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REVERBERATION TIMES AND REVERBERATION LEVELS

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Reverberation times and reverberation levels are the statistical parameters of the stochastical sound field in a room. Their theoretically derived and practically measured properties are discussed. Reverberation times and reverberation levels are place dependent, they are also to a certain extent influenced by the shape of the hall and the distribution of the absorption in it. This influence is however limited as regards the reverberation times.

1. Introduction

On 29 October 1898 W. C. SABINE observed that the product of total absorption and the duration of residual sound was a constant. The well known SABINE reverberation formula had been discovered.

There have been times that the reverberation time (*RT*) was highly valued, even overvalued, as the determining factor in roomacoustics; there have also been times that it was undervalued as not being of real importance for the subjective quality of the acoustics of a room.

We now know that the value of the *RT* is important, even very important, but certainly not the only important factor.

The sound energy density at different places in a room — mostly refered to in the form of the sound level — in absolute values, as well as relative to the sound power of the source, can be considered even more determinant for the acoustics of a room.

Reverberation times and the sound energy densities are related to each other, be it in a complicated way. We must always study them in their mutual relationship.

2. The sound field in a room

All known deductions of reverberation time formulae are based on models, on certain conceptions of what happens in a room. Most models are raymodels, mostly ending up in EYRING type of formulae; also well known is the "eigen" modes model etc.

All these models try to approximate reality, none of them is convincing and all are based on assumptions that can not be completely verified.

A rigorously valid reverberation formula can be found in differential form, but does not give an as simple and straightforward solution as the SABINE'S formula. The SABINE formula as well as the EYRING formula prove to be special cases or solutions of this more general formula. One may however, using this differential form deduce under what conditions the SABINE formula may be valid, and what may be expected, in general, if other conditions prevail.

The sound energy at a certain point in a room consists of kinetic energy and potential energy

$$E = \frac{1}{2}\frac{P^2}{K} + \frac{1}{2}\varrho v^{-2} \tag{1}$$

after differentiation

$$+\frac{dE}{dt} = \varrho \vec{v}\,\frac{\partial \vec{v}}{\partial t} + \frac{1}{K}\,p\,\frac{\partial p}{\partial t}. \tag{2}$$

With the use of the equations of continuity and inertia one finds

$$-\frac{dE}{dt} = \vec{v}\operatorname{grad} p + p\operatorname{div}\vec{v} = \operatorname{div} p\vec{v}. \tag{3}$$

Using GAUSS's theorem one can now write

$$-\frac{dE}{dt} = \iint_S p\cdot\vec{v}\vec{n}\,ds. \tag{4}$$

This is, the total sound energy E in a certain volume will decay as the scalar product of the normal component of the particle velocity on the surface times the pressure integrated over the boundary of the volume.

One may readily assume that the on the sufraces mean incident intensities (over time and total boundaries taken) will be proportional to the sound energy in the volume. Thus, in the mean again, an exponential decay may be expected.

From the formula (4) one learns, that if there is an ideal diffuse, that is a completely homogeneous and isotropic sound field, that will stay during the whole decay process ideally diffuse, the SABINE formula can be deduced. This can however only be the case if everywhere and in every direction in a room, sound will be equally absorbed. This may be found if air absorption is the only sound absorbing machanism present in the room — not a very interesting case from the practical point of view.

Normally the sound absorption will take place at the room boundaries, and therefore an ideal diffuse sound field will no longer be possible, because of the finite sound velocity. It will take some time before the depletion of the sound field, taking place at the boundaries will have been equally divided over the whole room and will be felt in the incoming sound at the absorbing boundary. The sound energy falling on the boundary will, in the mean being older and thus stronger than the mean, take over the whole hall of the in the hall present sound energy. Taking this effect into account results in the EYRING formula if one assumes, according to EYRING that one may substitute the "mean free path" for the distance sound will transverse between two reflections and that all boundaries have the same absorption coefficients under all conditions. If one calculates the influence of what we will call the EYRING effect one finds, as long as the mean absorption coefficient is smaller then ca. 0.7, that in a very good approximation we can write

$$T_r = T_S - 0.1\, c \frac{V}{S_{\text{tot}}}, \tag{5}$$

wherein T_r is the expected reverberation time and T_S the SABINE reverberation time; V the volume; S_{tot} the total surface of the boundaries and c a constant that in the derivation of EYRING becomes (1). This constant proves to depend also on the shape of the room and the absorption distribution over the boundaries. If one takes also into account that the real distances between two reflections in a room vary from zero to much larger then the mean free path, then c proves to be in most cases around 0.5 to 0.6.

If the value for c is found to be smaller than ca 0.5, then we must question the validity of the geometrical model on which the EYRING reverberation time formula is based. The EYRING effect is certainly not the only example of an uneven sound energy density distribution in a hall.

The uneven distribution throughout the hall of the sound sources, the uneven distribution of the absorption etc. etc., all this results in an uneven sound energy distribution that in general will result in an reverberation time formula in the form

$$T_r = T_S\left(1 + \frac{a_1}{T_S} + \frac{a_2}{T_S^2} \ldots\right). \tag{6}$$

Wherein $a_1, a_2, \ldots$ are coefficients (positive or negative) that depend on the unevenes of the sound distribution.

Until now we have assumed that the sound energy density distribution stays during whole the decay process the same. Only in this case one may expect a logarithmic decay.

As regards the instantaneous sound pressure distribution, large variations in time and in space are seen. This alone does however not mean that the sound energy density varies in the same way, for the sound pressure represents in general only the potential energy, the kinetic part of the total energy may and will in most cases vary as a complement to the potential energy. Nevertheless the sound energy density may also vary in the short term, as a consequence of the stochastic character of the sound field, as it will be discussed in the following section. If however, in the long run the sound energy distribution stays still the same, then the short time variations have for the reverberation time no other meaning then that they influence the accuracy with which the reverberation time can be measured. If on the other hand the sound energy density changes with time, then the decay will no longer be exponential although in most practical cases it will still stay nearly exponential.

3. Accuracy of reverberation times and reverberated sound level measurements

The sound field in a room may almost always — that is if the dimensions of the room are not small with regard to the wavelength and/or the walls etc. are not nearly 100 % absorbing — be considered to have a stochastic nature [1]. The sound pressure at any point in such a room will be a variable that is specified by the so called chi-square probability distribution function. Using this probability distribution it is possible to calculate the accuracy with which sound levels and reverberation times in a room can be determined [2].

The sound level may be determined with a standard deviation

$$\sigma_L \approx \frac{12.5}{\sqrt{2ft}} \tag{7}$$

and the standard deviation with which the reverberation time can be determined is

$$\frac{\sigma_{T_{60}}}{T_{60}} \approx \frac{12.5}{S} \sqrt{\frac{3}{2ft_0}}, \tag{8}$$

wherein σ_L — the standard deviation in the sound level, $\sigma_{T_{60}}$ — the standard deviation in the reverberation time, f — the frequency bandwidth, t — the measuring time (in case of sound level), t_0 — the measuring time (in case of reverberation time), s — the so called dynamic span, T_{60} — the reverberation time.

Between t_0, S and T_{60} exists the relation $t_0/T_{60} = S/60$. In Fig. 1 the meaning of these quantities is illustrated.

Using regression techniques, we can now measure reverberation times much more precisely than hitherto was possible.

The standard deviation that can then be determined will in every case be different but must in the mean agree with the above given values. This has been confirmed in practice.

It is thus possible to determine the reverberation times at a certain place in a room with standard deviations of ca. 1 to 7 %, depending on frequency etc. In the mid frequencies in general an accuracy better than 2 % can easily be reached.

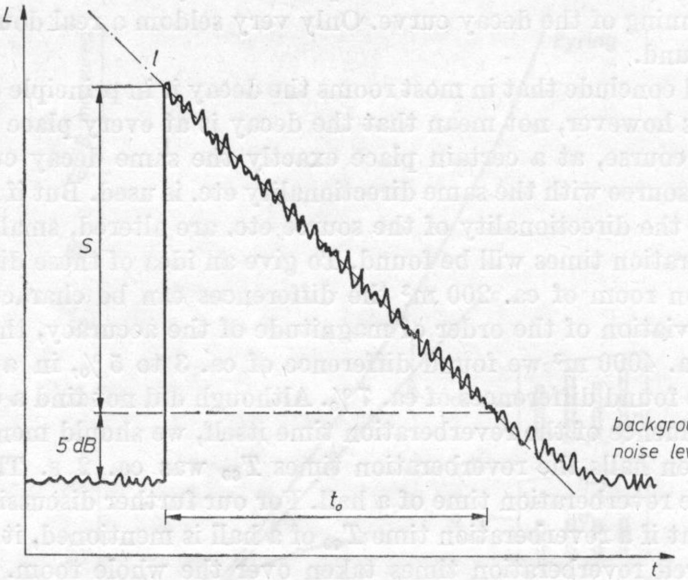

Fig. 1. Determination of the time t dependence of the decay of level L, by the regression technique. The figure illustrates the meaning of the dynamic span S and the measuring time t_0

4. Measured reverberation times

A precondition for the validity of the before given formulae is the assumption of an exponential decay.

Of course one may define the reverberation decay as an essentially exponential decay and consider every departure from it as an inaccuracy, but it is difficult to agree with such a point of view.

In general one considers a non-exponential decay as a consequence of many coupled system giving rise to a combination of exponential decays.

We have developed a procedure based on repeated integration to determine the different exponential components that may be found in a reverberation decay.

Beside real components (real reverberation times) we also had to take into account complex components (complex reverberation times). Moreover, as a set of exponential decays does not form a orthogonal system one should take a certain arbitrariness in the determination of the different decay times as unavoidable.

The results of the application of these procedures were nevertheless very interesting. In most of a very large number of cases that the reverberation times was determined this way, one important component was found that proved to agree reasonably well with the reverberation time as determined by regression techniques. And if there were more components they related in most cases to the very beginning of the decay curve. Only very seldom a real double or triple decay was found.

One could conclude that in most rooms the decay is in principle exponential.

That does however, not mean that the decay is at every place in the room the same. Of course, at a certain place exactly the same decay curve will be found when a source with the same directionality etc. is used. But if the distance to the source, the directionality of the source etc. are altered, small differences in the reverberation times will be found. To give an idea of these differences, in a reverberation room of ca. 200 m³ the differences can be characterised with a standard deviation of the order of magnitude of the accuracy, thus 1 to 2 %. In a hall of ca. 4000 m³ we found difference of ca. 3 to 5 %, in a hall of ca. 100.000 m³ we found differences of ca. 7 %. Although did not find a clear indication of the influence of the reverberation time itself, we should mention that in the above given halls the reverberation times T_{60} was ca. 2 s. There is thus clearly not one reverberation time of a hall. For our further discussions, it must be stressed that if a reverberation time T_{60} of a hall is mentioned, it is the mean of the measured reverberation times taken over the whole room.

As we will discuss in the next section, one finds never or almost never an even sound distribution in a room. We may thus expect to find the SABINE formula not completely validated by experiments. The absorption coefficients of materials, measured using the SABINE formula seldomly agree with those measured using other techniques.

But on the other hand, no indication could be found of the preferation of the EYRING formula.

We tried to verify the by EYRING predicted more than proportional shortening of the reverberation time if the amount of absorbing material in the room is increased.

At first we did our experiments in two reverberation rooms of different size. Although the accuracy was high enough to detect the EYRING effect no such effect could be found.

The results of the measurements (at 1000 Hz) in one of the reverberation rooms are given in Fig. 2. Vertically the apparent (calculated using the SABINE formula) mean absorption coefficient is given, if a part of the total material as indicated horizontally was placed in the reverberation room. The absorbing material was equally divided on walls and floor. The different patches of material were placed at a distance from each other in such a way that no influence of the edge effect was to be expected. To be sure that the absorption was halved etc. a rotation system was used.

We did a large number of reverberation measurements under all kinds of circumstances in the well known *Espace de Projection* in the Centre Pompidou in Paris. This hall has walls and a ceiling that is made of rotatable panels that have an absorbing, reflecting and diffuse reflecting side. Moreover the ceiling

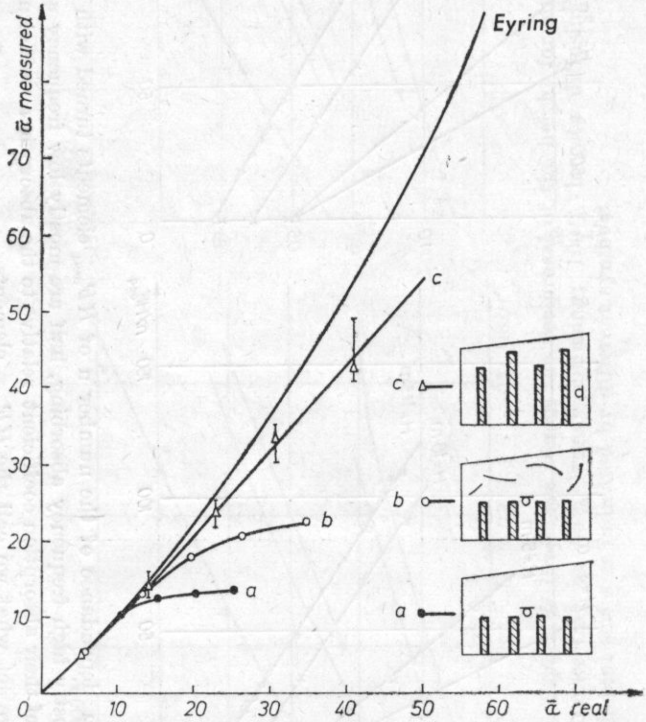

Fig. 2. The apparent (measured) mean absorption coefficient in dependence of the "real" absorption coefficient, as determinded according to the standardized reverberation method. To be sure that the relative points on the horizontal scale are correct, a method of halving and rotating of the material was used. In the first experiment (a) the walls were only partially covered. Bringing in some more material and diffusing elements (b) resulted in a better agreement with what was expected on basis of the Sabine's reverberation law. Covering all walls and the floor with material gave curve (c). Volume of the room 200 m³. Impulse type source, filtered octave of 1000 Hz

consists of three parts that all three can independently be lowered from maximum height (11.5 m) to ca. 2 m (lowest height permitted for safety reasons). The floor area is 24×15.5 m². Between the three ceiling parts heavy iron curtains can be lowered. Thus absorption and shape of the room can be varied. It is also possible to divide the hall in two or three coupled rooms.

Earlier measurements in this room, did not show an Eyring effect. We tried again, measuring at different heights and for a large number of situations. The results are given in Fig. 3 to 5 for the frequencies 500, 1000 and 2000 Hz.

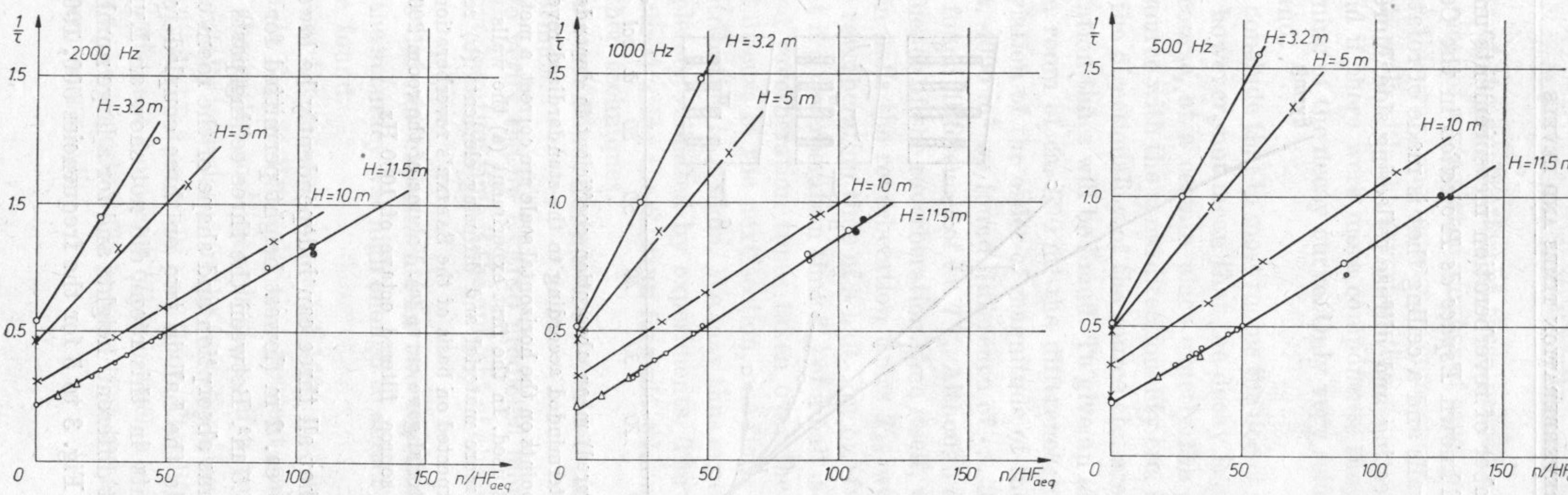

Fig. 3–5. The reciprocal $1/\tau$ of the reverberation time in dependance of the number n of HF_{aeq} elements turned with their absorbing sides to the hall. Half of the turnable elements (periactes) are mostly high frequency absorbing, half are mostly low frequency absorbing. If one counts the low frequency absorbing periactes as in proportion of their absorption coefficient relative to the absorption coefficient of the high frequency elements we get what we call the HF_{aeq} elements

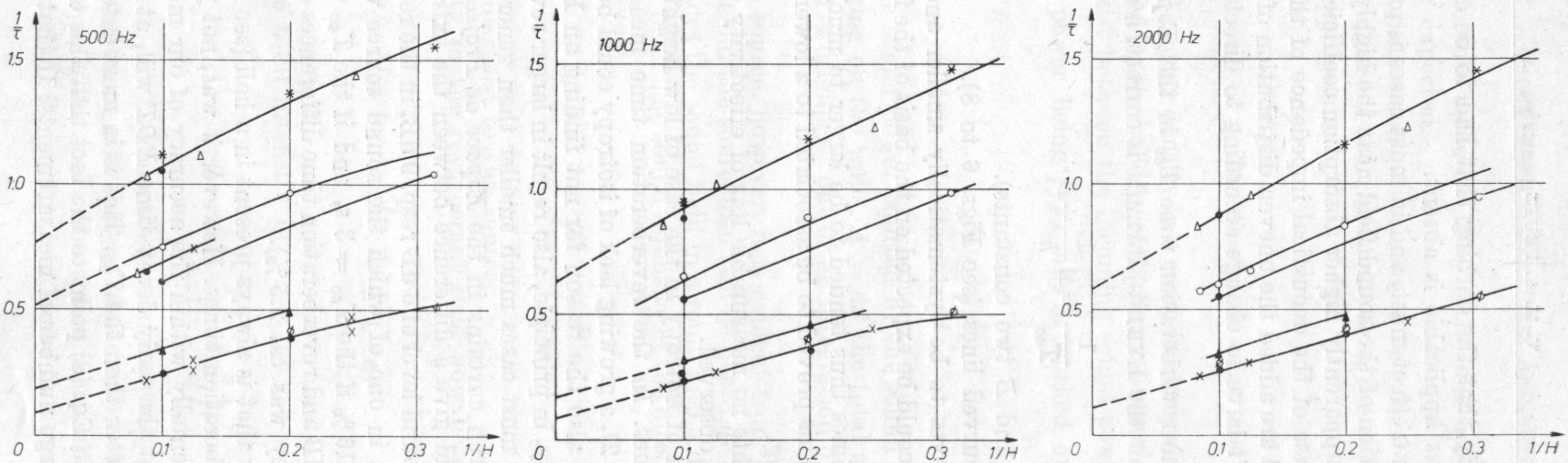

Fig. 6–8. The reciprocal of the reverberation time of the *Espace de Projection* versus the reciprocal of the height for different wall and ceiling configuration (100 % absorbing; 50 % absorbing and 50 % reflecting or 50 % largely diffusing; 100 % largely diffusing c.q. 100 % reflecting). Measured in different series as indicated by different symbols

One can easily verify that the curbing that had to be expected if indeed the EYRING formula was applicable is absent.

Still a certain EYRING effect must, and in most cases is present. In a reverberation room the depletion of the sound field near the highly absorbant 10 m^2 surface can result in an apparently higher absorption coefficient of the material then expected on the base of the acoustical impedance of the material, but if more patches of material are added the uneven distribution of the sound energy density evidently doesn't have to change according to direction expected from EYRING.

The reciprocal of the reverberation time T_{60} in the *Espace de Projection* must be, if the SABINE (or the EYRING) formula is correct, a straight line

$$\frac{1}{T_{60}} = A\frac{1}{h} + B$$

(h being the height, A and B two constants).

Instead we found curved lines (see Figs. 6 to 8).

The value of A proves to be systematically smaller and the value of B proves to be larger then could be expected on the basis of the formula and other measured data.

The reverberation times thus tended to be larger to much larger for lower heights, than expected. This proved to be associated to a lower effectivity of the absorption on the ceiling.

For the higher heights no measurable less of effectivity of the absorption on the ceiling could be observed.

This indicates a lack of isotropy in the case of low heights as the origin of longer reverberation times. In the reverberation time measurements in the reverberation room (Fig. 2), a growing lack of isotropy could be considered to be at least for a large part also the reason for not finding an EYRING effect.

Non-homogenity may, in principle, also result in larger reverberation times, however this effect is in most cases much smaller than generally expected.

Lowering the two iron curtains in the *Espace de Projection* to 5 m resp. 2.80 m from the floor did give a difference between the extreme parts of the hall in the steady state sound level of 6 dB resp. 9 dB, in the reverberation times for these extreme parts, in one of which the sound source was located, only a difference of 7 % resp. 10 % if the $T_{60} = 3$ s, and if the $T_{60} = 1$ s level differences of 10 dB resp. 17 dB and reverberation time differences of 10 % resp. 12 % were found (the accuracy was ca. 2.5 %).

The non-homogenity that is always present in a hall (see next section) will certainly influence reverberation times. However it was, not possible to determine this effect experimentally within the accuracy of our measurements.

One may expect that the early decay time *EDT* will, at least near to the source be somewhat shorter then the T_{60}. This is in most cases also found, but we must with reference to Eq. (8) point to the fact that this can only be found as the mean of a very large number of measurements. If that is taken into account then a difference up to ca. 10 % in the area near the source (see also next section) may be expected.

There is certainly much information to be gained out of the first part of the decay curve; to extract it other methods then the statistical reverberation analysis are then appropriate.

5. The reverberant sound level

If the sound energy density in a room is everywhere homogeneous and isotropic, as is needed for the validity of the SABINE reverberation formula, then equilibrium must exist between the incoming sound power P ($= -dE/dt$) and the outgoing sound power being $\iint p \vec{v} \cdot \vec{n} \, ds$ or worked out in case of homogenity and isotropy

$$P = I \frac{A}{4}, \tag{9}$$

wherein P — the sound power emitted by the sound source (s), I — the sound intensity impinging on the walls, $A = \bar{a}S$ the total absorption in the hall.

The factor 1/4 originates out of an integration over all angles of incidence on the walls.

Near to the source, however, it is evident from experience that the sound coming directly from the source can still be heard distinctly. As a logical consequence, one divides the sound field in a room into a direct sound field and a reverberant sound field. The sound intensity of the direct field can be given by the well known formula

$$I_{dir} = \frac{P}{4\pi D^2}, \tag{10}$$

wherein D the distance to the source.

The intensity in the reverberant field now becomes

$$I_R = \frac{P}{\frac{A(1-\bar{a})}{4}} \tag{11}$$

resulting for the total intensity in

$$I = P\left(\frac{1}{4\pi D^2} + \frac{4}{R}\right), \tag{12}$$

wherein $R = A(1-\bar{a})$ the so called roomconstant, $\bar{a}$ is the mean absorption coefficient. It is better to take instead of the mean absorption coefficient $\bar{a}$, the so called a_{proj}, that is the mean absorption when all absorbing surfaces in the room are projected on the surface of a body that results if one projects out of the sound source in all directions rays that are proportional to the square of the directivity factor Q in that direction.

In case of a real point source, the body is a sphere. Now one sees directly that absorption found near to the source has more influence on the level of the reverberant sound, then the same amount of absorption further away.

The second term in the expression at the right side in Eq. (12) proves in practice to be dependent on the distance to the source also, that is it decreases with distance. In most cases a power function is found experimentally, see Fig. 9 and 10.

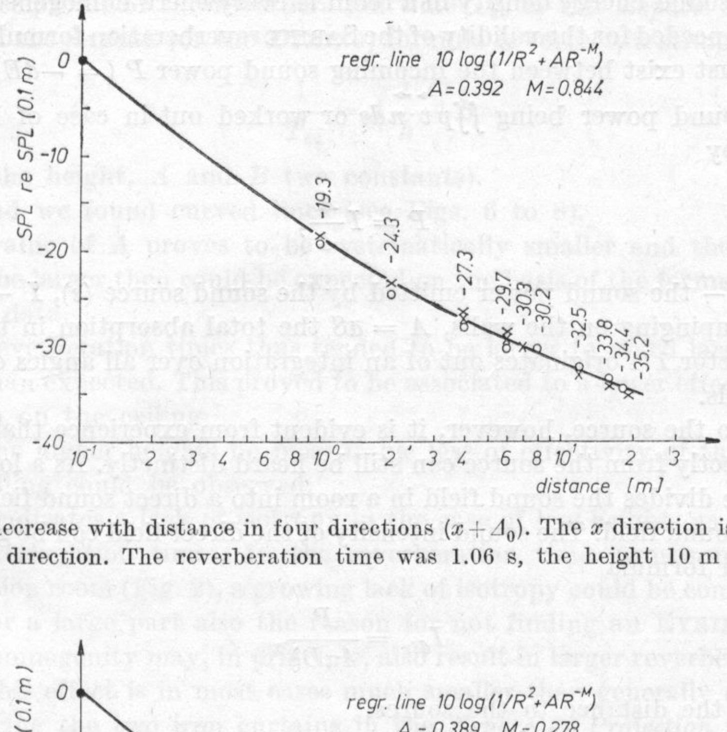

Fig. 9. The decrease with distance in four directions $(x + \Delta_0)$. The x direction is the vertica direction. The reverberation time was 1.06 s, the height 10 m

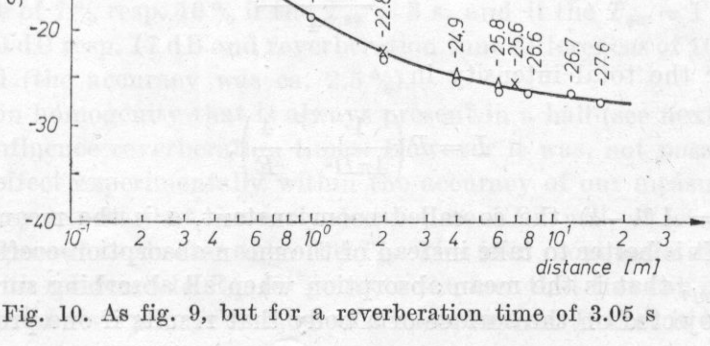

Fig. 10. As fig. 9, but for a reverberation time of 3.05 s

More precise observation shows that the basic mechanism is an exponentially decreasing propagation superimposed on a power function originating out of sound expansion. At the wall boundaries a reflection takes place, as can be seen in Fig. 12. The results in the overall decrease what gives the impression of a power function.

In a very large low room the exponential decrease is still very clear in the so called "plateau" (see Fig. 11). As can be seen from Fig. 9, the decrease with dis-

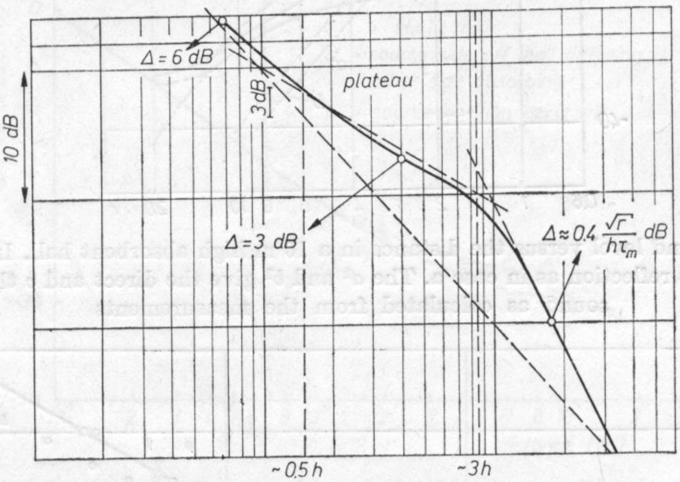

Fig. 11. Schematic presentation of the decrease with the distance to the source of the sound level in a large low space. Δ is the decrease in dB for each doubling of distance. V the volume of the room in m³, h the height in m and τ_m the measured reverberation time (r: = V)

ance is essentially the same in all directions. This has also been found in all kinds of rooms. An experimental relation between the decrease when doubling the distance ΔL in rectangular rooms and different roomconstants has been found (see Fig. 12)

$$\Delta L \cong 0.4 \frac{\sqrt{V}}{H T_{60}}. \tag{13}$$

From Fig. 12 and Fig. 13 we learn that this Eq. (13) is the best fit to experimental data only; it is however, an useful equation for practical calculations.

In any case, we learn that the reverberant sound field in a room in a steady state situation is far from homogeneity and isotropy.

The distribution of the absorption in a room with normal dimensions has almost no measurable influence on the reverberation time, absorption in corners is only a very little bit more efficient than in the middle of a wall. We also studied the case when one half of the *Espace de Projection* was absorbant, the other half diffusely reflecting. There was no significant difference in reverberation times observed. The reverberant sound levels in the room, especially the

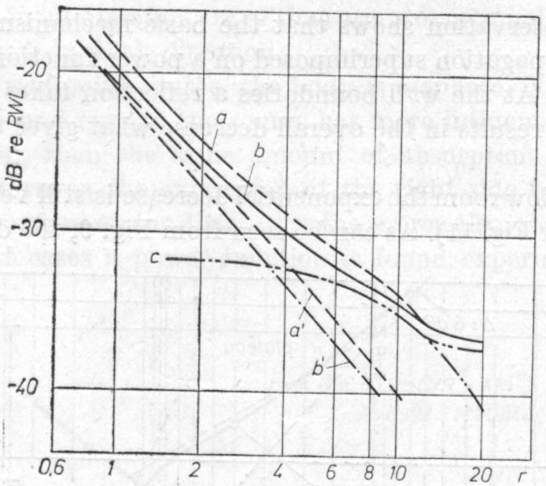

Fig. 12. The sound level versus the distance in a 10 m high absorbent hall. In case b there is no strong first reflection as in case a. The a^1 and b^1 give the direct and c the reverberant sound as calculated from the measurements

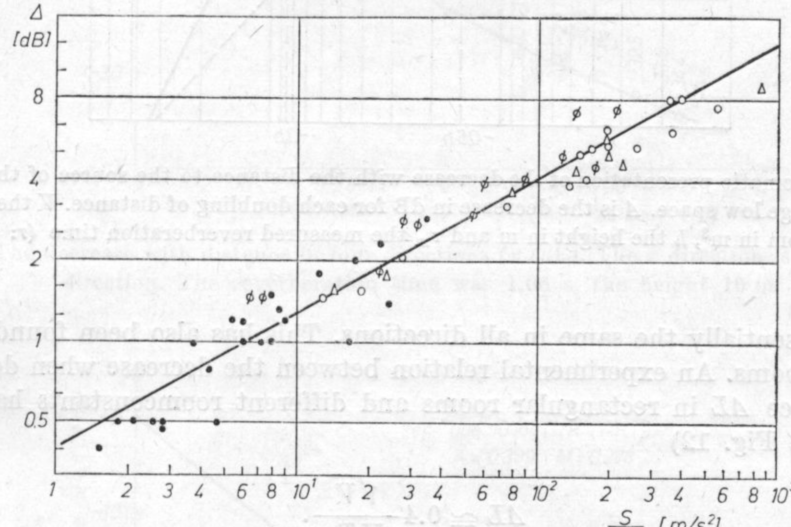

Fig. 13. The decrease per doubling of distance Δ vertically versus S/hT_m^2 the floor area S over the height h, times the square of the measured reverberation time T_m, as measured in a large number of rectangular rooms, halls etc.

decrease with distance showed a typical, but not very large difference (see Fig. 14).

The decaying sound field has in general the tendency to become more and more homogeneous, if not also isotropic. This can be deduced from i.a. Fig. 15. This figure shows the decay of a sound field at different places in the same room. The different decay curves at different distances of the sound sources (a pistol shot) have, as regards the starting points of the different curves, time lags as can be seen in the figure. The curves fall after a short time almost perfectly on each other, indicating that the decaying sound field becomes spatially equal in intensity. The same has been experienced in a large quantity of rooms of simple shape. In rooms having, a more complicated shape larger, to much larger, level differences have been found. It would, however, be outside the scope of this paper to discuss this.

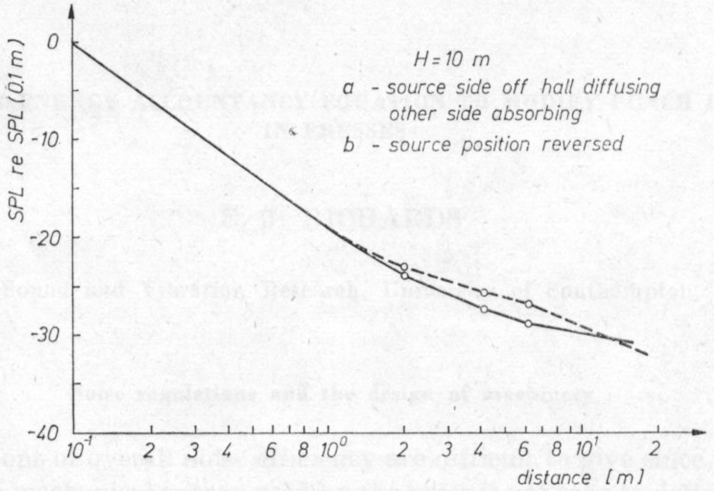

Fig. 14. The sound level versus distance for two loudspeaker position in the *Espace de Projection* of IRCAM Paris. One side of the hall is absorbent the other side diffusely reflecting (T_{60} = 1.6 s)

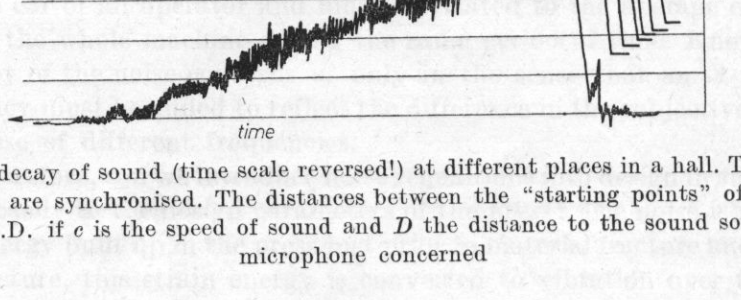

Fig. 15. The decay of sound (time scale reversed!) at different places in a hall. The different decay curves are synchronised. The distances between the "starting points" of the curves agree with c.D. if c is the speed of sound and D the distance to the sound source of the microphone concerned

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