 6[\mathrm{rad}] ; \mathrm{r}_{\mathrm{b}}=2[\mathrm{~mm}] ;$ $\mathrm{e}=0[\mathrm{~mm}] ; \mathrm{n}=5500[\mathrm{rpm}] ; \mathrm{x}_{\mathbf{0}}=\mathbf{2 0}[\mathrm{cm}] ; \mathrm{k}=1500[\mathrm{kN} / \mathrm{m}]$

Camshaft runs at a shaft speed halved $\left(\mathrm{n}_{\mathrm{c}}=\mathrm{n} / 2\right)$.
If we more reduce camshaft speed by three times ( $\mathrm{n}_{\mathrm{c}}=\mathrm{n} / 6$ ), we can reduce and the preload of the valve spring ( $\mathrm{x}_{0}=5[\mathrm{~cm}]$ ); see the dynamic diagram in the Fig. 20. However, in this case, the cam profile should be tripled (see the Fig. 21).


Fig. 20. The dynamic diagram at the cosine tripled profile at the cam with translated follower with roll; $\quad \mathrm{r}_{0}=15[\mathrm{~mm}] ; \quad \mathrm{h}=10[\mathrm{~mm}] ; \quad \varphi_{\mathrm{u}}=\pi / 6[\mathrm{rad}]$; $\mathrm{r}_{\mathrm{b}}=2[\mathrm{~mm}] ; \quad \mathrm{e}=0[\mathrm{~mm}] ; \quad \mathrm{n}=5500[\mathrm{rpm}] ; \quad \mathrm{x}_{0}=5[\mathrm{~cm}] ;$ $\mathrm{k}=1500[\mathrm{kN} / \mathrm{m}]$.


Fig. 21. The cosine tripled profile at the cam with translated follower with roll; $r_{0}=15[\mathrm{~mm}]$, $h=10[\mathrm{~mm}], \varphi_{\mathrm{u}}=\pi / 6[\mathrm{rad}], \mathrm{r}_{\mathrm{b}}=2[\mathrm{~mm}], \mathrm{e}=0[\mathrm{~mm}]$.

It tries increase the yield again; angle of climb is reduced to the value $\square_{\mathrm{u}}=\pi / 8[\mathrm{rad}]$.

The $\mathrm{r}_{0}$ (the radius of the base circle of the cam) is 0.013 [ m ].

The $h$ (the maximum displacement of the tappet) is 0.009 [ m ].

The angle of lift, $\square_{\mathrm{u}}$ is $\pi / 8$ [rad].
The radius of the tappet roll is $\mathrm{r}_{\mathrm{b}}=0.002[\mathrm{~m}]$.
The misalignment is $\mathrm{e}=0 \quad[\mathrm{~m}]$. The cosine profile can be seen in the fig. 22.


Fig. 22. The cosine profile at the cam with translated follower with roll; $\mathrm{r}_{0}=13[\mathrm{~mm}], \mathrm{h}=9[\mathrm{~mm}]$, $\varphi_{\mathrm{u}}=\pi / 8[\mathrm{rad}], \mathrm{r}_{\mathrm{b}}=2[\mathrm{~mm}], \mathrm{e}=0[\mathrm{~mm}]$.

The obtained mechanical yield (obtained by integrating the instantaneous efficiency throughout the climb and descent) is 0.538 or $\eta=54 \%$. The dynamic diagram can be seen in the fig. 23 (the dynamic setting are not normal). Valve spring preload 30 cm no longer poses today. Instead, achieve a long arc very-very hard $(\mathrm{k}=1600000[\mathrm{~N} / \mathrm{m}])$, require special technological knowledge.


Fig. 23. The dynamic diagram at the cosine profile at the cam with translated follower with roll; $\mathrm{r}_{0}=13[\mathrm{~mm}] ; \quad \mathrm{h}=9[\mathrm{~mm}] ; \quad \varphi_{\mathrm{u}}=\pi / 8[\mathrm{rad}] ; \mathrm{r}_{\mathrm{b}}=2[\mathrm{~mm}] ;$ $\mathrm{e}=0[\mathrm{~mm}] ; \mathrm{n}=5000[\mathrm{rpm}] ; \mathrm{x}_{0}=30[\mathrm{~cm}] ; \mathrm{k}=1600[\mathrm{kN} / \mathrm{m}]$

Camshaft runs at a shaft speed halved $\left(\mathrm{n}_{\mathrm{c}}=\mathrm{n} / 2\right)$. If we more reduce camshaft speed by four times $\left(\mathrm{n}_{\mathrm{c}}=\mathrm{n} / 8\right)$, we can reduce and the preload of the valve spring, $x_{0}=9[\mathrm{~cm}]$ and the elastic constant of the valve spring, $k=15000[\mathrm{~N} / \mathrm{m}]$; see the dynamic diagram in the Fig. 24. However, in this case, the cam profile should be fourfold (see the Fig. 25).


Fig. 24. The dynamic diagram at the cosine fourfold profile at the cam with translated follower with roll; $\mathbf{r}_{0}=13[\mathrm{~mm}] ; \quad \mathrm{h}=9[\mathrm{~mm}] ; \quad \varphi_{\mathrm{u}}=\pi / 8[\mathrm{rad}] ; \quad \mathbf{r}_{\mathrm{b}}=2[\mathrm{~mm}] ;$ $\mathrm{e}=0[\mathrm{~mm}] ; \mathrm{n}=5000[\mathrm{rpm}] ; \mathrm{x}_{0}=9[\mathrm{~cm}] ; \mathrm{k}=15[\mathrm{kN} / \mathrm{m}]$


Fig. 25. The cosine fourfold profile at the cam with translated follower with roll; $r_{0}=13[\mathrm{~mm}], h=9[\mathrm{~mm}]$, $\varphi_{\mathrm{u}}=\pi / 8[\mathrm{rad}], \mathrm{r}_{\mathrm{b}}=2[\mathrm{~mm}], \mathrm{e}=0[\mathrm{~mm}]$.

With the same angle of climb $\square_{\mathrm{u}}=\pi / 8[\mathrm{rad}]$, can increase performance even further, if the size tappet race take a greater value ( $\mathrm{h}=12[\mathrm{~mm}]$ ). The $r_{0}$ (the radius of the base circle of the cam) is 0.013 [m].

The h (the maximum displacement of the tappet) is $0.012[\mathrm{~m}]$. The angle of lift, $\square_{\mathrm{u}}$ is $\pi / 8$ [rad]. The radius of the tappet roll is $r_{b}=0.002$ [m]. The misalignment is $\mathrm{e}=0[\mathrm{~m}]$. The cosine profile can be seen in the fig. 26.


Fig. 26. The cosine profile at the cam with translated follower with roll; $r_{0}=13[\mathrm{~mm}]$, $h=12[\mathrm{~mm}], \varphi_{u}=\pi / 8[\mathrm{rad}], \mathrm{r}_{\mathrm{b}}=2[\mathrm{~mm}], \mathrm{e}=0[\mathrm{~mm}]$.

For correct operation it is necessary to decrease the speed of the camshaft four times, and all four times multiplication of the cam profile. Camshaft runs at a shaft speed halved ( $n_{c}=n / 2$ ). If we more reduce camshaft speed by four times $\left(\mathrm{n}_{\mathrm{c}}=\mathrm{n} / 8\right)$, we can reduce and the preload of the valve spring, $x_{0}=9[\mathrm{~cm}]$. The elastic constant of the valve spring is $k=1500000[\mathrm{~N} / \mathrm{m}]$. See the dynamic diagram in the Fig. 27. However, in this case, the cam profile should be fourfold. The obtained mechanical yield is 0.60 or $\eta=60 \%$.


Fig. 27. The dynamic diagram at the cosine fourfold profile at the cam with translated follower with roll; $\mathrm{r}_{0}=13[\mathrm{~mm}] ; \mathrm{h}=12[\mathrm{~mm}] ; \varphi_{\mathrm{u}}=\pi / 8[\mathrm{rad}] ; \mathrm{r}_{\mathrm{b}}=2[\mathrm{~mm}]$; $\mathrm{e}=0[\mathrm{~mm}] ; \mathrm{n}=5000[\mathrm{rpm}] ; \mathrm{x}_{0}=9[\mathrm{~cm}] ; \mathrm{k}=1500[\mathrm{kN} / \mathrm{m}]$

For now is necessary to stop here.
If we increase $h$, or decrease the angle $\square_{u}$, then is tapering cam profile very much. We must stop now at a yield value, $\eta=60 \%$.

## Nomenclature

$M \quad$ the mass of the mechanism, reduced at the valve
$K$ the elastically constant of the system
$k \quad$ the elastically constant of the valve spring
$c=\frac{1}{2} \cdot \frac{d M}{d t} \begin{aligned} & \text { the coefficient } \\ & \text { amortization }\end{aligned}$ of the system's
$F_{0}$ the elastically force which compressing the valve spring
$F_{r} \quad$ the resistant force
$F_{m} \quad$ the motor force
$x$ the effective displacement of the valve
$x_{0} \quad$ the valve (tappet) spring preload
$x$, the reduced valve (tappet) speed
$x$,' the reduced valve (tappet) acceleration
$y \equiv s \quad$ the theoretical displacement of the tappet reduced at the valve, imposed by the cam's profile
$y^{\prime} \equiv$ ' the velocity of the theoretical displacement of the tappet reduced at the valve, imposed by the cam's profile
$m_{T}=m_{2}$
$m_{3}$
$m_{5}$
$J_{l=} J_{c}$
$J_{4} \quad$ the inertia mechanical moment of the valve rocker
$\dot{y}_{2} \quad$ the tappet velocity, or the second movement-low, imposed by the cam's profile
$\dot{x} \quad$ the real (dynamic) valve velocity
$\ddot{x}$
$i=i_{25}$
$\dot{y}$

D
$\omega_{m} \quad$ the nominal (average) angular velocity of cam
$\omega=\omega_{l}=\quad$ the angular (real) rotation speed of the cam (or camshaft)
the angular rotation speed of the valve rocker
The pressure angle
The additional pressure angle roll radius
basic radius
horizontally misalignment
vertical misalignment position vectors
the reduced mechanical moment of inertia
the first derivative of the reduced mechanical moment of inertia the average reduced moment of inertia

## 4. Discussion

In this paper one presents an original method to determine the dynamic parameters at the camshaft (the distribution mechanisms). It makes the synthesis, of the rotary cam and tappet with translational motion with roll, with a great precision.

The authors introduce a new pressure angle, alpha, and a new method to determine the two pressure angles, alpha and delta.

The presented method is the most elegant and direct method to determine the kinematics and dynamic parameters.

The dynamic synthesis can generate a cam profile which will work without vibrations.

Processes robotization increasingly determine and influence the emergence of new industries, applications in specific environmental conditions, approach new types of technological operations, handling of objects in outer space, leading teleoperator in disciplines such as medicine, robots that covers a whole larger service benefits our society, modern and computerized. In this context, this paper seeks to contribute to the scientific and technical applications in dynamic analysis and synthesis of cam mechanisms.

In 1971 K . Hain proposes an optimization method to cam mechanism to achieve the optimum output transmission angle (maximum) and minimum acceleration [11].

In 1979 F . Giordana investigates the influence of measurement errors in kinematic analysis of cam [10].

In 1985 P . Antonescu presents an analytical method for the synthesis mechanism flat tappet cam and tappet rocker mechanism [2].

In 1987 F.I. Petrescu presents a new dynamic model with general applications [3].

In 1988 J. Angelas and C. Lopez-Cajun presents optimal synthesis mechanism oscillating flat tappet cam and [1].

In 1991 B.S. Bagepalli presents a generalized model of dynamic cam-follower pairs [4].

In 1999 R.L. Norton studying the effect of valve-cam ramps on the valve-train dynamics [12].
Z. Chang presents in 2001 [5] and 2011 [6] a study on dynamics of roller gear cam system, considering clearances.

In 2002 D. Taraza synthesized analyzes the influence of the cam profile, the variation of the angular velocity distribution tree, and the parameters of power load consumption and emissions of internal combustion engine [16].

In 2005 [13] and 2008 [14], F.I. Petrescu and R.V. Petrescu present a synthesis method of rotary cam profile, and translational or rotary tappet, flat or with roll, to obtain high yields output.

In 2009 K . Dan makes some research on dynamic behavior simulation technology for camdrive mechanism in single-cylinder engines [7].

In 2009 M. Satyanarayana makes a dynamic experiment in cam-follower mechanism [15].

In 2011 Z. Ge makes the design and dynamic analysis of the cam mechanisms [8-9].

## 5. Conclusions

The distribution mechanisms work with small efficiency for about 150 years; this fact affects the total yield of the internal heat engines. Much of the mechanical energy of an engine is lost through the mechanism of distribution. Multi-years the yield of the distribution mechanisms was only 4$8 \%$. In the past 20 years it has managed a lift up to the value of $14-18 \%$; car pollution has decreased and people have better breathing again. Meanwhile the number of vehicles has tripled and the pollution increased again.

Now, it's the time when we must try again to grow the yield of the distribution mechanisms.

The paper presents an original method to increase the efficiency of a mechanism with cam and follower, used at the distribution mechanisms.

This paper treats only one module: the mechanism with rotary cam and translated follower with roll (the modern module B ).

At the classical module C we can increase again the yield to about $30 \%$. The growth is difficult. Dimensional parameters of the cam must be optimized; optimization and synthesis of the cam profile are made dynamic, and it must set the elastic (dynamic) parameters of the valve (tappet) spring: k and $\mathrm{x}_{0}$.

The law used is not as important as the module used, sizes and settings used. We take the classical law cosine; dimensioning the radius cam, lift height, and angle of lift.

To grow the cam yield again we must leave the classic module C and take the modern module B . In this way the efficiency can be as high as $60 \%$.

Yields went increased from $4 \%$ to $60 \%$, and we can consider for the moment that we have gain importance, since we work with the cam and tappet mechanisms.

If we more increase $h$, or decrease the angle $\square_{\mathrm{u}}$, then is tapering cam profile very much. We must stop now at a yield value, $\eta=60 \%$.

It can synthesize high-speed cam, or highperformance camshafts.

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## التركيب الاينـاميكي للحدبات

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يقام هذا البحث طريقة جديدة لوصف النركيب الهنسي للحدبات الدوارة وانتقال التابع اللملصق اثناء الاندفاع ـ يستخدم الأسلوب التقليدي في وصف التركيب الهنسي وتقليل سر عة التابع المندفع، وفي هذا الوضع فان الأسلوب الكلاسيكي يصبح كطريقة تركيب هندسية وحركية . تستخدم طريقة اللتركيب الهندسية الجديدة اللحددات الهندسية (بدون السر عات)، ولكن يمكن استخدام ومعاينة سرعة التابع المندفع المتحققة والتي يمكن تنتأها من خلال قو انين الحركة النتجة من شكل الحدبة . عندها يتم التحليل الديناميكي لشكل الحدبة و التعديل على محدداتها لحساب التصرف الديناميكي الجيد في العمل .حيث يمكن من خلال هذا الوضع ادر اكك التركيب الديناميكي الامثل للحصول على الاداء الاعتياد.

