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Exploring the Difference Paralindelöf in Topological Spaces

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ABSTRACT. This study investigates emerging concepts for defining and categorizing topological spaces based on various features. Paralindelöf spaces are one such idea that is required to understand the compactness and covering features of topological spaces. This study is the first to introduce D-paralindelöf spaces, a novel type of topological space defined combining D-sets and paralindelöf spaces. The study's goal is to offer precise definitions for paralindelöf spaces and D-paralindelöf spaces, while also investigating their properties and linkages with other forms of topological spaces. The study contains various theoretical conclusions, definitions, and features that are rigorously proven by extending previous theorems on paralindelöf spaces. It is further backed by extensive illustrative examples.

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1. Introduction

In topological structures, there is a mathematical way of studying the characteristics of sets and their relationships is provided by the topological spaces. Topological spaces of lindelöfness, paralindelöfness, metalindelöf, difference metalindelöfness, difference lindelöfness, and their variants are among the many topological concepts that are essential to understanding the topological structure of spaces. We explore the deeper implications and connections among these concepts in this research, providing insight into the complicated content of topological spaces.

In topological spaces, the lindelöf spaces are a basic and interesting type of topological space that exhibits special characteristics in the study of topology. Lindelöf spaces present a significant viewpoint on the form of open covers. The notion of lindelöf was provided by Alexandroff and Urysohn (1929), see Engelking [1], which if each open cover of a topological space has a countable subcover, then the space is said to be lindelöf.

Moreover, a generalization about the extensively studied lindelöf space in topology is called paralindelöf spaces, where the notion introduced and studied by Tall [2] is that the topological space is called paralindelöf if every open cover contains an open locally countable refinement. The concept was introduced to explore the interaction between local covering qualities. This notion provides an advanced representation of the extent and structure of open covers in a topological space by using locally countability and lindelöf spaces.

Additionally, several generalizations of lindelöf spaces were introduced, one of them by Engelking [1] called metalindelöf spaces. If each open cover of a topological space has an open point countable refinement, then the space is called metalindelöf. Also, the core of metalindelöfness is captured in this definition, which ensures the presence of a specially refined covering that observes lindelöfness's global countability as well as metacompactness's intrinsic local finiteness. Metalindelöf spaces are distinguished by the complex interplay of these characteristics, which provide an exclusive viewpoint on the structure of open sets.

Also, in topological spaces, the difference sets allow us to define and explore new key topological notions, especially the properties and notions that relate to covering, separation axioms, and compactness. Many papers studied and used several topological spaces with this notion as a generalization of the spaces by using a special open cover called D −cover. Such as the notion of D −metalindelöf spaces established by Oudetallah, et al. [3], which showed every

D-cover of topological space has a point countable parallel refinement. Also, Qoqazeh, et al. [4] defined the notion of D −lindelöf spaces, in which every D −cover of a topological space has a countable subcover.

In the same way, we will use the concepts of paralindelöf spaces and difference sets (called D −sets) to define and study new key topological notions called D −paralindelöf spaces.

Alexandroff and Urysohn (1924) introduced the notion of compactness, see Engelking [1]. Also, Alexandroff and Urysohn (1929) contributed the notion of lindelöf spaces, see Engelking [1]. The compactness concept, which is more often used and implies the presence of a finite subcover, is weakened by the lindelöf notion. Later, Dieudonné [5] first proposed paracompact spaces as a conceptual generalisation of compactness. Since then, several fields of topology and analysis have greatly benefited from paracompactness and its tools, including refinement techniques and locally finite or closure−preserving conditions.

More applications of the paracompactness concepts with various topological spaces can be obtained from the studies [6-22].

Lindelöf spaces have been generalised in a number of ways in literary works, and each of these generalisations is examined independently for different purposes. To generalise lindelöfness, Frolik [23] was first suggested and researched the concept of weakly lindelöf spaces, and it was further explored by several authors. Burke [24] discussed the closed mapping characteristics of the paralindelöf spaces and associated it with topological spaces. He demonstrated that a paralindelöf property is maintained under perfect mappings but not under closed mappings. Further, the perfect mapping results of σ −paralindelöf spaces and the spaces with σ −locally countable basis are provided.

Additionally, by using open sets, Tong (1982)[25] proposed the concept of difference sets. He then utilized this idea to speeity and study an advanced separation axiom named $D_i(i = 0,1,2)$ spaces. Fleissner [26] was responsible for creating the paralindelöf concept. About the topic, Balasubramanian [27] initialized and investigated certain concepts of nearly−lindelöf spaces, which is in between lindelöf and weakly−lindelöf spaces. Also, Dissanayake and Willard [28] were first suggested the notion of almost lindelöf spaces. Therefore, Ganster, et al. [29] defined and studied σ −lindelöf spaces.

Moreover, Cammaroto and Santoro [30] contributed the notions of nearly regular−lindelöf, almost regular−lindelöf, and weakly regular−lindelöf spaces on using regular covers. In addition, as a generalisation of regular−lindelöf spaces, nearly regular−lindelöf, almost regular−lindelöf, and weakly regular−lindelöf spaces are all studied by Fawakhreh and Kilicman [31], they proposed characterizations and certain attributes for these spaces. They are also researched in relation to one another.

Additionally, Barr, et al. [32] introduced the conditions for a topological space that guarantee its product with each lindelöf space is lindelöf, and presented a Alster's condition, which they defined to as spaces satisfy his condition as Alster spaces, makes it a key tool. In addition, they examine a few variations upon scattered spaces that is significant to this study.

Thereafter, Kilicman and Salleh [33] proposed the idea of pairwise weakly lindelöf bitopological spaces, researched it, and found some findings. Additionally, they looked at some of the properties of pairwise weakly lindelöf subspaces and subsets and demonstrated that a pairwise weakly lindelöf property is not inherited. Alfaham and Al-Awadi [34] introduced the notions of paralindelöf and semi−paralindelöf bitopological spaces. They also determined some of these notions' features and provided an explanation of their relationship.

Al-Fatlawee [35] also defined a topological m −paralindelöf, countable paralindelöf, m − σ −paralindelöf, and countable σ −paralindelöf and examined several of these notions' characteristics before relating them. They provided the connection between normal spaces and paralindelöf spaces. In addition, Juhasz, et al. [36] introduced and analysed a type of weakly linear lindelöf spaces. While Dhanabalan and Padma [37] introduced and examined some product spaces and μ −paralindelöf spaces within generalized topological spaces. Furthermore, Qoqazeh, et al. [4] defined the notion of D −lindelöf spaces. Bani-Ahmad, et al. [38] defined and analysed the new type of D −perfect functions, a type of perfect function in topological spaces.

Moreover, Sarsak [39] presented and studied, as extended topological spaces (GTSs), the notion of $\mu - \beta$ −lindelöf sets as a subclass of both highly μ −lindelöf sets and μ −semi−lindelöf sets. Additionally, he studied and introduced a novel class of generalized open sets in GTSs known as $\tau\mu - \beta$ −open sets and uses them to derive additional features of $\mu - \beta$ −Lindelöf sets. While Song and Xuan [40] studied the topological characteristics of star weakly lindelöf spaces, they investigated the links between star almost lindelöf spaces, star weakly lindelöf spaces, and the spaces of star lindelöf.

2. Preliminaries

This Within this section, the main associated definitions, properties, examples, figures, and theorems to our study are provided as follows:

Definition 2.1 *[1] Let (Z, ℓ) be a topological space. Then $\tilde{O} = \{O_\omega : \omega \in \Omega, O_\omega \subseteq Z\}$ is called cover of Z if and only if $Z = \bigcup_{\omega \in \Omega} O_\omega$. Observe the following Figure 2.1 that represent the cover of space Z.*

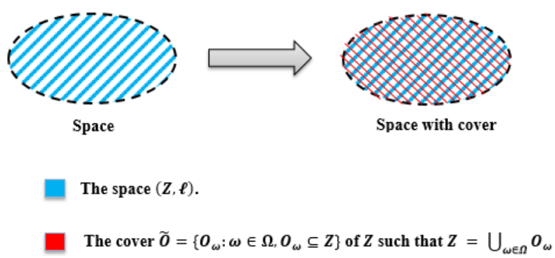


Figure 2.1 The cover of the topological space.

Moreover, there are types of covers of the topological space as follows [1]:

- Open cover: A collection of open sets that covers the space is called an open cover of a space.
- Closed cover: A collection of closed sets that covers the space is called a closed cover of a space.
- Finite cover: A collection of finite sets that covers the space is called a finite cover of a space.
- Countable cover: A collection of countable sets that covers the space is called a countable cover of a space.

Definition 2.2 *[1] Let (Z, ℓ) be a topological space and $\tilde{O}$ be a cover of Z. Then a subcover of $\tilde{O}$ of Z is a set $\tilde{P} \subseteq \tilde{O}$ such that $\tilde{P}$ is also cover for Z. Observe the following Figure 2.2 that represents the subcover of space Z.*

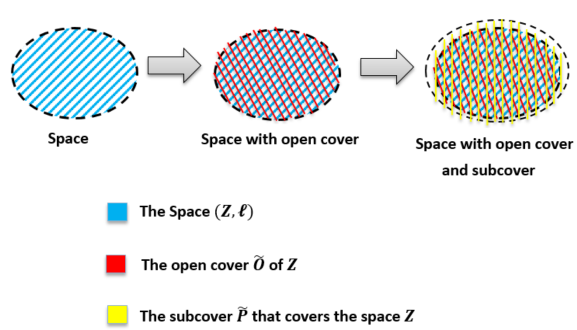


Figure 2.2 The subcover of the topological space.

Moreover, there are types of subcovers of the topological space as follows:

- Finite subcover [1]: A collection of finite subsets that covers the space is called a finite subcover of a space.
- Countable subcover [1]: A collection of countable subsets that covers the space is called a couuntable subcover of a space.

Definition 2.3 *[1] A new cover in the same space that has each set as a subset of some set from the old cover is called a refinement of the cover of space Z. i.e., the cover $\tilde{P} = \{P_\lambda : \lambda \in \Lambda\}$ is a refinement of the cover $\tilde{O} = \{O_\omega : \omega \in \Omega\}$ if and only if for all $P_\lambda \in \tilde{P}$, there exist $O_\omega \in \tilde{O}$ such that $P_\lambda \subseteq O_\omega$.*

Definition 2.4 *[41] When all open cover of (Z, ℓ) has a point finite open refinement, then a topological space (Z, ℓ) is said to be a metacompact space.*

Definition 2.5 *[42] If $F \subseteq Z$ such that $D_z \cap F \neq \emptyset$, for all $z \in Z$ and every difference set D_z containing z, then F is called a difference dense. Observe the following Figure 2.3 that represents the difference dense set in the space (Z, ℓ):*

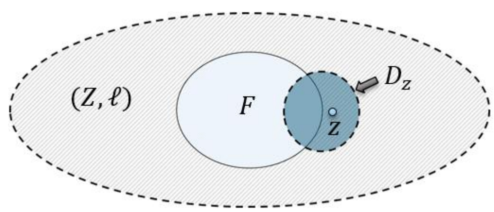


Figure 2.3 The difference dense in the topological space.

Definition 2.6 *[42] If a topological space (Z, ℓ) has a countable subset of difference dense, then it is said to be difference separable.*

Definition 2.7 *[42] Let (Z, ℓ) and (S, ρ) be any two topological spaces are required to be considered. A function $\Psi: Z \longrightarrow S$ is continuous if the preimage of each open set is open in, or alternatively, if the preimage of each closed set is closed. In other words, if $R \in \rho$, then $\Psi^{-1}(R) \in \ell$. Observe the following Figure 2.4 that represents the continuous function of the topological spaces.*

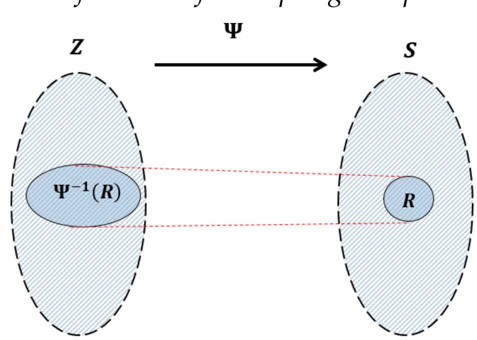


Figure 2.4 The continuous function of the topological spaces.

Definition 2.8 *[1] If each open cover of a topological space (Z, ℓ) has a countable subcover, then the topological space is said to be Lindelöf space.*

In the field of general topology, the lindelöf spaces are significant type having special covering characteristics. Observe the Flow Chart 3.1 that represents the impact relation of lindelöf spaces with other topological spaces.

Definition 2.9 *[2] If each open cover of a topological space (Z, ℓ) has an open locally countable refinement, then the topological space is said to be paralindelöf space.*

Definition 2.10 *[25] Whenever there are H and G are two open sets in a way that $H \neq G$ and $F = H - G$, then it is said that a subset F of a topological space (Z, ℓ) is a difference set. We define F as a $D-$set generated by H and G. If $F = H$ and $G = \phi$, then each open set H that differs from Z is a $D-$set. Observe the following Figure 2.5 that represents the difference sets of the topological space (Z, ℓ).*

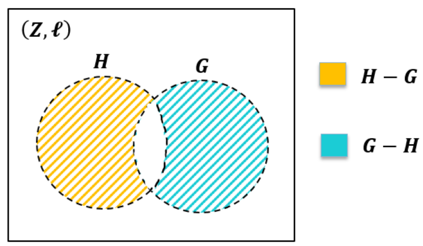


Figure 2.5 The difference sets of the topological space.

Definition 2.11 *[25] Let (Z, ℓ) and (S, ρ) be any topological spaces. If $\Psi: (Z, \ell) \longrightarrow (S, \rho)$ be closed, continuous, surjective, and proper function (or $\Psi^{-1}(s)$ is compact in Z, for all $s \in S$). Then the function Ψ is said to be perfect.*

Definition 2.12 *[1] Let (Z, ℓ) and (S, ρ) be any topological spaces and $\Psi: Z \longrightarrow S$. If the image $\Psi(K)$ is closed in S for any closed set K in Z, then the function Ψ is said to be closed. Observe the following Figure 2.6 that represents the closed function of topological spaces.*

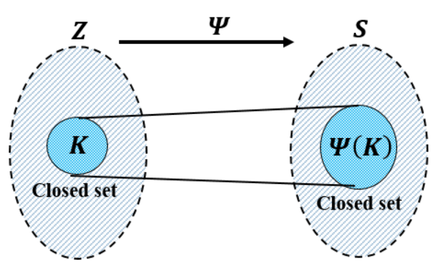


Figure 2.6 The closed function of topological space.

Definition 2.13 *[1] If each open cover of a topological space has a finite subcover, then the space is called compact.*

Definition 2.14 *[1] Let's say there are two topological spaces (Z, ℓ) and (S, ρ). The topological space is thus represented by the Cartesian product $(Z \times S, \ell \times \rho)$.*

To illustrate more, let $Z = \mathbb{R}$, $S = \mathbb{R}$, and $O \in \ell$, $P \in \rho$. Then observe the following Figure 2.7 that represents the Cartesian product $(\mathbb{R} \times \mathbb{R}, O \times P)$ of the topological spaces.

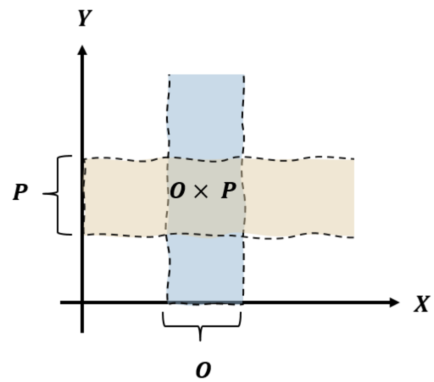


Figure 2.7 The Cartesian product of topological space.

Definition 2.15 *[43] If each open cover of a topological space has an open point countable refinement, then the space is called metalindelöf.*

Definition 2.16 *[44] Let (Z, ℓ) and (S, ρ) be any topological spaces and $\Psi: Z \longrightarrow S$. If the image $\Psi(H)$ is open in S for any open set H in Z, then the function Ψ is said to be open. Observe the following Figure 2.8 that represents the open function of topological spaces.*

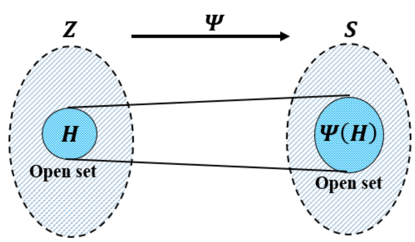


Figure 2.8 The open function of topological space.

Definition 2.17 *[44] When all open cover of (Z, ℓ) has a locally finite open refinement, then a topological space (Z, ℓ) is said to be a paracompact space.*

Definition 2.18 *[44] If every point in a topological space (Z, ℓ) has a neighborhood that intersects only a finite number of the sets in the collection, then the collection of subsets of Z is said to be locally finite.*

Definition 2.19 *[45] If every point in a topological space (Z, ℓ) has a neighborhood that intersects only a countable number of the sets in the collection, then the collection of subsets of Z is said to be locally countable.*

Definition 2.20 *[35] When all every countable open cover in a topological space (Z, ℓ) has a locally countable open refinement. Then the space is said to be countable paralindelöf.*

Definition 2.21 *[4] When all O_ω is a difference set for all $\omega \in \Omega$, then a cover $\tilde{O}$ of a topological space (Z, ℓ) is said to be $D-$cover.*

Definition 2.22 *[4] When all countable difference cover of a topological space (Z, ℓ) has a finite subcover, then the space is said to be $D-$countably compact.*

Definition 2.23 *[4] If each $D-$cover of a topological space (Z, ℓ) has a finite subcover, thereafter the space is said to be $D-$compact.*

Definition 2.24 *[3] If each $D-$cover of topological space has a point countable parallel refinement, then the space is called $D-$metalindelöf space.*

Definition 2.25 *[3] If each $D-$cover of topological space has a point finite parallel refinement, then the space is called $D-$metalcompact space.*

Definition 2.26 *[4] When all of difference cover of a topological space (Z, ℓ) has a countable subcover, then the space is said to be $D-$lindelöf.*

It is important to note that a topological space (Z, ℓ) will include specific topological properties if it has a $D-$topological property. We have the following properties [4]:

- A topological space (Z, ℓ) is lindelöf if $D-$lindelöf is true for it.
- A topological space is called compact when (Z, ℓ) is $D-$compact.

Theorem 2.27 *[4] A difference compact space's continuous image is a difference compact space.*

Theorem 2.28 *[4] Each $D-$compact space is a compact space.*

Theorem 2.29 *[4] Any open cover is a $D-$cover.*

Remark 2.30 *[4] Any open set is a $D-$set, but the converse need not to be true.*

Example 2.31 Let $Z = \{a, b, c\}$ and $\ell = \{\phi, Z, \{a\}, \{a, b\}\}$. Then $D = \{a, b\} - \{a\} = \{b\}$ is $D-$set. But D not open set.

Definition 2.32 *[38] If Ψ is a continuous, closed function and $\Psi^{-1}(s)$ is a difference compact function for any $s \in S$, then the function is said to be a $D-$perfect function.*

Definition 2.33 *[46] If every open set is clopen, then the topological space (Z, ℓ) is called locally indiscrete.*

3. Notions and Properties of Difference Paralindelöf Spaces

In this section, the notion of difference paralindelöf space in topological spaces and discusses how it relates to other types of spaces are introduced. Also, describe some of their characteristics. This would establish the groundwork for presenting some theoretical findings related to the topic at hand.

Definition 3.1 If any $D-$cover of a topological space (Z,ℓ) has an open locally countable refinement, then the space is said to be $D-$paralindelöf space.

To illustrate more, let $\tilde{O} = \{O_\omega : \omega \in \Omega\}$ be any $D-$cover of the topological space (Z,ℓ) has an open locally countable refinement $\tilde{P} = \{P_\lambda : \lambda \in \Lambda\}$. i.e. each O_ω is a $D-$set for all $\omega \in \Omega$ such that $Z = \bigcup_{\omega\in\Omega} O_\omega$ has an open locally countable refinement $\tilde{P} = \{P_\lambda : \lambda \in \Lambda\}$, which the open refinement $\tilde{P}$ is a locally countable if every point x_i of the space Z has a neighborhood N_i such that $P_\lambda \cap N_i \neq \emptyset$ is countable for all $\lambda, i \in \Lambda$. Then observe the following Figure 3.1 that represents the $D-$paralindelöf space (Z,ℓ):

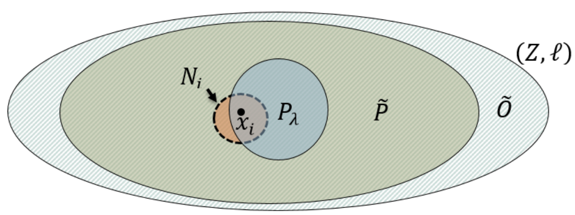
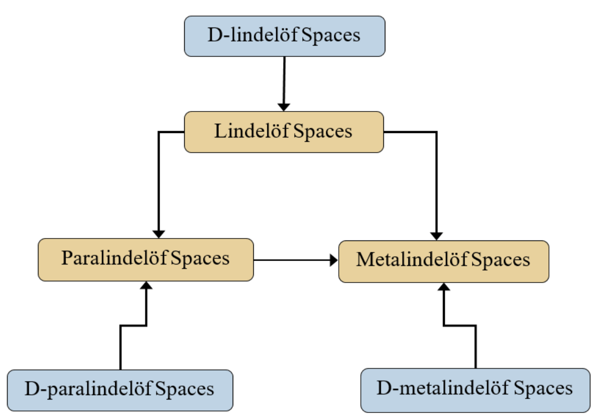


Figure 3.1 The difference paralindelöf space.

In topological spaces, the difference paralindelöf space represents a basic class characterized by special coverings. Observe the following of study's results flow chart that represents the relation of difference paralindelöf spaces with other topological spaces.



Flow chart 3.1 The study's results flow chart.

Theorem 3.2 *Every D −paralindelöf space (Z, ℓ) is paralindelöf.*

Proof. Let (Z, ℓ) be a D −paralindelöf space and $\tilde{O} = \{O_\omega : \omega \in \Omega\}$ be an open cover of (Z, ℓ). Then by Theorem 2.29, the cover $\tilde{O}$ is a D −cover, and so it has an open locally countable refinement. Hence, we get the result that the space (Z, ℓ) is paralindelöf.

Example 3.3 Given that the space $(\mathbb{R}, \ell_u)$ is a D −paralindelöf, it implies that the topological space is paralindelöf.

The next illustration in the Example 3.2.2 shows why the above theorem's converse is not always true.

Example 3.4 A topological space $(\mathbb{R}, \ell_{cof})$ is paralindelöf, but it is not a D −paralindelof space. Since every set of the form $\mathbb{R} - \{n\}$, for all $n \in \mathbb{R}$ is an open set in a topological space $(\mathbb{R}, \ell_{cof})$, then let $O = \mathbb{R} - \{n\}$ and $P = \mathbb{R} - \{m\}$ be any two open sets, for all $n, m \in \mathbb{R}$ such that $F = O - P = \{m\}$ is a D −set that is not open. Now, $\tilde{F} = \{\{m\}: m \in \mathbb{R}\}$ is a D −cover of the space $(\mathbb{R}, \ell_{cof})$ that is no of a locally countable refinement. Suppose that $\tilde{F} = \{\{m\}: m \in \mathbb{R}\}$ has a locally open countable refinement $\{\{m_1\}, \{m_2\}, \dots\}$, then $\mathbb{R} \subseteq \bigcup_{i=1}^{\infty} m_i$, and hence we get that $\mathbb{R}$ is a countable set, is true. Which is a contradiction with the fact that $\mathbb{R}$ is uncountable set.

The contrapositive of the given Theorem 3.2 is shown by the illustration in the following example:

Example 3.5 The topological space $(\mathbb{R}, \ell_{l.r})$ is not paralindelöf, so it is not a D −paralindelöf space.

The purpose of the next Theorem 3.2.2 is to demonstrate that the converses of the previous Theorem 3.2 might hold valid in under conditions.

Theorem 3.6 *If any topological space (Z, ℓ) is locally indiscrete and paralindelöf, then it is a D −paralindelöf space.*

Proof. Let $\tilde{O} = \{O_\omega : \omega \in \Omega\}$ be any D −cover of (Z, ℓ) and (Z, ℓ) be a locally indiscrete and paralindelöf space. Then each D −set O_ω is clopen for all ω ∈ Ω. So $\tilde{O}$ is open cover of (Z, ℓ). Since (Z, ℓ) is a paralindelöf space, then $\tilde{O}$ has an open locally countable refinement. Hence, we get the result that (Z, ℓ) is a D −paralindelöf space.

Example 3.7 (i) Let $Z = \mathbb{R}$ and ℓ_{ind} be a topology on Z. The topological space $(\mathbb{R}, \ell_{ind})$ is locally indiscrete and paralindelöf, hence by applying the Theorem 3.6 we get that $(\mathbb{R}, \ell_{ind})$ is clearly D −paralindelöf topological space.

(ii) Let $Z = \mathbb{R}$ and $\ell = \{\phi, \mathbb{R}, \mathbb{R} - \{3\}\}$ be a topology on Z. Then (Z, ℓ) is locally indiscrete and paralindelöf space. Hence, by applying the Theorem 3.6 we get that (Z, ℓ) is clearly D −paralindelöf topological space.

Theorem 3.8 *Given that $F \subseteq Z$ and (Z, ℓ) is a topological space, (F, ℓ_F) is only D −paralindelöf as and only as every D −cover of F by difference sets in Z has an open locally countable refinement.*

Proof. ⇒) Suppose that (F, ℓ_F) is D −paralindelöf topological space, for all $F \subseteq Z$ and $\tilde{O} = \{O_\omega : \omega \in \Omega\}$ is D −cover of F by difference sets in Z. Let $O_\omega^* = O_\omega \cap F$ be a difference set in F, for all $\omega \in \Omega$. Then $\tilde{O}^* = \{O_\omega^* : \omega \in \Omega\}$ is a difference cover of F by difference sets in F. Now, since the topological space (F, ℓ_F) is D −paralindelöf, then the difference cover $\tilde{O}^*$ has an open locally countable refinement $\{O_{\omega 1}^*, O_{\omega 2}^*, \dots\}$ for the subset F. Because of this, the family $\{O_{\omega 1}, O_{\omega 2}, \dots\}$ is an open locally countable refinement of $\tilde{O}$ in Z for F, where $o_{\omega i}^* = o_{\omega i} \cap F$, $\forall\, i = 1,2, \dots$. Hence, we get the required result.

⇐) Suppose that there is an open locally countable refinement for any difference cover of F by difference sets in Z. Assuming that $\tilde{F} = \{F_\omega : \omega \in \Omega\}$ is a difference cover of F by difference sets in F. Therefore, there is a difference set of O_ω in Z such that $F_\omega = O_\omega \cap F$ for all $\omega \in \Omega$. At this point, we get that $\tilde{O} = \{O_\omega : \omega \in \Omega\}$ is a difference cover of F by difference sets in Z. On this basis of the supposition that $\tilde{O}$ has an open locally countable refinement $\{O_{\omega 1}, O_{\omega 2}, \dots\}$. Since $F_\omega \subseteq O_\omega$, for all $\omega \in \Omega$, $i = 1, 2, \dots$, the family $\{F_{\omega 1}, F_{\omega 2}, \dots\}$ is an open locally countable refinement of $\tilde{F}$ for F. Hence, we get the result that the space (F, ℓ_F) is D −paralindelöf.

Let's look at next corollaries, which every open cover is a D −cover.

Corollary 3.9 *Let (F, ℓ_F) be any D −paralindelöf topological space. Then all open cover of F by open sets in Z has an open locally countable refinement.*

Corollary 3.10 *A topological space (F, ℓ_F) is paralindelöf if every D −cover of F by D −sets in Z has an open locally countable refinement.*

Proof. Let (Z, ℓ) be a paralindelöf topological space. Then by applying the Theorem 3.2.1 that every D −paralindelöf space is a paralindelöf, we get that the second trend from the earlier Theorem 3.8 is the direct cause of this corollary.

Theorem 3.10 *Let (Z, ℓ_1) and (Z, ℓ_2) be any two topological spaces, respectively. If $\ell_1 \subseteq \ell_2$ and (Z, ℓ_2) is D −paralindelöf space, then (Z, ℓ_1) is a D −paralindelöf space.*

Proof. Let (Z, ℓ_2) is D −paralindelöf space and $\ell_1 \subseteq \ell_2$ such that $\tilde{O} = \{O_\omega : \omega \in \Omega\}$ be a difference cover of the topological space (Z, ℓ_1). Since $\ell_1 \subseteq \ell_2$, then we get that $\tilde{O}$ is also a D −cover of the space (Z, ℓ_1). And since (Z, ℓ_2) is D −paralindelöf space, then the difference cover $\tilde{O} = \{O_\omega : \omega \in \Omega\}$ has a refinement that is open locally and countable. Hence, we get the result that (Z, ℓ_1) is D −paralindelöf space.

Theorem 3.11 *Every closed subspace (F, ℓ_F) of a D −paralindelöf topological space (Z, ℓ) is a D −paralindelöf closed subspace.*

Proof. Let (Z, ℓ) be a D −paralindelöf topological space and (F, ℓ_F) be a closed subspace of the space (Z, ℓ). Then $\tilde{O} = \{O_\omega : \omega \in \Omega\}$ is difference cover of F by the difference sets in Z. Since $F \subseteq Z$, then $\tilde{O} \cup \{Z - F\}$ is a D −cover of Z. Since Z is D −paralindelöf space, then the difference cover $\tilde{O} \cup \{Z - F\}$ has an open locally countable refinement $\tilde{O}^* - \{Z - F\}$. Therefore, since $F \subseteq Z$ we get that $\tilde{O}^* - \{Z - F\}$ is also an open locally countable refinement of $\tilde{O}$ for F. Hence, we get the required result that the closed subspace (F, ℓ_F) is D −paralindelöf subspace.

Theorem 3.12 *Every D −paralindelöf space (Z, ℓ) has closed subspaces, each of them is a paralindelöf subspace.*

Proof. Consider F to be closed subset of Z and Z be a D −paralindelöf topological space. Let $\tilde{O} = \{O_\omega : \omega \in \Omega\}$ be an open cover of F by open sets in Z. Then $\tilde{O} - \{Z - F\}$ is an open cover of Z. Since Z is a D −paralindelöf space, then by applying the Theorem 3.2 we get that Z is paralindelöf space such that the open cover $\tilde{O} - \{Z - F\}$ of Z has a locally countable refinement $\tilde{O}^* - \{Z - F\}$. Now, since F is a subset of Z, then the open cover $\tilde{O}$ of F has an open locally countable refinement $\tilde{O}^* - \{Z - F\}$. Hence, we get the result that each closed subspace of paralindelöf space (Z, ℓ) is a paralindelöf subspace.

Theorem 3.13 *If the space (Z, ℓ) is D −separable and D −paralindelöf, then it is a D −lindelöf space.*

Proof. Suppose that (Z, ℓ) is not a D −lindelöf space. Then $\tilde{O} = \{O_\omega : \omega \in \Omega\}$ is difference cover of Z such that $\tilde{O}$ has no countable subcover. However, let $\tilde{P} = \{P_\lambda : \lambda \in \Lambda\}$ be an open uncountably locally refinement of $\tilde{O}$ and F be a countable difference dense subset of Z such that $P_\lambda \cap F \neq \emptyset$, for all $\lambda \in \Lambda$, then F is an uncountable set because $\tilde{P}$ is uncountable, which is a contradiction, and hence we get the result that (Z, ℓ) is a D −lindelöf space.

We can get the following corollary by the same work.

Corollary 3.14 *If the space (Z, ℓ) is D −separable and D −paralindelöf, then it is a lindelöf space.*

Definition 3.15 *If every countable D −cover of a topological space (Z, ℓ) has an open locally countable refinement, then the space (Z, ℓ) is called a countably D −paralindelöf.*

Theorem 3.16 *If the topological space (Z, ℓ) is a D −lindelöf and countably D −paralindelöf, then the space (Z, ℓ) is D −paralindelöf.*

Proof. Let $\tilde{O} = \{O_\omega : \omega \in \Omega\}$ be any difference cover of (Z, ℓ) and let (Z, ℓ) be D −lindelöf and countably D −paralindelöf space. Since (Z, ℓ) is a D −lindelöf space, then the difference cover $\tilde{O}$ has a countable subcover $\tilde{P} = \{P_i\}_{i=1}^{\infty}$. Since (Z, ℓ) is countably D −paralindelöf, then $\tilde{P}$ has an open locally countable refinement of $\tilde{O}$. Hence, we get the result that (Z, ℓ) is a D −paralindelöf space.

By the same work we get the following corollaries.

Corollary 3.17 *If the topological space (Z, ℓ) is a D −lindelöf countably and D −paralindelöf, then the space (Z, ℓ) is a paralindelöf space.*

Corollary 3.18 *If the topological space (Z, ℓ) is a lindelöf countably and D −paralindelöf, then the space (Z, ℓ) is a paralindelöf space.*

Example 3.19 The space (Z, ℓ_{dis}) is countably D −paralindelöf and D −lindelöf, so by applying the Theorem 3.16 we obvious get that (Z, ℓ_{dis}) is a D −paralindelöf topological space.

4. Product of Difference Paralindelöf Spaces

In this section, further of complex properties relating to the concept of mappings and the product of two difference paralindelöf spaces are explaned. As a result, the difference paralindelöf spaces are essential topics for study in areas associated with topology. Their finiteness and tightness give rise to important findings and applications in mathematics by offering a framework for comprehending and interpreting the structure of spaces. several theoretical findings relating to the concept of mappings and the product of two D −paralindelöf spaces will be established.

Theorem 4.1 Let (Z, ℓ) and (S, ρ) be two topological spaces. If $\Psi: Z \longrightarrow S$ is D −perfect function and the space Z is a locally indiscrete, then Z is a D −paralindelöf space, if S is so.

Proof. Let $\Psi: Z \longrightarrow S$ be a D −perfect function such that the space (S, ρ) be a D −paralindelof space and (Z, ℓ) be a locally indiscrete space. Then consider $\tilde{O} = \{O_\omega : \omega \in \Omega\}$ to be a D −cover of Z. Since Ψ is a D −perfect function, then for each $s \in S$, $\Psi^{-1}(s)$ is a D −compact of Z. So, there is a finite subset ρ_s of Ω, in a way that $\Psi^{-1}(s) \subseteq \cup_{\omega \in \rho_s} H_\omega$, and since Z is locally indiscrete, then each difference set O_ω is clopen for all $\omega \in \Omega$. So, the difference cover $\tilde{O}$ is an open cover of Z. Now, let $\tilde{P} = \{P_s : s \in S\}$ be a difference cover of S such that $P_s = S - \Psi\left(Z - \cup_{\omega \in \rho_s} H_\omega\right)$ is difference sets of S and $\Psi^{-1}(P_s) \subseteq \cup_{\omega \in \rho_s} H_\omega$, where $s \in P_s$. Since S is D −paralindelöf, then $\tilde{P}$ has an open locally countable refinement $\tilde{P}^* = \{P_s^*: s \in S\}$, where P_s^* is a difference sets of S. Because Ψ is a D −perfect, so the set $\{\Psi^{-1}(P_s^*): s \in S\}$ is an open locally countable refinement of Z. Hence, we get the result that the space Z is D −paralindelöf.

Theorem 4.2 *Let $\Psi: (Z, \ell) \longrightarrow (S, \rho)$ be a D −perfect function. Then Z is paralindelöf space if S is D −paralindelöf.*

Proof. Similar to the proof of Theorem 4.1, this is theorem also are simple to prove.

Theorem 4.3 *Let (Z, ℓ) and (S, ρ) be two topological spaces. If $\Psi: Z \longrightarrow S$ is D −perfect function, such that Z is a locally indiscrete and S is a countable set, then Z is countably D −paralindelöf space, if S is so.*

Proof. Let Ψ be a D −perfect function such that the space (Z, ℓ) be a locally indiscrete space and (S, ρ) be a countably D −paralindelöf space, where S is a countable set. Then consider $\tilde{O} = \{O_\omega: \omega \in \Omega\}$ be any countable D −cover of Z. Since Ψ is a D −perfect function, then for each $s \in S$, $\Psi^{-1}(s)$ is a D −compact of Z. So, there is a finite subset of Ω, where $\Psi^{-1}(s) \subseteq \cup_{i=1}^{m} H_i$, and since Z is a locally indiscrete, then each difference set O_ω is clopen for all $\omega \in \Omega$. So, the difference cover $\tilde{O}$ is an open cover of Z and H_i is a difference open of Z, $\forall\, i \in \Omega$. Now, let $\tilde{P} = \{P_s: s \in S\}$ be a countable difference cover of S such that $P_s = S - \Psi(Z - \cup_{i=1}^{m} H_i)$ is countable difference sets of S and $\psi^{-1}(P_s) \subseteq \cup_{i=1}^{m} H_i$, where $s \in P_s$. Also, since S is a countably difference paralindelöf, then the countable difference cover $\tilde{P}$ has an open locally countable refinement $\tilde{P}^* = \{P_{s1}, P_{s2}, \ldots\}$, where P_s^* is a difference sets of S, and since Ψ is a D −perfect function, then the set $\{\Psi^{-1}(P_s^*): s \in S\}$ is an open locally countable refinement of Z. Hence, we get the result that the space Z is a countably D −paralindelöf.

During this study, we get the following new theory that related to paralindelöf spaces.

Theorem 4.4 *Let (Z, ℓ) and (S, ρ) be two topological spaces and the function $\Psi: (Z, \ell) \longrightarrow (S, \rho)$ be a D −perfect. If (S, ρ) is a paralindelof space, then (Z, ℓ) is so.*

Proof. Let Ψ be a D −perfect function and the space (S, ρ) be a paralindelöf. Then consider $\tilde{O} = \{O_\omega: \omega \in \Omega\}$ be any open cover of (Z, ℓ), so by applying the theorem 2.29 we get the fact that the open cover $\tilde{O}$ is a difference cover of Z. Since Ψ is a D −perfect function, then for each $s \in S$, $\Psi^{-1}(s)$ is a D −compact of Z. So, there is a finite subset of Ω, where $\Psi^{-1}(s) \subseteq \cup_{k=1}^{m} \{O_k: k \in \Omega\}$, and O_k is a difference open of Z, $\forall\, k \in \Omega$. Now, let $\tilde{T} = \{T_s: s \in S\}$ be an open cover of S such that $T_s = S - \Psi(Z - \cup_{k=1}^{m} O_k: k \in \Omega)$ is an open set of S and $\psi^{-1}(T_s) \subseteq \cup_{k=1}^{m} O_k$, where $s \in T_s$. Also, since S is a paralindelöf space, then the open cover $\tilde{T}$ has an open locally countable refinement

$\tilde{B}^* = \{B_\lambda: \lambda \in \Lambda\}$, where B_λ^* is an open set of the space S. Let $F = \{\Psi^{-1}(B_\lambda)O_k: \lambda \in \Lambda, k \in \Omega\}$, then the set F is an open locally countable refinement of the open cover $\tilde{O}$. Hence, we get the result that the space Z is a paralindelöf.

Theorem 4.5 Let (Z, ℓ) and (S, ρ) be two topological spaces such that the space Z is a D −compact and the space S is a D −paralindelöf. Then $Z \times S$ is a D −paralindelöf space.

Proof. By using the fact that the projection function $J: Z \times S \longrightarrow S$ is continuous and $J^{-1}\{s\} \cong Z$ is D −compact, for all $s \in S$, then $J: Z \times S \longrightarrow S$ is a D −perfect function. Now, since the space S is D −paralindelöf, then $Z \times S$ is also D −paralindelöf space.

Corollary 4.6 *Let (Z, ℓ) and (S, ρ) be two topological spaces such that the space Z is a compact and the space S is a paralindelöf. Then $Z \times S$ is a paralindelöf space. i.e., The product between compact space and paralindelöf space is a paralindelöf.*

Example 4.7 A topological space $(\mathbb{R}, \ell_{cof})$ is both compact and paralindelöf space, so the cartesian product between them $(\mathbb{R} \times \mathbb{R}, \ell_{cof} \times \ell_{cof})$ is also a paralindelöf space.

Theorem 4.8 *Let (Z, ℓ) and (S, ρ) be two topological spaces. Let $\Psi: (Z, \ell) \longrightarrow (S, \rho)$ be a continuous, closed, and onto function such that S is a locally indiscrete space. Then, the space S is a D −paralindelöf if Z is so.*

Proof. Let $\Psi: (Z, \ell) \longrightarrow (S, \rho)$ be a continuous, closed, and onto function such that S is a locally indiscrete space and Z is D −paralindelöf space. Then consider $\tilde{O} = \{O_\omega: \omega \in \Omega\}$ be a D −cover of S. Since S is locally indiscrete space, then each difference set O_ω is clopen for all $\omega \in \Omega$. So, the difference cover $\tilde{O}$ is an open cover of S .Also, since Ψ is continuous, closed, and onto function, then also the set $\tilde{O} = \{\Psi^{-1}(O_\omega): \omega \in \Omega\}$ is an open cover of Z. Now, since Z is a D −paralindelöf space, then there exists an open locally countable refinement $\tilde{O}^* = \{\Psi^{-1}(O_\omega^*): \omega \in \Omega\}$ of $\tilde{O}$. Hence, we get the result that the space S is D −paralindelöf.

With the same strategy we get the following corollary.

Corollary 4.8 *Let (Z, ℓ) and (S, ρ) be two topological spaces. Let $\Psi: (Z, \ell) \longrightarrow (S, \rho)$ be a continuous, closed, and onto function such that S is a locally indiscrete space. Then, the space S is a D −paralindelöf if Z is paralindelöf.*

5. Conclusion and Future Works

In this section, the conclusion of the main achievements that presented in both Section 3 and Section 4 are presented. Furthermore, a few future studies that are connected to our study of difference paralindelöf in topological spaces are proposed.

5.1. Main Achievement

This research embarks on an exploration of paralindelöfness, which serves as the basis for defining a distinctive class of covering properties known as difference paralindelöf spaces within topological contexts. This generalization brought to light a new concept that uses difference sets and paralindelöf spaces to represent the D −paralindelöf spaces that have a certificate cover type called a difference cover with an open locally countable refinement.

This research main goal is to clarify the notion of different paralindelöf spaces. By delving into their inherent properties and intricate relationships with other topological spaces, the research aims to shed light on their characteristics. The research involves the examination of various examples to illustrate the concepts introduced.

Furthermore, the research extends its scope by generalizing well-established theorems, thereby contributing to the expanding body of knowledge surrounding difference paralindelöf spaces. Also, this research enriches our understanding of these spaces and their connections, enhancing the broader field of topology and its implications for mathematical exploration.

Moreover, some preliminary and basic notions were provided related to the notion of difference paralindelöf spaces, such as paralindelöf spaces, difference sets, paracompact spaces, countably paralindelöf spaces, etc. Also, some of their characteristics and illustrated figures were presented in section 2 Furthermore, in section 3, we provided the definition and established the figure form of the notion of difference paralindelöf in topological spaces, which supported us in illustrating and understanding the main notions and properties. After all, we introduced the flow chart 3.1, which represented the relationship of difference paralindelöf spaces with other topological spaces. Our study has used some of the basic properties of topological structures as functions and the Cartesian product to introduce new properties and results of the notion of difference paralindelöf spaces.

5.2. Future Works

Developments in the study of paralindelöf spaces are still occurring. Considering their specific characteristics, connections to other topological concepts, and applications in other mathematical contexts, there are still unanswered questions and guesses. With possible applications in many mathematical fields, more investigation into these spaces should provide a deeper understanding of the properties and structure of topological spaces.

The studies of the difference of paralindelöf spaces often result in the development of new obstacles and the investigation of novel research avenues. By participating in the current study about the characterization, categorization, and features of D −paralindelöf spaces, we make a significant contribution to the direction of topological and related research. The study of D −paralindelöf spaces lies at the interface of topology and other branches of mathematics. Such as in fuzzy sets, researchers can generalize the fuzzy paralindelöf spaces to be fuzzy D −paralindelöf spaces. Also, in algebra field, there is a compact (topological) group, which we can generalize to a D −paralindelöf group and provide mathematicians with a way to expand their knowledge about both topological and group algebra. This also acts as an effective basis for future research.

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