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NONLINEAR EVOLUTION OF THE ACOUSTIC WAVE IN A SEMI-IDEAL GAS

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The method of deriving the evolution equation, based on projecting is applied for the evaluation of the sound velocity and the parameters of nonlinearity for real gases and liquids. The method yields in a coupled system of interacting modes: leftwards and rightwards acoustic and heat modes in the one-dimensional flow problem. The general form of the caloric equation of state allows to get the coefficients of nonlinear equations in the general form. As an example, the sound velocity and the nonlinear parameter B/A for a variety of semi-ideal gases were calculated and the results compared with experimental data.

Notations

x	– space coordinate [m],
t	– time [s],
ρ	– density [kg/m^3],
p	– pressure [N/m^2],
v	– velocity [m/s],
T	– absolute temperature [K],
e	– internal energy per unit mass [J/kg],
$\rho_0, p_0, v_0, e_0, T_0$	– unperturbed values,
$\acute{\rho}, \acute{p}, \acute{v}, \acute{e}, \acute{T}$	– perturbations,
$x_*, t_*, \rho_*, p_*, v_*$	– dimensionless variables,
λ	– characteristic scale of disturbance,
α	– coefficient responsible for amplitude of acoustic wave,
$D_1..D_5$	– dimensionless coefficients in evolution equations,
$E_1..E_5$	– coefficients in caloric equation of state,
c	– linear sound velocity [m/s],
B/A, C/A	– acoustic parameters of nonlinearity,
$c_{v(p)}$	– heat capacity under constant (volume) pressure per unit mass [J/kg·K],
R	– the universal gas constant [J/mol·K],
μ	– molar mass [kg/mol],
f_{osc}	– number of oscillation degrees of freedom of a gas molecule,
θ_i	– characteristic temperature of oscillation [K],
γ	– adiabatic gas constant (c_p/c_v).

1. Introduction

The projecting method serves for deriving nonlinear evolution equations for the interacting modes. Modes as basic types of motion of the concrete problem, to be defined by this method as eigenvectors of the corresponding linear problem. The main physical idea hence is to fix relations between the perturbations of wave variables. For linear flows, the modes are independent and may be extracted from the overall perturbation by operators projecting to the eigenspaces. The operators may be constructed by means of the eigenvectors and are applied when either linear or nonlinear dynamics is considered. Acting projectors on the full nonlinear system of gas dynamic equations leads to coupled nonlinear evolution equations which may be related with known evolution equations.

Examples of acoustic-gravity waves in the atmosphere and electromagnetic waves are studied in [1]. Nonlinear evolution equations for the bubbly liquid dynamics are derived in [2], and the acoustics in the exponentially stratified atmosphere is investigated in [3]. In the present paper, we apply the projecting technique for deriving the nonlinear evolution equation for one progressive acoustic mode. The caloric and thermal equations of state are incorporated in their general forms which allow to treat an arbitrary fluid. We, however, go to the representation of the equations as multivariable Taylor series: these formulas are convenient for practical purposes. Therefore the sound velocity, the nonlinear parameter B/A and some nonlinear parameters of higher order (C/A, ...) depend on the coefficients of the Taylor series of the equations of state. A similar approach is developed in [4 – 6] on a different theoretical basis and applications. Beginning from the results in [4], the theory allows to study the direct links of the acoustic parameters with the thermodynamic ones and in turn the modeling of the intermolecular forces. Let also mention the developing techniques and quality of the measurements of the nonlinear constants (see e.g. [7, 8]). The results give hope of a progress in this difficult problem of the condensed matter physics.

Though a wide variety of gases and fluids may be treated in this way, we start from the examples of semi-ideal gases which account for oscillatory degrees of freedom. The motivation is simplicity, that help to explain the main ideas, as well as the existence of explicit formulae for the state equations. One arrives at the sound velocity and the nonlinear parameter B/A in an explicit form and it is easy to calculate both these values over a wide range of equilibrium states of a gas. The important thing is the existence of available experimental data with which the results of calculations could be compared.

2. Basic equations

Let us repeat briefly the ideas and results of the projecting method. We consider an one-dimensional fluid flow without thermal conduction and internal friction. A basic system thus represents conservation laws of momentum, energy and mass:

$$\begin{aligned} \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} &= 0, \\ \rho\frac{\partial e}{\partial t} + \rho v\frac{\partial e}{\partial x} + p\frac{\partial v}{\partial x} &= 0, \\ \frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} &= 0. \end{aligned} \tag{2.1}$$

We should complete (2.1) with the caloric equation of state $e(p,\rho)$. Let $\acute{e}$ has the form of the Taylor series of two variables:

$$\begin{aligned} \rho_0\acute{e} = E_1\acute{p} + \frac{E_2 p_0}{\rho_0}\acute{\rho} + \frac{E_3}{p_0}\acute{p}^2 + \frac{E_4 p_0}{\rho_0^2}\acute{\rho}^2 + \frac{E_5}{\rho_0}\acute{p}\acute{\rho} \\ + \frac{E_6}{p_0\rho_0}\acute{p}^2\acute{\rho} + \frac{E_7}{\rho_0^2}\acute{p}\acute{\rho}^2 + \frac{E_8}{p_0^2}\acute{p}^3 + \frac{E_9}{\rho_0^3}p_0\acute{\rho}^3 + \ldots, \end{aligned} \tag{2.2}$$

E_1, ..., E_9 are dimensionless coefficients. The system (2.1), (2.2) is valid for a wide variety of fluids and we are not restricted to any special cases of internal energy on pressure and density since we use the caloric $e = e(p,\rho)$ equation of state in a general form.

The equivalent system $(v_*, \acute{p}_*, \acute{\rho}_*, \acute{r}_*, \acute{t}_*)$ in dimensionless variables:

$$v = \alpha c v_*, \qquad \acute{p} = \alpha c^2\rho_0\acute{p}_*, \qquad \acute{\rho} = \alpha\rho_0\acute{\rho}_*, \quad x = \lambda x_*, \qquad t = t_*\lambda/c, \tag{2.3}$$

where c is the linear sound velocity, as follows from (2.1), (2.2)

$$c = \sqrt{\frac{p_0(1-E_2)}{\rho_0 E_1}},$$

λ means the characteristic scale of disturbance along x and α is the coefficient responsible to the amplitude of the acoustic wave, may be written in the matrix form (asterisks for dimensionless variables will be later omitted):

$$\frac{\partial}{\partial t}\Psi + L\Psi = \widetilde{\Psi} + \widetilde{\widetilde{\Psi}} + O(\alpha^3), \qquad \Psi = \begin{pmatrix} v \\ \acute{p} \\ \acute{\rho} \end{pmatrix}, \tag{2.4}$$

where

$$L = \begin{pmatrix} 0 & \frac{\partial}{\partial x} & 0 \\ \frac{\partial}{\partial x} & 0 & 0 \\ \frac{\partial}{\partial x} & 0 & 0 \end{pmatrix}, \qquad \widetilde{\Psi} = \alpha \begin{pmatrix} -v\frac{\partial v}{\partial x} + \acute{\rho}\frac{\partial \acute{p}}{\partial x} \\ -v\frac{\partial \acute{p}}{\partial x} + \frac{\partial v}{\partial x}\left(\acute{p}D_1 + \acute{\rho}D_2\right) \\ -v\frac{\partial \acute{\rho}}{\partial x} - \acute{\rho}\frac{\partial v}{\partial x} \end{pmatrix},$$

$$\widetilde{\widetilde{\Psi}} = \alpha^2 \begin{pmatrix} -\acute{\rho}^2\frac{\partial \acute{p}}{\partial x} \\ \frac{\partial v}{\partial x}\left(\acute{\rho}^2 D_3 + \acute{p}^2 D_4 + \acute{\rho}\acute{p}D_5\right) \\ 0 \end{pmatrix}, \tag{2.5}$$

where the symbols $D_1..D_5$ denote dimensionless coefficients in following forms:

$$\begin{aligned}
D_1 &= \frac{1}{E_1}\left(-1+2\frac{1-E_2}{E_1}E_3+E_5\right),\\
D_2 &= \frac{1}{1-E_2}\left(1+E_2+2E_4+\frac{1-E_2}{E_1}E_5\right),\\
D_3 &= \frac{1}{1-E_2}\left(1-3E_9-2E_4-\frac{E_7(1-E_2)}{E_1}+\frac{E_5}{E_1}(1+E_2+2E_4)+\frac{E_5^2(1-E_2)}{E_1^2}\right),\\
D_4 &= \frac{(1-E_2)}{E_1^3}\left(\frac{4E_3^2(1-E_2)}{E_1}-E_6E_1+2E_3E_5-3E_8(1-E_2)-2E_3\right),\\
D_5 &= \frac{1}{E_1^2}\left(\frac{4E_5E_3(1-E_2)}{E_1}+2E_3(1+E_2)-2E_6(1-E_2)-2E_1E_7-E_1E_5\right.\\
&\qquad\left.+E_5^2+4E_3E_4-E_1-E_5\right).
\end{aligned}$$

The second-order nonlinearity column $\widetilde{\Psi}$ will contribute to the B/A parameter, and the third-order one $\widetilde{\widetilde{\Psi}}$ will yield in C/A.

3. Projecting technique

For a linear flow, we may find a solution as the sum of plane waves, every plane wave being a solution of the linearized system (2.1), (2.2). Let us introduce plane waves $\sim \exp(i\omega t - ikx)$ with amplitudes V_k, P_k and R_k. The eigenvalues of the corresponding system of equations for Fourier transformed components in the linear problem, are determined from the equation:

$$\begin{vmatrix} i\omega & -ik & 0\\ -ik & i\omega & 0\\ -ik & 0 & i\omega \end{vmatrix} = 0.$$

The solution of this equation serve as dispersion relations for the right- and left-progressive and stationary components. Eigenvectors in the k-presentation look as:

$$\Psi_{1,2} = \begin{pmatrix} \pm 1\\ 1\\ 1 \end{pmatrix} R_{k_{1,2}}, \qquad \Psi_3 = \begin{pmatrix} 0\\ 0\\ 1 \end{pmatrix} R_{k_3}.$$

Therefore, returning to the (x,t) representation connections for the specific variables appear and we write it down as:

$$v_{1,2} = \pm\rho_{1,2}, \qquad p_{1,2} = \rho_{1,2}, \qquad v_3 = 0, \qquad p_3 = 0. \tag{3.6}$$

In this way we defined the components of the right, left and stationary modes of a wave in the linear model. From these relations (3.6) the projectors follow immediately:

$$P_1 = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0\\ 1 & 1 & 0\\ 1 & 1 & 0 \end{pmatrix}, \quad P_2 = \frac{1}{2}\begin{pmatrix} 1 & -1 & 0\\ -1 & 1 & 0\\ -1 & 1 & 0 \end{pmatrix}, \quad P_3 = \begin{pmatrix} 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & -1 & 1 \end{pmatrix}. \tag{3.7}$$

The matrices (3.7) have general properties of orthogonal projectors:

$$P_1 + P_2 + P_3 = \widetilde{I},$$

$$P_1P_2 = P_2P_3 = P_3P_1 = \widetilde{0}, \qquad P_1P_1 = P_1, \quad \text{etc.},$$

where the $\tilde{I}$ and $\tilde{0}$ projectors separate the chosen mode from the overall field in a unique way: $\Psi_1: \ P_1\Psi = \Psi_1$, $P_2\Psi = \Psi_2$, $P_3\Psi = \Psi_3$. Projectors P_1, P_2, P_3 do commute both with L and $\partial/\partial t$, that allows to generate the equations of modes interaction acting by P_i on the basic system (2.4).

4. Nonlinear coupled evolution equations

In the nonlinear problem we preserve the same notations for the modes. We consider (now approximately defined) rightwards, leftwards and stationary modes of the nonlinear problem with the eigenvectors Ψ_1, Ψ_2 and Ψ_3 as it was accepted in the linear model and assume that the relation equations (3.6) also holds. Thus, the defined modes are strictly directed and stationary in the linear limit and form a system of coupled nonlinear equations when the projectors act on both sides of (2.4). Marking these modes by indices 1, 2, 3 correspondingly for the quasi-rightwards, leftwards and stationary one, we get finally the system:

$$P_n\frac{\partial}{\partial t}\Psi + P_nL\Psi - P_n\widetilde{\Psi} - P_n\widetilde{\widetilde{\Psi}} + O(\alpha^3) = 0, \tag{4.8}$$

or another one (for density only):

$$\begin{aligned}\frac{\partial\rho_n}{\partial t} &+ c_n\frac{\partial\rho_n}{\partial x} + \frac{\alpha}{2}\sum_{i,m=1}^{3} Y^n_{im}\rho_i\frac{\partial\rho_m}{\partial x} \\ &+ \frac{\alpha^2}{2}\sum_{i,m=1}^{3} {T^I}^n_{im}\rho_i\rho_m\frac{\partial\rho_1}{\partial x} + \frac{\alpha^2}{2}\sum_{i,m=1}^{3} {T^{II}}^n_{im}\rho_i\rho_m\frac{\partial\rho_2}{\partial x} + O(\alpha^3) = 0,\end{aligned} \tag{4.9}$$

where

$$c_n = \left\{\begin{array}{rl} 1 & \text{for } \ n = 1 \\ -1 & \text{for } \ n = 2 \\ 0 & \text{for } \ n = 3 \end{array}\right\}$$

and the matrices of constants the Y, T^I and T^{II} for the first mode are:

$$\left|\begin{array}{cccc} Y^1_{i,m} & m=1 & m=2 & m=3 \\ i=1 & -D_1-D_2+1 & D_1+D_2-1 & 0 \\ i=2 & -D_1-D_2-3 & D_1+D_2-1 & 0 \\ i=3 & -D_2-1 & D_2-1 & 0 \end{array}\right|$$

$$\left|\begin{array}{cccc} {T^I}^1_{i,m} & m=1 & m=2 & m=3 \\ i=1 & -D_3-D_4-D_5+1 & -D_3-D_4-D_5+1 & -D_3+1 \\ i=2 & -D_3-D_4-D_5+1 & -D_3-D_4-D_5+1 & -D_3+1 \\ i=3 & -D_3-D_5+1 & -D_3-D_5+1 & -D_3+1 \end{array}\right|$$

$$\left|\begin{array}{cccc} {T^{II}}^1_{i,m} & m=1 & m=2 & m=3 \\ i=1 & D_3+D_4+D_5+1 & D_3+D_4+D_5+1 & D_3+1 \\ i=2 & D_3+D_4+D_5+1 & D_3+D_4+D_5+1 & D_3+1 \\ i=3 & D_3+D_5+1 & D_3+D_5+1 & D_3+1 \end{array}\right|$$

The other matrices look similarly. There are also equivalent equations for pressure and velocity.

The system (4.9) allows to calculate all possible interactions of modes. One may specify a class of initial (boundary) conditions, define the dominant modes and later solve the system approximately. Here we are interested in the evolution equation for one progressive (say, rightwards) mode. Physically it means, that this mode is dominant initially: $\rho_1 \gg \rho_2$, $\rho_1 \gg \rho_3$, and we account self-interaction only in the evolution equation for this mode:

$$\frac{\partial\rho_1}{\partial t} + c_1\frac{\partial\rho_1}{\partial x} + \varepsilon\rho_1\frac{\partial\rho_1}{\partial x} + \delta\rho_1^2\frac{\partial\rho_1}{\partial x} = 0, \tag{4.10}$$

where $\varepsilon = \frac{\alpha}{2}(-D_1 - D_2 + 1)$, and $\delta = \frac{\alpha^2}{2}(-D_3 - D_4 - D_5 + 1)$. The parameters B/A, C/A are well known nonlinear parameters of the nonlinear acoustics equation:

$$p = p_0 + A\frac{\rho-\rho_0}{\rho_0} + \frac{B}{2}\frac{(\rho-\rho_0)^2}{\rho_0^2} + \frac{C}{6}\frac{(\rho-\rho_0)^3}{\rho_0^3} + \left(\frac{\partial p}{\partial s}\right)\bigg|_{\rho,s=s_0}(s-s_0) + \ldots$$

where s is entropy. For our accounting the last expression is neglected — we assume an adiabatic process. The coefficients A, B, C can be expressed as:

$$\begin{aligned} A &= \frac{1-E_2}{E_1}p_0, \qquad B = -(D_1+D_2+1)\frac{1-E_2}{E_1}p_0\,, \\ C &= ((D_1+D_2+1)(D_1+2) - 2(D_3+D_4+D_5))\frac{1-E_2}{E_1}p_0\,. \end{aligned}$$

5. Semi-ideal gases: theory and experiment

We stress once more that the system (2.1) + (2.2) and the subsequent formula for the operators are suitable for gases and liquids treated by the general caloric equation of state. The case of ideal gas is considered with coefficients:

$$E_1 = E_4 = E_7 = \frac{1}{\gamma-1}\,, \qquad E_2 = E_5 = E_9 = -\frac{1}{\gamma-1}\,, \qquad E_3 = E_6 = E_8 = 0.$$

and: $B/A = \gamma - 1$, $C/A = (\gamma-1)(\gamma-2)$. To find some corresponding coefficients for the semi-ideal gas, we have to accept the energy of oscillation in the molecules([1]) [9]:

$$c_{v,\mathrm{sid}} = c_{v,\mathrm{id}} + c_{\mathrm{osc}} + \Delta c_{\mathrm{rot}} + \Delta c_{\mathrm{el}}\,. \tag{5.11}$$

We use the Einstein–Planck formula for the vibrational specific heat:

$$c_{\mathrm{osc}} = R\sum_{i=1}^{f_{\mathrm{osc}}}\left(\frac{\theta_i}{T}\right)^2\frac{e^{\theta_i/T}}{\left(e^{\theta_i/T}-1\right)^2}. \tag{5.12}$$

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(1) We neglect electron excitations energy because it concerns very high temperatures, and we omit the energy of rotation — it is significant for very low temperatures and light gases only.

Using the above formula, we get the equation for the internal energy for semi-ideal gases [9]:

$$e = e_{\rm id} + \frac{R}{\mu}\sum_{i=1}^{f_{\rm osc}} \frac{\theta_i}{e^{\theta_i/T}-1}\,, \qquad e_{\rm id} = \frac{p}{\rho}\,\frac{1}{(\gamma-1)}\,, \tag{5.13}$$

where c, e, μ, $f_{\rm osc}$ mean respectively: molar heat, internal energy per unit mass, molar mass and number of oscillation degrees of freedom of a gas molecule. θ_i it is characteristic temperature of oscillation (T — absolute temperature) and γ — adiabatic gas constant (c_p/c_v, in classical theory we take $\gamma = 5/3$ for a monoatomic ideal gas, 1.4 for a diatomic one and 4/3 for other gases).

Below, we present a comparison of the values founded for a few gases treated first as ideal ones and then as semi-ideal ones. To calculate the sound velocity c and B/A values we use the following formulas:

$$c = \sqrt{RT_0\frac{(1-E_2)}{E_1}}, \qquad \frac{B}{A} = -D_1 - D_2 - 1, \tag{5.14}$$

where the coefficients $E_1..E_5$, which have been used (see (2.2)), have the general forms:

$$\begin{gathered} E_1 = \left.\frac{\partial e}{\partial p}\right|_{p_0,\rho_0}\rho_0, \qquad E_2 = \left.\frac{\partial e}{\partial \rho}\right|_{p_0,\rho_0}\frac{\rho_0^2}{p_0}, \qquad E_3 = \frac{1}{2}\left.\frac{\partial^2 e}{\partial p^2}\right|_{p_0,\rho_0}\rho_0 p_0, \\ E_4 = \frac{1}{2}\left.\frac{\partial^2 e}{\partial \rho^2}\right|_{p_0,\rho_0}\frac{\rho_0^3}{p_0}, \qquad E_5 = \left.\frac{\partial^2 e}{\partial \rho p}\right|_{p_0\rho_0}\rho_0^2 \end{gathered} \tag{5.15}$$

and for the concrete semi-ideal gas model with account of oscillation degrees of freedom (the model described above):

$$\begin{aligned} E_1 &= -E_2 = \frac{1}{\gamma-1} + \sum_{i=1}^{f_{\rm osc}}\left(\frac{\theta_i}{T_0}\right)^2 e^{\theta_i/T_0}\left(e^{\theta_i/T_0}-1\right)^{-2}, \\ E_3 &= \frac{1}{2}\sum_i^{f_{\rm osc}}\left(\frac{\theta_i}{T_0}\right)^2 e^{\theta_i/T_0}\left(e^{\theta_i/T_0}-1\right)^{-2}\left(-2-\frac{\theta_i}{T_0}+2\frac{\theta_i}{T_0}e^{\theta_i/T_0}\left(e^{\theta_i/T_0}-1\right)^{-1}\right), \\ E_4 &= \frac{1}{\gamma-1} - \frac{1}{2}\sum_i^{f_{\rm osc}}\left(\frac{\theta_i}{T_0}\right)^3 e^{\theta_i/T_0}\left(e^{\theta_i/T_0}-1\right)^{-2}\left(1-2e^{\theta_i/T_0}\left(e^{\theta_i/T_0}-1\right)^{-1}\right), \\ E_5 &= -\frac{1}{\gamma-1} + \sum_i^{f_{\rm osc}}\left(\frac{\theta_i}{T_0}\right)^2 e^{\theta_i/T_0}\left(e^{\theta_i/T_0}-1\right)^{-2}\left(1+\frac{\theta_i}{T_0}+\frac{\theta_i}{T_0}\left(e^{\theta_i/T_0}-1\right)^{-1}\right). \end{aligned}$$

For calculating C/A we need the next coefficients: $E_6, ..., E_9$, which are higher order derrivatives of e.

The results of calculations are presented in Table 1.

Now, we can notice that for any monoatomic gases (for example He) we have the ideal gas model without oscillations, and for the diatomic ones (N_2, CO) there is a very small difference in the sound velocities (about 10^{-2} m/s). Next, for some poliatomic gases (CO_2, CH_4) the difference is noticeable, especially for CO_2, even for the low temperatures.

Table 1. (2)

Gas	Model of ideal gas		Model of semi-ideal gas		Experimental data	
	B/A	c [m/s]	B/A	c [m/s]	B/A^a	c [m/s]^b
He	0.67	972.9	0.67	972.9	0.66	971^c
N_2	0.40	336.9	0.40	336.9	0.40	334.0
CO	0.40	337.0	0.40	337.0	0.40	336^d (338)
CO_2	0.33	262.2	0.24	255.0	0.31	256.7
CH_4	0.33	434.7	0.29	431.3	0.30	430

[a] All values are taken as $\gamma - 1$ from [9].
[b] All values are taken from [10].
[c] The experimental value is taken from [11].
[d] The first value is taken from [9].

It is necessary to add that the experimental data are taken from various sources, so we are sure of the temperature measurements only (273 K), but data on pressure are often not available and we often do not know the other measurement parameters. (For example: c value in the case of the gas CO.)

For an ideal gas $B/A \equiv \gamma - 1$ [12], but it must be stressed that for a semi-ideal gas and real gases c_v is a function of temperature and a new γ' has a new thermodynamic sense. The experimental data of γ for monoatomic gases are almost the same as the theoretical values, but for poliatomic gases the experimental values are lower than theoretical ones, which results from the classical approach to the ideal gas [13]. Some experimental data of γ, taken from other sources, for example [14], would be more close to our theoretical values:

$$B/A_{CO_2} = 0.28, \qquad B/A_{CH_4} = 0.26,$$

but they describe some gases under somewhat different measurement conditions. However, finding various experimental data, we can notice the temperature and pressure sensibility of the γ parameter.

Below we present also diagrams of the temperature dependence of c and B/A for some gases:

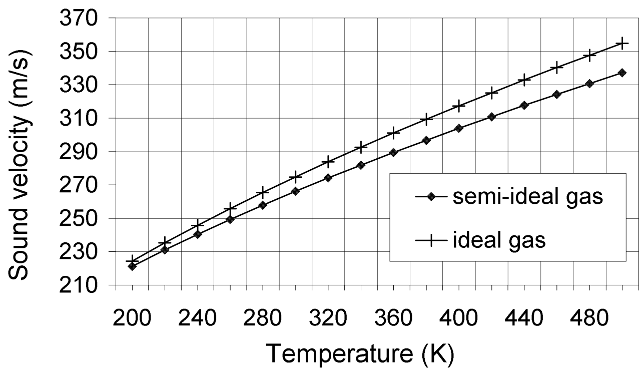

Fig. 1. Comparison of theoretical values of sound velocities for CO_2 gas.

(2) All values in Table 1 are obtained for $T = 273.15$ K.

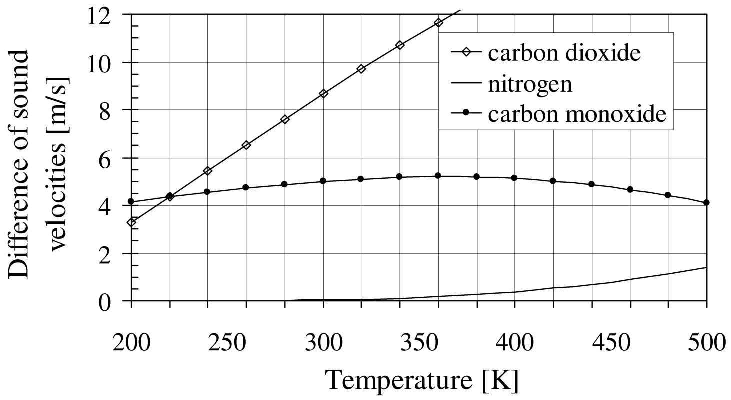

Fig. 2. Difference of theoretical sound velocities: $c_{\mathrm{id}} - c_{\mathrm{sid}}$ for N_2, CO and CO_2 gases.

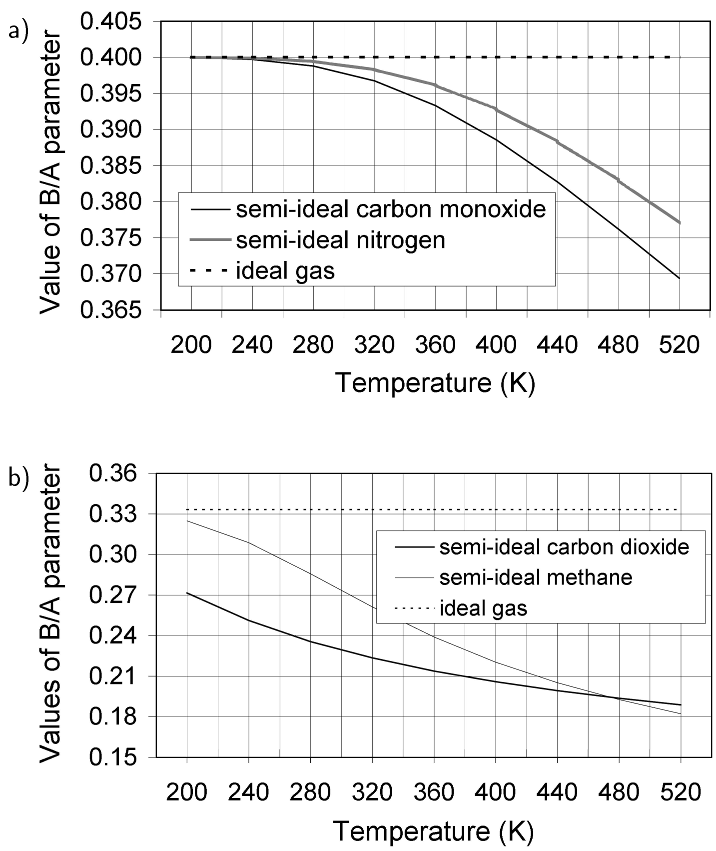

Fig. 3. Temperature dependence of theoretical values of B/A for: a) diatomic gases CO and N_2 and b) polyatomic ones: CO_2 and CH_4.

6. Conclusions

A comparison of the theoretical values of the sound velocity and the nonlinear parameter B/A for an ideal and semi-ideal gas demonstrate that both of the approaches give different results for the considered gases. For some polyatomic gases (CO_2, CH_4) the theoretical values of c at 0°C, are in the semi-ideal model of gas closer to the experimental ones. In the case of the B/A parameter, we have a less clear situation, but one can notice that for the mentioned polyatomic gases the ideal gas model is valid.

The method presented in this paper was applied for deriving the evolution equation for one progressive acoustic mode only, though its application is considerably extensive. After adding some thermoconducting and viscous expressions to the basic system of equations, and adding a thermic equation of state, we could get some new projectors for a thermoviscous flow. Also the formula of the equation of state written in the general form allows to apply the method to different liquids.

References

[1] S.B. Leble, *Nonlinear waves in waveguides with stratification*, Springer-Verlag, Berlin 1990.

[2] A.A. Perelomova, *Projectors in nonlinear evolution problem: Acoustic solitons of bubbly liquid*, Applied Mathematics Letters, **13** (2000).

[3] A.A. Perelomova, *Nonlinear dynamics of vertically propagating acoustic waves in a stratified atmosphere*, Acta Acustica, **84**, 6, 1002–1006 (1998).

[4] B. Hartmann, Journal of the Acoustical Society of America, **65**, 6, 1392–1396 (1979).

[5] B. Sharma, Journal of the Acoustical Society of India, **17**, 3-4, 172–175 (1989).

[6] B. Sharma, Pramana–Journal of Physics, **37**, 6, 489–496 (1991).

[7] S. Takahashi, Japanese Journal of Applied Physics, **34**, 2790–2792 (1995).

[8] K. Zakhariasz, *The acoustic method for measurement of the nonlinearity parameter B/A of the liquid using parabolic model* [in Polish], Proceedings of the XV Symposium on Hydroacoustics, R. Salamon, L. Kilian [Eds.], Politechnika Gdańska, 1998.

[9] J. Szargut, *Thermodynamics* [in Polish], PWN, Warsaw 1976.

[10] A.P. Babichev, N.A. Babushkina, A.M. Bratkovski *et al.*, *Physical values* [in Russian], Energoatomizdat, Moscow 1991.

[11] S. Szczeniowski, *Experimental physics* [in Polish], PWN, Warsaw 1972.

[12] S. Makarov and M. Ochmann, *Nonlinear and thermoviscous phenomena in acoustics*, Part I. *Acustica*, Acta Acustica, **82**, 579–606 (1996).

[13] A.I. Anselm, *Foundations of statistical physics and thermodynamics* [in Russian], Nauka, Moscow 1973.

[14] *Handbook of fluid dynamics*, Mc Graw–Hill Book Company Inc., London 1961.