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Improved evaluation of uncertainty for indirect measurement

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ABSTRACT
The paper gives formulae for uncertainty evaluation of an indirect measurement based on direct measurements made by different types of measuring devices. The first type of the measuring devices has the specifications of a total error (e.g. digital instruments), while the second type has the specifications of offset, gain and linearity errors (e.g. analog to digital converters). The choice of a device range and the configuration of measuring circuits for decreasing uncertainty are considered. The conversion of the specifications for the first type to the specifications for the second type is discussed.

Section: RESEARCH PAPER

Keywords: Indirect measurement; accuracy specifications; measurement uncertainty

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1. INTRODUCTION

It is well-known [1] that the result of an indirect measurement is a function of several variables:

$$X = f(X_1, X_2, ..., X_n) . \tag{1}$$

The values of these variables are found by direct measurements. The maximum possible absolute error of the indirect measurement as a function of maximum possible errors for the direct measurements ($|\Delta X|_1 ... |\Delta X_n|$) can be found from (1) approximately as [1]:

$$|\Delta X| = \left|\frac{\partial f}{\partial X_1}\right| |\Delta X_1| + ... + \left|\frac{\partial f}{\partial X_n}\right| |\Delta X_n| . \tag{2}$$

It is usually suggested that the accuracy of (2) is found by means of high order derivatives in Taylor's expansion of f [1]. Another way is to use simulation methods giving changes of $X_1, ..., X_n$ in (1).

According to modern metrology, (2) can be explained as the uncertainty with 100 % confidence level (the worst-case uncertainty) [2]. The worst-case uncertainty means that errors higher than found by (2) are absent. In practice, a real maximum possible absolute error of the indirect measurement can be a little higher than one found by (2). One reason of this fact has already been discussed (errors of Taylor's expansion). Other reasons are elevated values of the errors of the direct measurements with regard to $\Delta X_1, ..., \Delta X_n$. The worst-case method supposes that elevated errors of the indirect measurements with regard to (2) are negligible, for example, lower than five percent from (2).

The result found by (2) is usually much higher than the real values. The reason is that all the errors of the direct measurements are calculated independently. The exception to the rule is given in [2]. All the direct measurements are supposed to use an analog to digital converter (ADC) with the same values of the maximum offset error U_0, the maximum gain error U_G, and the maximum linearity error U_{inl}. The first two errors are supposed to have the same sign (positive or negative), while the linearity error can change the sign for different direct measurements. The uncertainty of the indirect measurements is found in accordance with [2], [3] for four cases: the standard uncertainty and the worst-case uncertainty (both absolute and relative). The absolute worst-case uncertainty (the maximum possible absolute error) of the indirect measurement with negligible quantization error is then [2]:

$$U(X) = U_0\left|\sum_{i=1}^{n} k_i\right| + U_G\left|\sum_{i=1}^{n} k_i X_i\right| + U_{inl}\sum_{i=1}^{n}\left|k_i\right|. \quad (3)$$

Coefficients k_i can have different signs. For example, the indirect measurement $X = X_1 - X_2$ gives $k_1 = -k_2 = 1$. Therefore, (3) can show a lower value than the one found by (2) for the devices with the same errors of each direct measurement.

Direct measurements of $X_1 ... X_n$ are often fulfilled by data acquisition devices. Three most popular sampling architectures are: multiplexed structures, simultaneous sample and hold structures and multi-ADC structures [4]. Corresponding structures are shown in Figure 1. For the same speed of the used ADC, the multi-ADC architecture gives a higher scan rate per channel and therefore is preferable according to the recommendations in [4]. The following abbreviations are used in Figure 1:

Mux – multiplexer,
Amp – instrumentation amplifier,
ADC - analog to digital converter,
SSH – simultaneous sample and hold.

The advantage of the multiplexed architecture in comparison with the multiplexed structure in terms of uncertainty is shown in Section 4.

Besides the values of direct measurements, the values reproduced by material measures (standard electric resistors, standard signal generators etc.) are also included in (1).

For several direct measurements, the following variants are discussed in terms of uncertainty: the application of the same ADC at the same range, or different ranges; the choice of the same ADC, or different ADCs at the same range.

For simplicity, several phenomena (dynamic errors, e.g.) ignored in [2] are not considered here either. Additionally, the quantization error and noise are supposed to be negligible. All these approximations do not usually influence the main conclusions given in the paper.

2. ACCURACY SPECIFICATIONS FOR DIFFERENT TYPES OF MEASURING DEVICES

Accuracy specifications of digital instruments (DI) are usually presented by the total (maximum) absolute error. As a first approximation, the maximum possible absolute measurement error for each variable X_i found by a digital instrument is

$$\left|\Delta X_i\right|_{DI} = (a + b\left|X_i\right|), \quad (4)$$

where a is a positive number of the same unit as X_i and b is a positive non-dimensional number.

The maximum absolute error of a digital instrument as a function of an input signal X is shown in Figure 2. Two values of the input signal (X_1 and X_2) are considered. The corresponding absolute errors (Δ_1 and Δ_2) are shown in Figure 2.

Sometimes a is given in % of the full scale ($X_{FS.i}$) and b is given in % of a reading.

Accuracy specifications of the ADC and data acquisition (DA) devices are usually presented by the maximum offset, gain and linearity errors. The quantization error and noise (the random error) are usually included in a and b for digital instruments but can be specified separately for other devices. For simplicity, we will not consider them in this paper. Then the maximum absolute error of the DA is

$$\left|\Delta X_i\right|_{DA} = U_0 + U_G\left|X_i\right| + U_{inl}. \quad (5)$$

For example, (5) is used for finding the maximum absolute error of a single measurement in [5]. The maximum absolute error of the ADC as a function of input signal X is shown in Figure 3. Positive errors are considered only as an example. Two values of the input signal (X_1 and X_2) are used. The linearity error is supposed to be zero at the ends of the range (X=0 and X=X_{FS}) but is equal to the maximum value U_{inl} with any sign at any other points. The linearity error is negative near X_1 and is positive near X_2 (the worst-case method). Let us

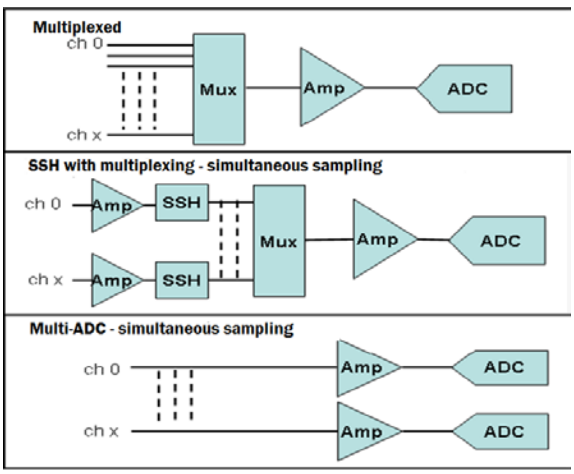

Figure 1. Simultaneous sampling architecture – simultaneous sample and hold (ssh).

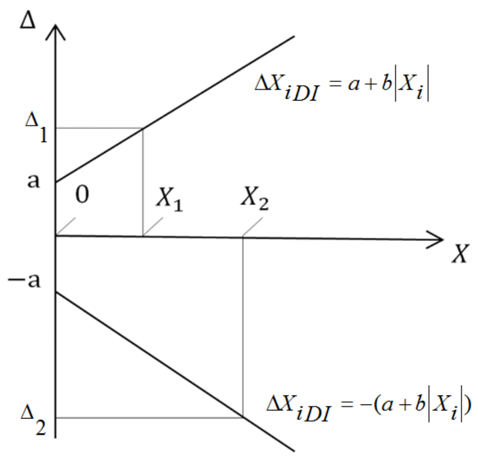

Figure 2. Maximum absolute measurement error vs. input signal for digital instruments.

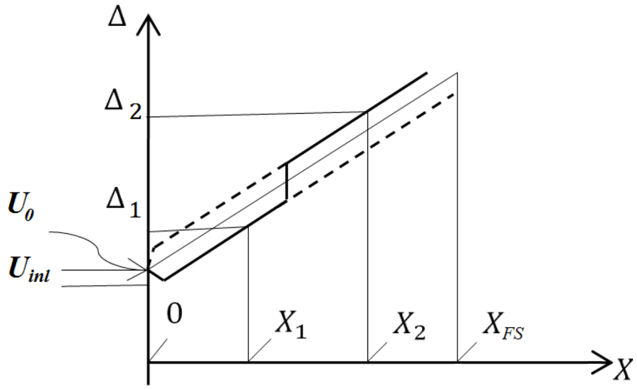

Figure 3. Maximum absolute error vs. input signal for ADC.

suppose that Δ_1 and Δ_2 errors are the same for a digital instrument (Figure 2) and an ADC (Figure 3). Then the difference between specifications of the two devices mentioned is not important for the evaluation of the direct measurement uncertainty. The situation changes dramatically if we investigate indirect measurements. Let us consider the simplest indirect measurement $X = X_2 - X_1$. The uncertainty of the measurement for the digital instrument (Figure 2) is $\left|\Delta X\right|_{DI} = |\Delta_1| + |\Delta_2|$. The uncertainty of the measurement for the ADC (Figure 3) is $\left|\Delta X\right|_{DA} = |\Delta_2 - \Delta_1|$. It is clear from Figures 2 and 3 that the second value can be much lower. Because of this result it is necessary to find ways to transform the specifications of digital instruments into the form specified for ADCs. If (4) and (5) show the same results, then a and b can be found with given U_0, U_G, U_{inl} as

$$a = U_0 + U_{inl}, \quad (6)$$

$$b = U_G. \quad (7)$$

Let a and b be specified. If we consider X=0, then $U_0 = a$ .It is possible to find the following inequalities (see Figure 2):

$$U_G \le b + 2a / \left|X_{FS}\right|, V_{inl} \le 2(a + b\left|X\right|), \quad (8)$$

where X_{FS} is the full scale of the DI. The evaluation of the linearity error by (8) is usually much higher than the true value. Therefore (8) does not give any advantages for calculation of the uncertainty of indirect measurements in comparison with initial equation (4).

Fortunately, some digital instruments have the additional specification of the linearity error. For example, the linearity error is specified in [6] as

$$U_{inl} = A_L X_{FS} + B_L \left|X\right|, \quad (9)$$

where A_L and B_L are positive non-dimensional numbers.

Now the maximum possible absolute error of the digital instrument can be written practically in the same way as it was given for ADCs or data acquisition devices:

$$\left|\Delta X_i\right|_{DI} = a + \left(\frac{2a}{\left|X_{FS}\right|} + b\right)\left|X_i\right| + U_{inl}. \quad (10)$$

In accordance with Figure 3, U_{inl} is supposed to be equal to zero at the ends of the scale, but can produce both positive and negative errors at any other points. The signs of errors produced by a and b are constant for the given indirect measurement. The linearity error found by (9) is approximately 10 times less than one found by (8) for the instrument 3401A [6].

If the linearity error for the DI is not specified, then it can be found approximately from the experiment [7].

The accuracy of material measures (standard electric resistors, standard signal generators etc.) can be specified by both (4) and (5). For example, the accuracy of voltage calibrators is usually specified as (4), while data acquisition devices with analog output [5] have specifications (5). It means that all the following theory can be used both for results of the direct measurements and quantities reproduced by material measures.

3. GENERAL FORMULAE FOR INDIRECT MEASUREMENT UNCERTAINTY

Direct measurements n used for the indirect measurement can be divided into three parts.

Then the absolute worst-case uncertainties of the indirect measurement

$$U(X) = U_0\left|\sum_{i=1}^{n_1} k_i\right| + U_G\left|\sum_{i=1}^{n_1} k_i\left|X_i\right|\right| + U_{inl}\sum_{i=1}^{n_1}\left|k_i\right| + \sum_{i=n_1+1}^{n_1+n_2} U_{0.i}\left|k_i\right| + \sum_{i=n_1+1}^{n_1+n_2} U_{G.i}\left|k_i X_i\right| + \sum_{i=n_1+1}^{n_1+n_2} U_{inl.i}\left|k_i\right| + \sum_{i=n_1+n_2+1}^{n}(a_i + b_i\left|X_i\right|)\left|k_i\right|, \quad (11)$$

The simplest indirect measurement is described by the function $X = X_2 - X_1$, where $k_1 = -k_2 = -1$. The absolute worst-case uncertainty of the indirect measurement when one device is used in the same range $(n_1 = n = 2)$ and $X_2 \ge X_1 \ge 0$ in accordance with (11) is

$$U_I(X) = U_G(X_2 - X_1) + 2U_{inl}. \quad (12)$$

The absolute worst-case uncertainty of the indirect measurement when two devices of the same type are applied in the same range ($n_2 = n = 2$, $U_{0.1} = U_{0.2} = U_0$, $U_{G.1} = U_{G.2} = U_G$, $U_{inl.1} = U_{inl.2} = U_{inl}$) in accordance with (11) is

$$U_{II}(X) = 2U_0 + U_G(X_1 + X_2) + 2U_{inl}. \quad (13)$$

The maximum difference of the results found by (12) and (13) is at $X_1 \approx X_2 \approx X_{FS}$:

$$\frac{U_{II}(X)}{U_I(X)} = 1 + \frac{U_0 + U_G X_{FS}}{U_{inl}}. \quad (14)$$

Let us use (14) to compare the uncertainties of the multiplexed and multi-ADC structures (discussed in Section 1) for the implementation of the function $X = X_2 - X_1$. Model PCI-6250, used in the multiplexed structure, includes the ADC with the following parameters [5]: $X_{FS} = 10$ V, $U_0 = 2 \cdot 10^{-4}$ V, $U_G = 6 \cdot 10^{-5}$, $U_{inl} = 6 \cdot 10^{-4}$ V. If we use the same ADC in the multi-ADC structure, then, according to (14), the worst-case uncertainty is 2.3 times more. It means that recommendations [4] can be not true for the uncertainty of the indirect measurement.

If $X_1 << X_2$, then two ranges can be used for each direct measurement. In this case, the absolute worst-case uncertainty of the indirect measurement $X = X_2 - X_1$ is

$$U_{III}(X) = U_{0.1} + U_{0.2} + U_{G.1} X_1 + U_{G.2} X_2 + U_{inl.1} + U_{inl.2}. \quad (15)$$

The application of one range for both direct measurements will be better if (12) gives a lower result in comparison with (15).

Let us consider the example PCI-6250 [5] used for the indirect measurement $X = X_2 - X_1$ with $X_1 \approx 5$ V and $X_2 \approx 10$ V. The corresponding specifications for $X_1 \approx X_{FS.1} = 5$ V are $U_{0.1} = 1 \cdot 10^{-4}$ V, $U_{G.1} = 7 \cdot 10^{-5}$ V and $U_{inl.1} = 3 \cdot 10^{-4}$ V. The specifications for $X_2 \approx X_{FS.1} = 10$ V were given before. Using the PCI-6250 at 10 V range only, we get from (12) that $U_I(X) = 1.5$ mV. If we use two ranges (5 V for X_1 and 10 V for X_2), the result is $U_{III}(X) = 2.15$ V. It means that the application of only one range gives 1.4 times better result, and the well-known recommendation to use the lowest range (5 V in this example) is valid only for the uncertainty of the direct measurements but can be incorrect for the uncertainty of the indirect measurements.

4. CONCLUSIONS

There are two main types of the specifications for measuring devices: the maximum possible total error (4) and the maximum offset, the gain, and the linearity errors (5). The inequalities (8) are offered to find the maximum gain and linearity errors approximately. The result of the evaluation of the linearity error by (8) can be much higher than the true value. If the linearity error is specified besides (4), then the gain error can be found from (4) by (8).

General formulae for the absolute (11) and the relative (12) worst-case uncertainties of the indirect measurement are found as functions of three parts of variables. These parts include application for the indirect measurement of one or several devices in one or several ranges with the specifications of the total error or the maximum offset, gain, and linearity errors separately. Only the first parts of (11) can give a lower value of the uncertainty for some types of the indirect measurements in comparison with the approach (2). The formulae published before [1], [2] are special cases of (11). The second part of (11) was not discussed in [1], [2]. Formula (11) can also be used to take into account application of material measures (standard electric resistors, standard signal generators etc.) for the indirect measurements.

Applications of the offered approaches are given in Section 4. The advantage of the multiplexed structure vs. the multi-ADC structure from the uncertainty point of view is shown though the multi-ADC structure is better from the dynamics point of view [4]. Conditions for the choice of one range of a device instead of two ranges for two direct measurements are found from the uncertainty point of view.

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