CONNECTING VOLUME EFFECTS ON DYNAMICS OF PNEUMATIC PRESSURE MEASUREMENT SYSTEMS

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Abstract:

The pressure sensors are often placed at a certain distance from the measured object. Beside the properties of the transmission fluid, dynamic characteristics of such a measurement system depends on the geometry and dimensions of the connecting elements. So far made research works have shown that the internal volume of the pressure sensor can have a large influence on the dynamic response. This paper is focused on theoretical analysis of the effects of the sensor volume on characteristic parameters of both the frequency and the time response of the system under discussion with a gas medium.

Keywords: pressure measurement; pressure transmission; dynamic response; sensor volume; connecting tube

1. INTRODUCTION

The dynamic response of the pressure measurement system depends on dynamic characteristics of its constructional components. Because the pressure sensors are often placed at a certain distance from the measured object, an additional pressure transmission system is formed in-between. The dynamic response of the pressure transmission depends on geometry, dimensions and stiffness of the connecting elements, as well as thermodynamic and transport properties of the transmission fluid.

The most frequently studied physical model of the pressure transmission system consists from a straight cylindrical connecting tube and an internal volume of the sensor (Figure 1). Following the work of Bergh and Tijdeman [1] the authors of this paper derived one of the latest, most sophisticated versions of the linear mathematical model, which is applicable for both pneumatic and hydraulic pressure measurement systems, and considers viscous, thermal and compliance effects [2]. This improved mathematical model was also experimentally validated for the case of pneumatic pressure measurement systems [3].
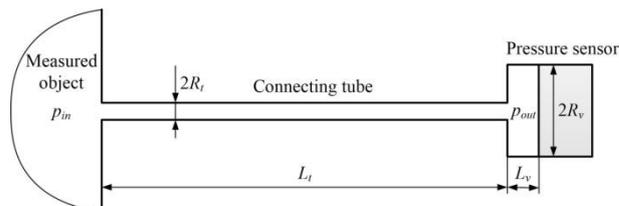

Figure 1: Physical model of the pressure transmission system

In some limited frequency ranges the dynamics of the pressure transmission system can be well described by low-order lumped-parameter mathematical models; e.g., the second-order model with two parameters (natural frequency, damping ratio) or the first-order model with one parameter (time constant) [4–7]. In [2] the authors derived analytical approximations of these lumped parameters by applying an asymptotic series expansion to the complete mathematical model.

The paper is organized as follows. Section 2 presents the mathematical model for the frequency and the step response of the pressure transmission system and its analytical approximations. Section 3 discusses effects of dimensions on characteristic parameters of dynamic response and exposes potential directions for optimal design of the pressure transmission systems. Section 4 shortly summarizes conclusions about the effects of the sensor volume.

2. MATHEMATICAL MODEL

The presented mathematical model deals with the pressure transmission system that consists of the cylindrical connecting tube with radius R_t and length L_t and the cylindrical sensor volume with radius R_v and length L_v (Figure 1). It is based on the following main assumptions: the internal radius of the tube is small enough in comparison with its length to neglect the end-flow effects, the dimensions of the sensor volume are small enough to neglect fluid-wave effects, and all the pressure variations in the system have small enough amplitudes for validity of the linear model.

2.1. Complete model

The complex frequency response function of the pressure measurement system under discussion can be written as:

$$G(i\omega) = \frac{P_{out}}{P_{in}} = \frac{1}{\cosh\Gamma + \dfrac{V_v}{V_t}\dfrac{n_t}{n_v}\Gamma\sinh\Gamma}, \qquad (1)$$

where $V_t = \pi R_t^2 L_t$ is the internal volume of the tube, $V_v = \pi R_v^2 L_v$ is the internal volume of the sensor, Γ is the propagation wavenumber, and n_t and n_v are the effective thermodynamic indexes for the gas in the tube and the sensor volume, respectively (see [2] for the complete expressions for Γ, n_t and n_v, including the wall compliance effects).

The time step response is calculated from the frequency response function by using the Fourier transform, considering the rectangular pulse of the input pressure signal. The Fourier transform of the rectangular pulse with the length T_{pulse} is defined as:

$$P_{in}(i\omega) = \int_{-\infty}^{\infty} p_{in}(t)e^{-i\omega t}dt = \frac{1 - e^{-i\omega T_{pulse}}}{i\omega}. \qquad (2)$$

The output signal in the frequency domain is determined as:

$$P_{out}(i\omega) = G(i\omega)P_{in}(i\omega), \qquad (3)$$

and the time response is calculated by its inverse Fourier transform:

$$p_{out}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} P_{out}(i\omega)e^{i\omega t}d\omega. \qquad (4)$$

With the intention to improve the efficiency of calculations, the procedure presented by equations (2–4) is implemented in calculations in this paper by using the discrete Fourier transform. The corresponding sampling frequency and the number of samples were adapted in a way to assure that errors in calculations of all presented response times are lower than 10^{-3}.

2.2. Analytical approximations

The study in this paper is limited to the pressure transmission systems with the gas medium, where the wall compliance effects are usually not of much importance, so the presented analytical approximations taken from ref. [2] do not consider the compliance effects.

For the connecting tubes with relatively small internal diameters (overdamped conditions) the pressure transmission system can be approximated with the first-order low-wavenumber model:

$$G_1(i\omega) = \frac{1}{1 + \tau i\omega}, \qquad (5)$$

which is defined with one parameter, i.e., the time constant τ:

$$\tau = \frac{8\mu L_t^2}{\rho c^2 R_t^2}\left(\frac{1}{2} + \frac{V_v}{V_t}\right), \qquad (6)$$

where c, ρ and μ are the speed of sound, density and dynamic viscosity of gas, respectively. The amplitude frequency response of the first order system is defined as:

$$\left|G_1(i\omega)\right| = \frac{1}{\sqrt{1 + (\tau\omega)^2}}, \qquad (7)$$

and the step response can be defined as:

$$p_{out,1}(t) = 1 - e^{-t/\tau}. \qquad (8)$$

For the connecting tubes with relatively large internal diameters (underdamped conditions) the pressure transmission system can be approximated with the second-order high-wavenumber model:

$$G_2(i\omega) = \frac{1}{1 + 2\zeta i\omega/\omega_n - \omega^2/\omega_n^2}, \qquad (9)$$

which is defined with two parameters, i.e., the natural frequency ω_n and the damping ratio ζ:

$$\omega_n = \frac{c}{L_t}\sqrt{\frac{1}{\dfrac{1}{2} + \dfrac{V_v}{V_t}}} \qquad (10)$$

$$\zeta = \frac{\sqrt{2}}{2R_t}\sqrt{\frac{\mu}{\rho\omega_n}}\left(\frac{\omega_n L_t}{c}\right)^2\left(\frac{1 + \delta_t}{2} + \frac{V_v}{V_t}(1 + \delta_v)\right), \quad (11)$$

where δ_t and δ_v represent the thermal effects for the gas in the tube and the sensor volume, respectively:

$$\delta_t = \frac{\gamma - 1}{\sqrt{Pr}}, \quad \delta_v = \frac{R_t}{L_v}\left(1 + \frac{L_v}{R_v}\right)\frac{\gamma - 1}{\sqrt{Pr}}, \qquad (12)$$

where γ is the heat capacity ratio and Pr is the Prandtl number. For $\zeta \ll 1$ the amplitude frequency response of the second order system can be approximated as:

$$\left|G_2(i\omega)\right| \approx \frac{1}{1 - \omega^2/\omega_n^2}, \qquad (13)$$

and the lower envelope of the oscillating step response can be approximated as:

$$p_{out,2}(t) \approx 1 - e^{-\omega_n \zeta t}. \qquad (14)$$

3. RESULTS

3.1. Frequency bandwidth

Characteristics of the frequency response are analysed in terms of the frequency bandwidth, which is defined as the range of frequencies over which the relative amplitude dynamic errors do not exceed 10^{-2} or 1%. The bandwidth limit frequency f_{lim} is calculated as the minimum frequency, where the amplitude ratio reaches 0.99 or 1.01. Figure 2 shows variations of the limit frequency for different dimensions of the pressure transmission system with dry air ($\rho = 1.19$ kg/m^3, $c = 343$ m/s, $\mu = 18.2$ µPa s, $\gamma = 1.4$, Pr = 0.71).
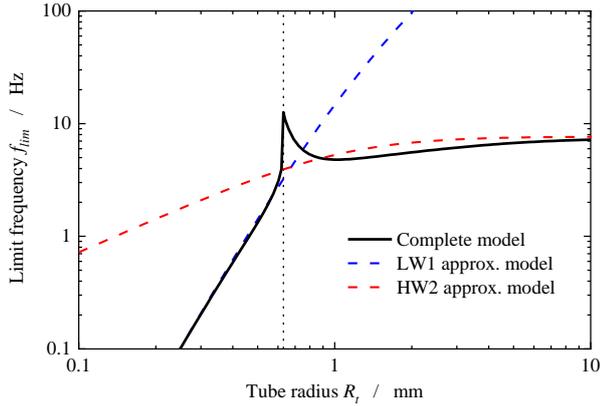
(a) Sensor volume $V_v = 1.77$ cm^3 ($L_v = 2.5$ mm, $R_v = 15$ mm)
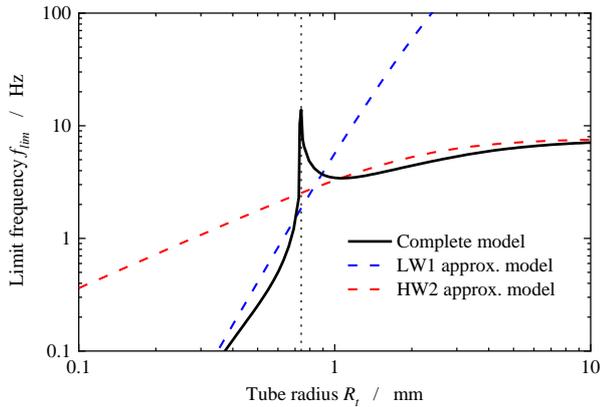
(b) Sensor volume $V_v = 7.07$ cm^3 ($L_v = 10$ mm, $R_v = 15$ mm)

Figure 2: Variations of the limit frequency with the tube radius for two different sensor volumes (tube length $L_t = 1$ m, dry air)

The dashed lines in Figure 2 represent the results of the first-order low-wavenumber (LW1) and second-order high-wavenumber (HW2) approximation models. The LW1 limit frequency is determined using equation (7):

$$\frac{1}{\sqrt{1+\left(\tau\omega_{lim}^{(L)}\right)^2}} = 1-e_r , \qquad (15)$$

which results in:

$$f_{lim}^{(L)} = \frac{\omega_{lim}^{(L)}}{2\pi} = \frac{1}{2\pi\tau}\sqrt{\frac{1}{\left(1-e_r\right)^2}-1} . \qquad (16)$$

The HW2 limit frequency is determined using equation (13):

$$\frac{1}{1-\omega_{lim}^{(H)2}/\omega_n^2} = 1+e_r , \qquad (17)$$

which results in:

$$f_{lim}^{(H)} = \frac{\omega_{lim}^{(H)}}{2\pi} = \frac{\omega_n}{2\pi}\sqrt{1-\frac{1}{1+e_r}} . \qquad (18)$$

The LW1 model predicts the limit frequency relatively well for thiner connecting tubes. The frequency bandwidth is inversely proportional to the time constant τ and, therefore, is inversely proportional to $(1/2 + V_v/V_t)$. The HW2 model predicts the limit frequency relatively well for thicker connecting tubes. The frequency bandwidth is proportional to ω_n and, therefore, is inversely proportional to $(1/2 + V_v/V_t)^{1/2}$.

The frequency bandwidth is the largest in the intermediate region of validity of the low-wavenumber and high-wavenumber approximate solutions. For the presented simulation results the largest limit frequency is found at the tube radius of 0.63 mm for the smaller sensor volume (Figure 2(a)) and at the tube radius of 0.74 mm for the larger sensor volume (Figure 2(b)). The optimal tube radius that results in the largest frequency bandwidth can be well predicted by the intersection of both approximate models.
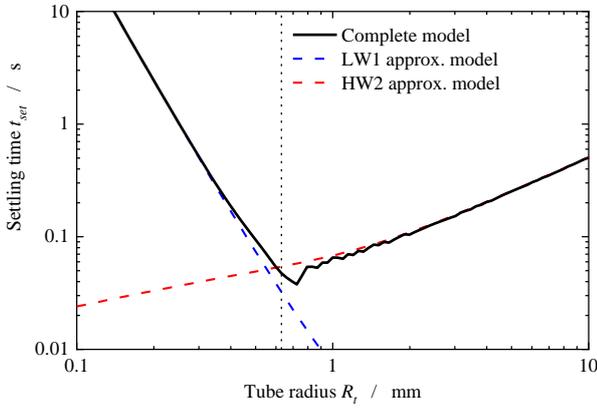
3.2. Settling time

Characteristics of the step transient response are analysed in terms of the settling time, which is defined as the minimum time that the normalized response needs to fully settle within 0.99 and 1.01; i.e., the relative dynamic errors of the response do not exceed 10^{-2} or 1%. Figure 3 shows variations of the settling time for different dimensions of the pressure transmission system.

The dashed lines in Figure 3 represent the results of the first-order low-wavenumber (LW1) and second-order high-wavenumber (HW2) approximation models. The LW1 settling time is determined using equation (8):
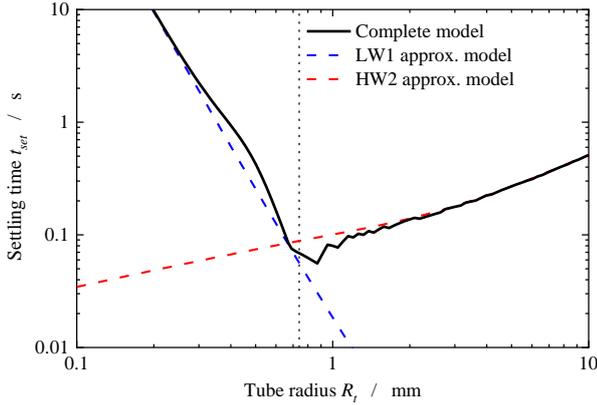
$$1-e^{-\frac{t_{set}^{(L)}}{\tau}} = 1-e_r , \qquad (19)$$

which results in:

$$t_{set}^{(L)} = -\tau \ln e_r . \qquad (20)$$

(a) Sensor volume $V_v = 1.77$ cm^3 ($L_v = 2.5$ mm, $R_v = 15$ mm)

(b) Sensor volume $V_v = 7.07$ cm^3 ($L_v = 10$ mm, $R_v = 15$ mm)

Figure 3: Variations of the settling time with the tube radius for two different sensor volumes (tube length $L_t = 1$ m, dry air)

The HW2 settling time is determined using equation (14):

$$1 - e^{-\omega_n \zeta t_{set}^{(H)}} = 1 - e_r, \qquad (21)$$

which results in:

$$t_{set}^{(H)} = -\frac{1}{\omega_n \zeta} \ln e_r. \qquad (22)$$

The LW1 model predicts the settling time relatively well for thiner connecting tubes. The settling time is proportional to the time constant τ and, therefore, is proportional to $(1/2 + V_v/V_t)$. The HW2 model predicts the settling time relatively well for thicker connecting tubes. The settling time is inversely proportional to $\omega_n \zeta$ and, therefore, is proportional to:

$$t_{set}^{(H)} \propto \frac{1}{\omega_n \zeta} \propto \frac{\left(\dfrac{1}{2} + \dfrac{V_v}{V_t}\right)^{5/4}}{\dfrac{1}{2}(1 + \delta_t) + \dfrac{V_v}{V_t}(1 + \delta_v)}; \qquad (23)$$

so if the thermal effects are neglected ($\delta_v \to 0$, $\delta_t \to 0$), the settling time is found proportional to $(1/2 + V_v/V_t)^{1/4}$.

Similar to the findings about the frequency bandwidth, the fastest response is found in the intermediate region of validity of the low-wavenumber and high-wavenumber approximate solutions. The smallest settling time occurs at a bit higher tube radius in comparison with the tube radius that results in the largest limit frequency (marked in Figure 3 with the vertical dot line).

4. SUMMARY

As could be expected, the dynamic response of the pressure transmission system is deteriorated with the increased sensor volume. These effects significantly depend on dimensions of the connecting tube. Both the limit frequency of the frequency response and the settling time of the step response are influenced by the sensor volume in terms of the magnitude of $(1/2 + V_v/V_t)$. Therefore, the effects of the sensor volume can be neglected if $V_v << ½ V_t$.

5. REFERENCES

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