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Optimal Warranty Length and Selling Price to Maximize the Profit

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Abstract

This study focuses on the problem of determining the optimal coverage period and selling price of warranted products from the manufacturer's perspective. We first consider how to maximize the profit per unit under the assumption that the product can be sold or the demand is independent of the warranty policy. Then we try to maximize the total profit for a planning period for the case where the demand for the product depends on the warranty coverage period and selling price. Since the warranty period and the selling price should be positively correlated, we first solve the profit maximization problem with the warranty depended demand under the constraint that the selling price is a linear function of the warranty coverage period (warranty based pricing). Furthermore, we investigate the case when such a constraint is removed (non-warranty-based pricing). Optimizing on two independent decision variables, the coverage period and the selling price, certainly improves the total profit. Under the two variable optimal conditions, it is observed that while the positive relationship between the optimal coverage period and the optimal price is confirmed, it is more complex than a linear one. We also find that the profit advantage for the non-warranty-based pricing over the warranty-based pricing is more significant for shorter coverage period. However, when the coverage period exceeds a threshold value, such a profit advantage becomes insignificant. The results of this study provide practitioners with useful insights in designing the profit optimal product warranty in highly competitive market.

Keywords: warranty, profit maximization, repair, replacement, renewable, optimization, price

1. Introduction

In today's highly competitive market of consumer products, product warranties become an important criterion that distinguishes good and poor quality products. A warranty satisfying customer well can enhance customer willingness of purchasing, hence stimulate customer demand. This is because that a comprehensive warranty that covers a longer period implies a higher quality of the product. Therefore, manufacturers try to offer a variety of warranties to expand the market share as consumers consider warranties as one of most important aspects in deciding which product to be purchased. However, a more attractive (or better customer service) warranty also implies higher service costs for manufacturers. As the product becomes more complex and expensive, the coverage of the warranty is required longer. An obvious trade-off between the benefit and cost of a warranty for a manufacturer is a critical issue for both academic researchers and practitioners. In the past decade, most of studies in the literature focused on the issue of how to minimize the warranty cost via preventive maintenance. Shafiee and Chukova (2013) provided a survey in this area from 2001 to 2011. There are relatively fewer works addressing both benefit and cost aspects of a product warranty. In fact, the marginal benefit can well exceed the marginal cost when the warranty coverage period is not long enough. For example, Murthy et al. (2004) pointed out that on average the marginal profit for product with a warranty is about 30% compared with 10% without a warranty. In addition,

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a warranty with more comprehensive and longer coverage period helps build up the company and product image that may have the same positive effects of advertisement on future demand. Thus, a good warranty benefits both consumer and manufacturers.

As mentioned before, there are not many studies on determining product warranty parameters with consideration of its effects on market demand. Glickman and Berger (1976) is one the earliest papers on the effects on consumer demand of warranty parameters. They addressed the problem of determining the optimal price and the warranty period and considered demand function of market statistic in which the demand decreases exponentially with price and increases exponentially with the warranty length (coverage period). Applications of the demand function of this form can be found in the literature such as Mitra and Jayprakash (1997). Ladany and Shore (2007) also consider a Cobb-Douglas-type demand function in determining the optimal warranty period, they presented a review on manufacturer's pricing strategies for supply chain with warranty period-dependent demand.

From the consumer's perspective, the longer the warranty period is, the better the after-sales service is. However, a longer warranty period implies a higher expected warranty cost to the manufacturer. This is particular true for the Renewing Free-Replacement Warranty (RFEW) under which the product must be replaced with a new one and a new full warranty period if the product fails within the warranty period offered. Since the purpose of REFW is to ensure a product to function for the entire warranty period, the length of the warranty period can be viewed as a measure of the product's quality and reliability. In addition, due to its cost implication to a manufacturer, we can assume that there is a positive correlation between the selling price and the warranty period. In this paper, we first find the optimal warranty period or selling price to maximize the profit of a sold product. Another issue we address is the impact of the RFEW on the customer demand. It is reasonable to assume that the warranty period and the customer demand can be positively correlated for a certain range of the selling price. To model such a relation, we utilize the Cobb-Douglas-type demand function of selling price and warranty period to determine the optimal or profit maximization warranty period and/or selling price for the manufacturer

2. Mathematical Formulation

Consider a product with a random life-time of X. Let $f(x)$, $F(x)$ (=$P(X \leq x)$), $\overline{F}(x)$ (=1–$F(x)$), and $r(x)$ ($= f(x)/\overline{F}(x)$) be the probability density function (pdf), cumulative distribution function (cdf), survival function (sf), and failure rate function (fr) of X, respectively. Let W be the period of the RFEW and c_p be the selling price of the product. We first assume that there is a positive linear relation between the selling price and W or $c_p = a+bW$. Such an assumption has been made in Ladany and Shore (2007). Under this assumption, we can find the optimal W to maximize the profit of a sold product.

2.1. Unit Profit Maximization for a Product Sold

Denote by Y the number of failures (replacements) of a sold product within the RFRW. According to the scheme of RFRW, the probability mass function of Y can be expressed as

$$P(Y=y)=\begin{cases}\overline{F}(W), & \text{if } \ y=0,\\ [F(W)]^{y}\,\overline{F}(W), & \text{if } \ y\geq 1.\end{cases} \tag{1}$$

Let c_r be the replacement cost when the product fails within the RFRW period. Based on (1), the expected cost of RFRW is

$$C(W)=c_r\cdot E(Y)=c_r\,\frac{F(W)}{\overline{F}(W)} \tag{2}$$

Thus the expected profit for the product sold is

$$\kappa(W)=c_p-C(W)=a+bW-c_r\,\frac{F(W)}{\overline{F}(W)} \tag{3}$$

Let W_0^* be the maximize of (3) or $\kappa\left(W_0^*\right)=\max_{W\geq 0}\kappa(W)$. If the failure rate is an increasing function (IFR), we can establish the following properties.

Property 1. *For a product sold with the* RFRW, *if the selling price is a linear function of the warranty period, the profit maximization W_0^* has the following properties*:

(i) *If* $r(0) \geq b/c_r$, *then* $W_0^* = 0$ *and* $\kappa(W_0^*) = \kappa(0) = a > 0$;

(ii) *If* $r(0) < b/c_r$, *then* $0 < W_0^* < \infty$ *exists and unique, and* $\kappa(W_0^*) > a$.

This proposition indicates that there exists a finite warranty period that maximizes the profit and the profit of selling one product is at least *a*. Whenever the initial failure rate exceeds a threshold (or $r(0) \geq b/c_r$), the RFRW should not be provided or $W_0^* = 0$ and the unit profit is still at least *a*.

2.2. Total Profit Maximization for Products Sold in a Planning Period

Now we solve the profit maximization problem when the impact of the RFRW on the customer demand is taken into account. Let $Q(W)$ denote the quantity sold in a planning period for products sold under RFRW. It is assumed that the demand depends on the warranty period W and is modeled as Cobb-Douglas-type function:

$$Q(W) = \alpha \cdot (c_p)^{\beta} (W)^{\gamma} \tag{4}$$

where $\alpha > 0$, $\beta < 0$, $\gamma > 0$. Denoting the total profit for the period by $\pi(W)$, we have

$$\pi(W) = Q(W) \times \kappa(W) \tag{5}$$

Denote the profit maximal warranty period by W^* or $\pi(W^*) = \max_{W \geq 0} \pi(W)$. We can obtain the following result.

Property 2. *For the product with an* RFRW, *if the selling price is a linear function of W and the demand function is given in* (4), *then there exists a unique and finite W that maximizes the total profit for a planning period*.

2.3. Total Profit Maximization Without Warranty-based Pricing

Now we assume that both the price and the warranty period are decision variables for maximizing the total profit for a planning period. The total profit function $\pi(c_p, W)$ can be written as

$$\pi(c_p, W) = Q(c_p, W) \times \kappa(c_p, W)$$

$$= \alpha \cdot (c_p)^{\beta} (W)^{\gamma} \left[c_p - c_r \frac{F(W)}{\overline{F}(W)} \right] \tag{6}$$

where $\alpha > 0$, $\beta < 0$ and $\gamma > 0$. We can show that there exists an optimal solution to (6).

Property 3. *For the profit function* $\pi(c_p, W)$ *given in* (6), *under the condition* $\beta + 1 < 0$, *if there exist* (c_p^*, W^*) *satisfying*

$$\left(\frac{-\gamma}{\beta + 1} \right) = \frac{W^* r(W^*)}{F(W^*)}$$

and

$$c_p^* = \frac{\beta c_r}{\beta + 1} \frac{F(W^*)}{\overline{F}(W^*)},$$

then (c_p^*, W^*) *maximize* $\pi(c_p, W)$ *or* $\pi(c_p^*, W^*) = \mathrm{Max}_{c_p > 0, W > 0}\, \pi(c_p, W)$.

An interesting finding here is that when both the price and warranty period are decision variables, the optimal warranty period W^* only depends on the demand function parameters of β and γ. It does not even depend on the replacement cost. Specifically, as γ increases both W^* and c_p^* increase; as β decreases both W^* and c_p^* decrease. These relations are intuitive.

Now we solve the profit maximization problem when the impact of the RFRW on the customer demand is taken into account. Let $Q(W)$ denote the quantity sold in a planning period for products sold under RFRW. It is assumed that the demand depends on the warranty period W and is modeled as Cobb-Douglas-type function:

3. Numerical Illustration and Sensitivity Analysis

Assume that lifetime of the product X has a Weibull distributed survival function

$$\overline{F}(x) = \exp\left\{ -(\lambda x)^{\theta} \right\} \tag{7}$$

where scale parameter and shape parameter are $\lambda = 1$ and $\theta = 2$, respectively. Clearly, such a distribution has the IFR property.

Our first numerical example is the one with the warranty-based pricing and is based on the set of parameters as $c_r = 1.0$, $a = 1.0$, $b = 0.2$, $\alpha = 10$, $\beta = -0.5$, and $\gamma = 0.5$. The unit profit function and its derivative can be shown. The decreasing first-order derivative of the unit profit function shows the concavity of the function. Thus the first-order derivative condition will give the global optimal point.

The total profit $\pi(W)$ and its derivative are shown below respectively, which indicate that a unique optimal warranty period exists although the profit function has both convex and concave sections.

Next we consider the case where both the selling price and the warranty period are decision variables for maximizing the total profit. The optimal c_p^*, W^* and $\pi(c_p^*, W^*)$ are computed under various β and γ, presented in Table 1.

Table 1 The optimal W^*, c^*_p and π (c_p^*, W^*) under various β and γ

γ	β			
	−1.1	−1.2	−1.3	−1.4
1.2	2.446 2175.7 12.330	1.680 47.451 7.176	1.262 8.4862 5.3544	0.935 2.4447 4.6083
1.4	2.645 6002.0 14.865	1.839 85.281 8.0364	1.423 14.247 5.6810	1.117 4.3440 4.6343
1.6	3.000 44561. 22.517	2.109 253.33 10.553	1.680 34.270 6.7785	1.386 10.198 5.0772

4. Conclusions

In this paper, we have studied the problem of joint determination of the optimal selling price of a product and the optimal warranty period offered at no extra charge with the product as an incentive to the consumer. Expected demand in our model was taken to be the Cobb-Douglas type demand function. Structural and insightful properties of the optimal results are obtained and compared analytically. Our study provides a useful quantitative tool for sellers to decide price and warranty length for products sold.

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References

[1] T. S. Glickman and P. D. Berger, "Optimal price and protection period decisions for a product under warranty," Management Science, vol. 22, pp.1381-1389, 1976.

[2] S. P. Ladany and H. Shore, "Profit maximizing warranty period with sales expressed by a demand function," Quality and Reliability Engineering International, vol. 23, pp. 291-301, 2007.

[3] A. Mitra and G. P. Jayprakash, "Market share and warranty costs for renewable warranty programs," International Journal of Production Economics, vol. 50, pp. 155-168, 1997.

[4] D. N. P. Murthy, O. Solem and T. Roren, "Product warranty logistics: Issues and challenges," European Journal of Operational Research, vol. 156, pp. 110-126, 2004.

[5] M. Shafiee and S. Chukova, "Product warranty logistics: issues and challenges," European Journal of Operational Research, vol. 229, pp. 561-572, 2013.