Nonlinear Distribution of Neashore Free Surface and Velocity

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ABSTRACT

The Edgeworth expansion with the measured skewness and kurtosis is shown to be capable to describe the measured nonlinear distributions of the surface elevation and horizontal velocity in the shoaling and surf zones. The moments involved in the energetics-based model are expressed in terms of the skewness and kurtosis and shown to be in agreement with available data. Stokes wave theory is applied to obtain a relationship between skewness and kurtosis. The relationship is adjusted empirically because of the limitation of Stokes wave theory in the shoaling and surf zones. The relative simple relationship for the higher moments obtained here may be applied for cross-shore sediment transport analysis.

INTRODUCTION

Nonlinear waves in nearshore regions are important in estimating sediment transport and designing coastal structures. The estimation of an extreme wave crest is an important factor in predicting the deck elevation of an offshore structure. The wave profile in the surf zone is significantly skewed because of wave breaking and bottom topography effects. Time dependent numerical models based on extended Boussinesq equations have been shown to be capable of predicting the nonlinear profile of the surface elevation outside the surf zone(*e.g.*, Nwogu 1993). Time dependent numerical models, however, require large computation time to calculate the wave profiles.

On the other hand, time-averaged models for random waves are more efficient computationally at the expense of the loss of the detailed temporal information(*e.g.*, Battjes and Janssen 1978; Thornton and Guza 1983; Mase and

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Kobayashi 1991). However, time-averaged models may not be accurate enough, because random wave are expressed as the superposition of regular waves or by a representative wave. Guza and Thornton (1985) has pointed out that both randomness and nonlinearity are necessary to predict the moments of the cross-shore fluid velocity on a beach. Hence, the purpose of this study is to develop a probabilistic model of the surface elevation and cross-shore velocity in the nearshore including both random and nonlinear effects.

Many studies have were performed to describe the nonlinear distribution of the free surface elevation. Ochi and Ahn (1994) and Kobayashi et al. (1998) used Siegert solution and the exponential gamma distribution, respectively, to describe the distribution of the surface elevation and velocity. These distributions agree fairly with the measured skewed distributions. However, it is difficult to obtain the relationships among the various moments of the distribution analytically. In contrast, attempts were made to describe the nonlinear distribution of the free surface elevation using perturbations of the Gaussian distribution(Longuet-Higgins 1963; Haung and Long 1980; Bitner 1980). These methods based the Gram-Charlier or Edgeworth approximation can be expanded as a function of skewness and kurtosis. Then, it is much easier express the various moments in terms of the skewness and kurtosis, although these approximations give negative density values for certain skewed distributions.

In the following, the probability density distributions and moments of the measured free surface elevation and horizontal velocity in a large wave flume are compared with the Edgeworth expansion and the measured skewness and kurtosis. Stokes wave theory is applied to derive a relationship between the skewness and kurtosis.

EXPERIMENTS

The experimental data used here was reported in Japanese by Shimizu *et al.*(1996). The experiment was conducted in a large wave flume that was 205m long, 3.4m wide, 6m high. The water depth in the flume was 4m. A sand beach of a 1:30 slope was installed at the end of the wave tank. Water surface displacements at 17 locations in Fig.1 were measured using capacitance type wave gages located in the still water depth range of 0.1-4.0m. Fluid velocities were measured with six electro-magnetic current meters. The current meters were set 15cm above the bottom. The current meters $C_1 - C_6$ in Fig.1 were located in the still water depths 2.25, 1.92, 1.58, 1.25, 0.92 and 0.48m, respectively. The measurements with a sampling frequency of 20Hz were performed for the duration of 819s.

Random waves based on the JONSWAP spectrum and random phases were generated using linear wave theory with a computer-controlled piston-type wave paddle. Two cases were reported by Shimizu *et al.*(1996) as summarized in Table 1 where $H_{1/3}$ and $T_{1/3}$ are the significant wave height and period above the horizontal bottom. In Table 1, the wave amplitude $a_0=H_{1/3}/2$, the water

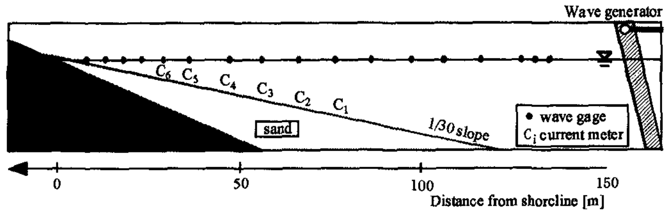


Figure 1 Experimental setup and locations of wave gages and current meters.

Table 1 Experimental conditions for two cases.

case No.	$H_{1/3}$	$T_{1/3}$	ka_0	kh_0
case1	0.43m	4.93s	0.049	0.91
case2	1.12m	3.06s	0.262	1.87

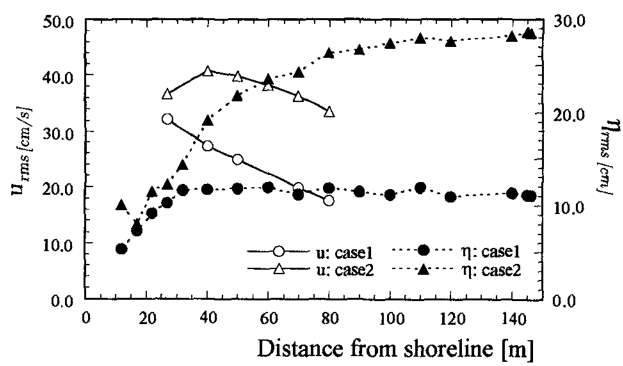


Figure 2 Measured cross-shore variations of *rms* values of horizontal fluid velocity u and surface elevation η.

depth h_0=4.0m on the horizontal bottom, and k is the linear wave number based on $T_{1/3}$ and h_0. The value of ka_0 and kh_0 in Table 1 indicate wave steepness and nondimensional water depth at the wave paddle. Case 2 was more nonlinear than case 1.

The surface displacement η and the horizontal fluid velocity u near the bottom are analyzed in the following. Fig.2 shows the spatial distributions of the root-mean-square(*rms*) values of the horizontal fluid velocity u and the surface elevation η. Solid lines with open symbols indicate the *rms* values of u and dashed lines with filled symbols indicate the *rms* values of η. Circles ○ are for case 1 and triangles $\triangle$ are for case 2. Fig.2 illustrates the difference in the width of the surf zone for case 1 and 2. The rms values of u increased landward, whereas the rms values of η decreased monotonically landward.

The moments μ_3, μ_4, μ_3^* and $\mu_4^\bullet$ for $f{=}\eta$ or u are defined as follows:

$$\mu_n = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{f_i-\bar{f}}{f_{rms}}\right)^n \tag{1}$$

$$\mu_n^* = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{f_i-\bar{f}}{f_{rms}}\right|^n \tag{2}$$

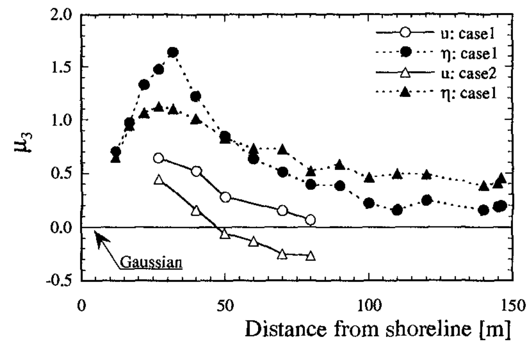
$$\mu_n^\bullet = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{f_i-\bar{f}}{f_{rms}}\right)\left|\frac{f_i-\bar{f}}{f_{rms}}\right|^{n-1} \tag{3}$$

where $\bar{f}$ is the mean value of f, f_{rms} is the rms value of f, and N is the number of data point. Eq.(1) with $n{=}3$ and 4 gives μ_3=skewness and μ_4=kurtosis. The moments are calculated after removing the high-frequency components with frequency larger than 4 times of the peak frequency to reduce the statistical sensitivity. The third order absolute moment μ_3^* and the fourth order signed moment $\mu_4^\bullet$ for the velocity are related to the energetics-based sediment transport model(Bailard 1981). The measured third and fourth moments are shown in Fig.3. The measured moments of η show systematic trends. The values of the moments of η are close to the Gaussian values, $\mu_3{=}0$, $\mu_4{=}3$, $\mu_3^*{=}1.6$ and $\mu_4^\bullet{=}0$, offshore. These values then increase landward before their decrease toward the shoreline. On the contrary, the measured moments of u are below the values of the Gaussian offshore. μ_4 and μ_3^* are reduced first and then increase landward, whereas μ_3 and $\mu_4^\bullet$ increase monotonically.

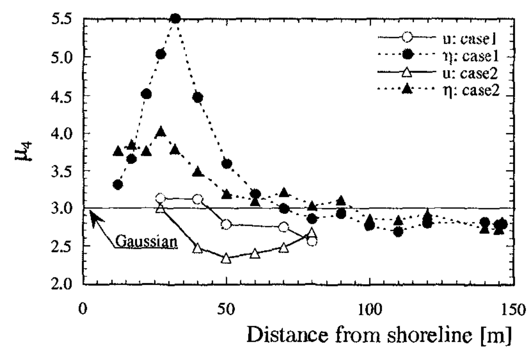
In summary, the spatial variations of the moments of the horizontal fluid velocity u and the surface elevation η deviate from the Gaussian values near and inside the surf zone. The measured moments of u are close to the Gaussian values but the moments of η exceed the Gaussian values significantly inside the surf zone. These results are consistent with the small scale tests on a 1:16 slope by Kobayashi et al. (1998).

STATISTICAL MODELING OF SURFACE AND VELOCITY MOMENTS

The data shown in Fig.3 indicates strong nonlinearity of η in the shoaling and surf zones. The horizontal velocities near the bottom have weaker nonlinearity. Hence, the Edgeworth expansion(Kendall and A.Stuart 1963) is applied to describe the probability density function(PDF) of the surface elevation and horizontal velocity. The Edgeworth expansion is adequate to describe random and weak nonlinear stochastic processes. The Edgeworth expansion is derived in the same manner as the Gram-Charlier expansion but the terms of the series are expressed in terms of cumulants.



(a) Measured cross-shore variations of skewness μ_3



(b) Measured cross-shore variations of kurtosis μ_4

Figure 3 Measured cross-shore variations of third and fourth of moments of horizontal fluid velocity u and surface elevation η.

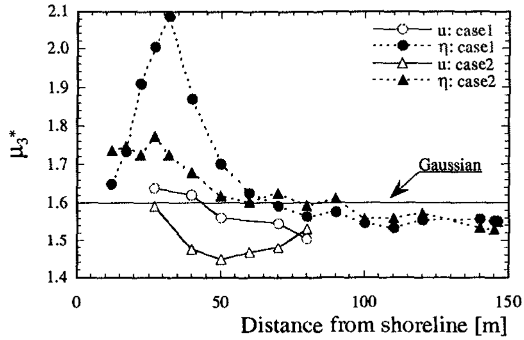
The PDF $p(x)dx$ of the normalized statistical variables x with zero mean and its standard deviation of unity can be described as(Kendall and A.Stuart 1963)

$$p(x)dx = \sum_{r=0}^{\infty} c_r H_r(x) G(x)\, dx \tag{4}$$

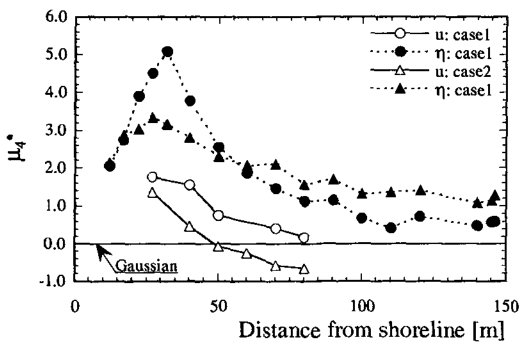
with

$$G(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) \tag{5}$$

where $G(x)$ is the Gaussian distribution, $H_r(x)$ is the Chebyshev-Hermite poly-



(c) Measured cross-shore variations of third order absolute moment μ_3^*



(d) Measured cross-shore variations of fourth order signed moment $\mu_4^\bullet$

Figure 3 Continued

nomial and c_r is the rth order coefficient of the Gram-Charlier expansion. Introducing the characteristic function, the Edgeworth expansion of type A is given by (*e.g.*, Longuet-Higgins 1963)

$$p(x)dx = G(x)\left\{1 + \frac{\kappa_3}{6}H_3(x) + \left[\frac{\kappa_4}{24}H_4(x) + \frac{\kappa_3^2}{72}H_6(x)\right] + \left[\frac{\kappa_5}{120}H_5(x) + \frac{\kappa_3\kappa_4}{144}H_7(x)\right] + \cdots\right\}dx \tag{6}$$

where κ_r is the rth order cumulant. The cumulants with r=3-6 are related to the rth order moments μ_r as follows:

$$\left.\begin{aligned} \kappa_3 &= \mu_3 \\ \kappa_4 &= \mu_4 - 3 \\ \kappa_5 &= \mu_5 - 10\mu_3 \\ \kappa_6 &= \mu_6 - 15\mu_4 - 10\mu_3^2 + 30 \end{aligned}\right\} \tag{7}$$

where the mean value μ_1 of x is equal to zero(κ_1=0) and the standard deviation of x is unity(κ_2=1). Therefore, μ_3 is skewness and μ_4 is kurtosis.

It must be noted that an asymptotic expansion does not have monotonic convergence for high order corrections. There are many studies about the truncation of (6)(Longuet-Higgins 1963; Haung and Long 1980; Ochi and Wang 1984; Mori and Yasuda 1996). They have shown that the first three terms of (6) are sufficient for describing the nonlinear property of the PDF of the surface elevation. Hence, the first three terms in (6) are used to represent the PDF of x denoting the normalized the surface elevation and horizontal velocity

$$p(x)dx = G(x)\left\{1 + \frac{\kappa_3}{6}H_3(x) + \left[\frac{\kappa_4}{24}H_4(x) + \frac{\kappa_3^2}{72}H_6(x)\right]\right\}dx \tag{8}$$

which requires κ_3=μ_3=skewness and κ_4=(μ_4-3) with μ_4=kurtosis . The truncation of high order terms in (6) gives the following relationships based on κ_5=0 and κ_6=0 for the high order moments:

$$\mu_5 = 10\mu_3 \tag{9}$$

$$\mu_6 = 10\mu_3^2 + 15\mu_4 - 30 \tag{10}$$

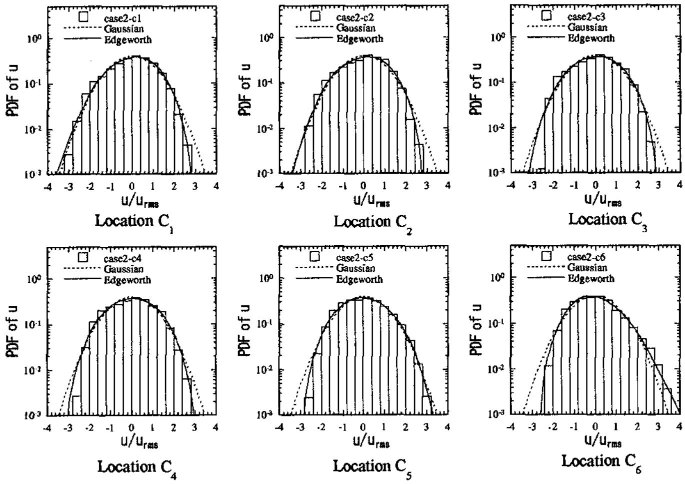
The other higher moments related to the energetics-based sediment transport model(Bailard 1981) can be calculated using (8)

$$\mu_3^* = \int_{-\infty}^{\infty} |x|^3 p(x)dx = \frac{1}{6\sqrt{2\pi}}(3\mu_4 - \mu_3^2 + 15) \tag{11}$$

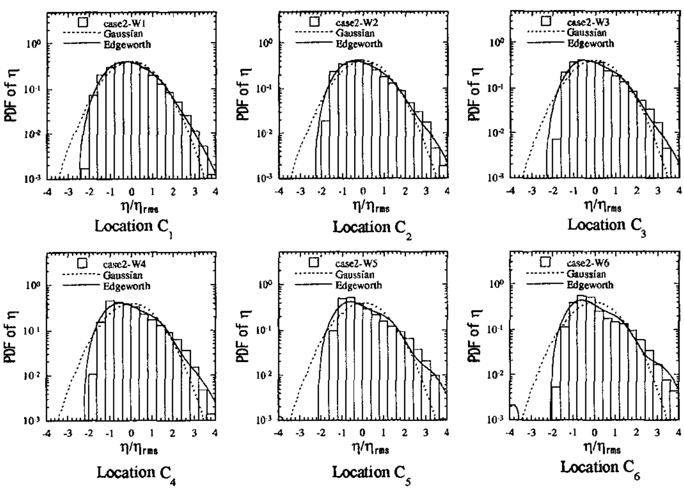
$$\mu_5^* = \int_{-\infty}^{\infty} |x|^5 p(x)dx = \frac{2}{3\sqrt{2\pi}}(15\mu_4 + 5\mu_3^2 - 21) \tag{12}$$

$$\mu_4^\bullet = \int_{-\infty}^{\infty} x|x|^3 p(x)dx = \frac{8}{\sqrt{2\pi}}\mu_3 \tag{13}$$

Fig.4 show the comparisons of the measured PDF of u and η with (8) for case 2 where the measured values of μ_3 and μ_4 are used in (8). The PDF of u is skewed negatively offshore but positively in the surf zone. The measured spatial variation of μ_3 shown in Fig.3a indicates the corresponding sign shift. This may be important for the cross-shore sediment transport. On the other hand, the PDF of η is always skewed positively as expected from nonlinear wave theory. The nonlinear PDF given by (8) agrees better with the data than the linear Gaussian distribution partly because of the additional input of μ_3 and μ_4.

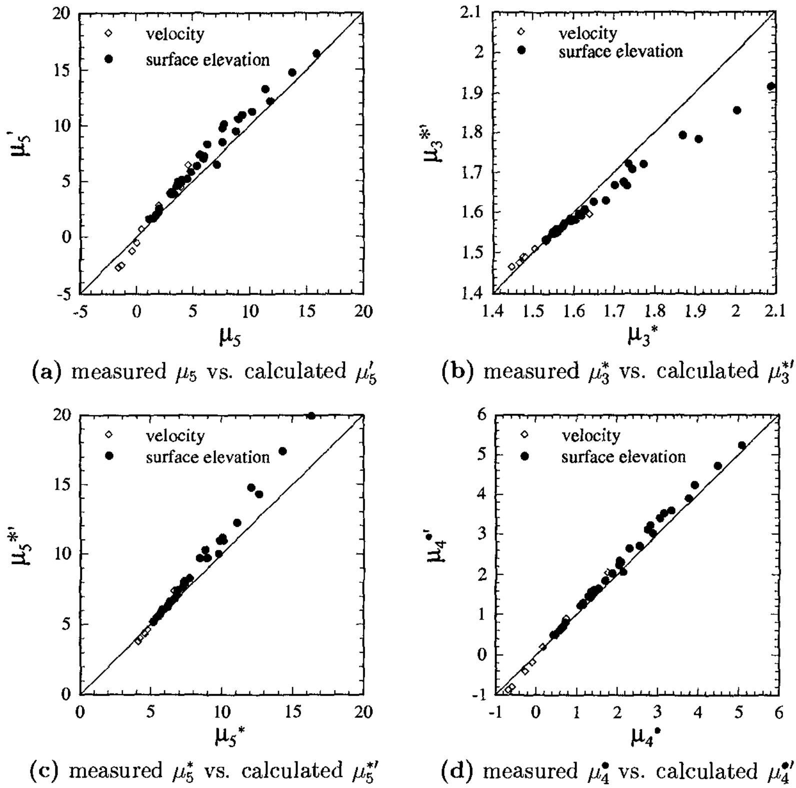


(a) PDF of horizontal velocity u.



(b) PDF of surface elevation η.

Figure 4 Measured PDF of u and η for case 2 in comparison with the theory.



(a) measured μ_5 vs. calculated μ_5' (b) measured μ_3^* vs. calculated $\mu_3^{*\prime}$

(c) measured μ_5^* vs. calculated $\mu_5^{*\prime}$ (d) measured $\mu_4^\bullet$ vs. calculated $\mu_4^{\bullet\prime}$

Figure 5 Comparison between measured μ_5, μ_3^*, μ_5^* and $\mu_4^\bullet$, and calculated μ_5', $\mu_3^{*\prime}$, $\mu_5^{*\prime}$ and $\mu_4^{\bullet\prime}$ for measured μ_3 and μ_4.

To check the validity of (9)-(13), the comparisons between the measured moments μ_n and the calculated moments μ_n' indicated by the prime are shown in Fig.5. Eq.(9) gives a simple relationship between μ_3 and μ_5. The correlation coefficient of in Fig.5a is 0.98, indicating good agreement between the measured μ_5 and the calculated μ_5' . The sixth moments μ_6 given by (10) for u and η show similar results(not shown). Fig.5b-d also show good agreement between the measured moments(μ_3^*, μ_5^* and $\mu_4^\bullet$) and the calculated moments($\mu_3^{*\prime}$, $\mu_5^{*\prime}$ and $\mu_4^{\bullet\prime}$) using (11)-(13). The PDF of u and η in the shoaling and surf zones can be described by the Edgeworth expansion given by (8). As a consequence, the associated moments can be estimated by (9)-(13).

Eq.(8) requires both skewness μ_3 and kurtosis μ_4 as input. An attempt is made to express μ_4 in terms of μ_3.

RELATIONSHIP BETWEEN SKEWNESS AND KURTOSIS

Tayfun (1980), Srokosz and Longuet-Higgins (1986) and Winterstein et al. (1991) derived the PDF of the surface elevation based on the assumption that random waves can be expressed as a summation of Stokes 2nd or 3rd waves. This assumption considers only the self wave interaction components, although there are random wave-wave interaction components. This approach is easy to apply and calculate the moments in comparison with the fully nonlinear random interaction method(*e.g.*, Sharma and Dean 1979). Admittedly, Stokes wave theory may not be valid in shallow water and the derived relationship will be interpreted in view of this limitation. The 3rd order Stokes wave in finite water depth h is given by:

$$k\eta = \frac{1}{2}(ak)^2 D_1 + ak\cos\theta + \frac{1}{2}(ak)^2 D_2 \cos 2\theta + \frac{3}{8}(ak)^3 D_3 \cos 3\theta \tag{14}$$

where a is the amplitude, k is the wave number, θ is the phase with $\theta{=}(kx - \omega t + \varepsilon)$ in which ω is the angular frequency, and the phase ε is assumed to random. Eq.(14) neglects the phase shift of the second and third harmonics which may be important for the wave profile pitched landward in the surf zone. D_i is the function of the nondimensional water depth kh:

$$\begin{aligned} D_1 &= \coth kh \\ D_2 &= \coth kh \left(1 + \frac{3}{2\sinh^2 kh}\right) \\ D_3 &= 1 + \frac{1}{\sinh^2 kh}\left(3 + \frac{3}{\sinh^2 kh} + \frac{9}{8\sinh^4 kh}\right) \end{aligned} \tag{15}$$

It is assumed that the PDF of the first order component $a\cos\theta$ is the Gaussian and that the amplitude a and the phase ε are independent of each other with ε being distributed uniformally between 0 to 2π. These assumptions yields the joint PDF of a and θ as

$$f(a,\theta) = \frac{a}{2\pi\sigma^2}\exp\left(-\frac{a^2}{2\sigma^2}\right), \tag{16}$$

where σ is the *rms* value of the first order component($a\cos\theta$). The moments λ with $n{=}0, 1, \cdots$ can be calculated using (14) and (16):

$$\lambda_n = \int_{a=0}^{\infty}\int_0^{2\pi} [\eta(a,\theta)]^n f(a,\theta)\, da\, d\theta \tag{17}$$

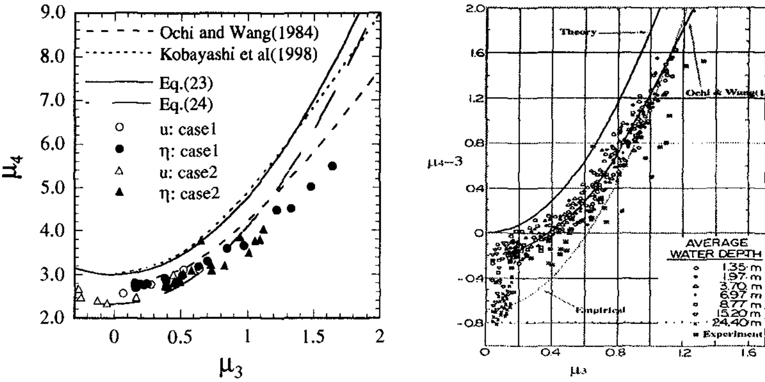
which yields

$$\lambda_0 = \sigma D_1 \alpha \tag{18}$$

$$\lambda_2 = \sigma^2\left[1 + (D_1^2 + D_2^2)\alpha^2 + O(\alpha^4)\right] \tag{19}$$

$$\lambda_3 = \sigma^3\left[3(D_1 + D_2)\alpha + \frac{1}{2}(4D_1^3 + 12D_1D_2^2 + 27D_2D_3)\alpha^3 + O(\alpha^4)\right] \tag{20}$$

$$\lambda_4 = \sigma^4\left[3 + 3(6D_1^2 + 6D_2^2 + 8D_1D_2 + 3D_3)\alpha^2 + O(\alpha^4)\right] \tag{21}$$



(a) Comparison with laboratory data

(b) Comparison with field data[Ochi and Wang, 1984]

Figure 6 Measured and predicted μ_4 as a function of measured μ_3.

where α is the wave steepness ak of the linear component. Dividing λ_3 by $\lambda_2^{3/2}$, the leading skewness contribution is given by

$$\mu_3 = 3\alpha(D_1 + D_2) \tag{22}$$

The low frequency component involving D_1 in (14) was retained by Vinje (1989) but was neglected by Tayfun (1980) and Winterstein *et al.*(1991). Dividing λ_4 by λ_2^2, neglecting D_1 and D_3 and substituting (22), the leading kurtosis contribution obtained

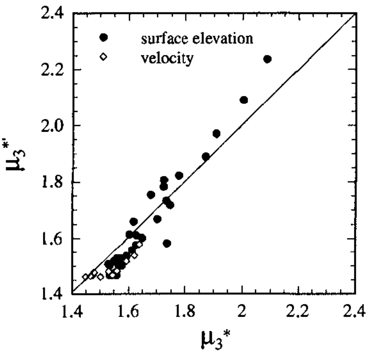
$$\mu_4 = 3 + \left(\frac{4}{3}\mu_3\right)^2 \tag{23}$$

The effect of the water depth D_2 on μ_4 is included in (23) through μ_3 given by (22) with D_1=1.

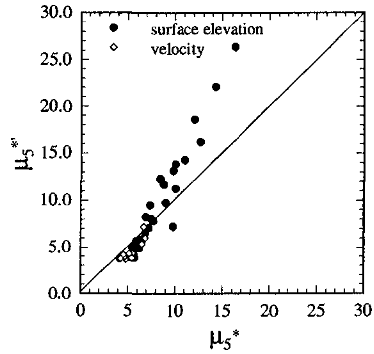
To examine the validity of (23), Fig.6a shows the comparison between the measured and calculated μ_4 as a function of the measured μ_3. Eq.(23)(solid line) follows the trend of the data point but overpredicts the kurtosis μ_4. The measured μ_4 corresponding to $\mu_3 \simeq 0$ is smaller than 3. It means that for zero skewness the kurtosis is smaller kurtosis than linear random wave theory. The field data of Ochi and Wang (1984) also showed similar tendency. Therefore, (23) is adjusted empirically as follows:

$$\mu_4 \;=\; \beta + \left(\frac{4}{3}\mu_3\right)^2 \tag{24}$$

where β is the empirical constant. Eq.(24) reduces to (23) for β=3. Eq.(24) with β=2.3 is shown as a dashed line in Fig.6. The value of β=2.3 is determined by a



(a) measured μ_3^* vs. calculated $\mu_3^{*\prime}$



(b) measured μ_5^* vs. calculated $\mu_5^{*\prime}$

Figure 7 Comparisons between measured μ_3^* and μ_5^* and calculated $\mu_3^{*\prime}$ and $\mu_5^{*\prime}$ for measured μ_3.

least-square method using the laboratory data in Fig.6a. Additional two lines are plotted in Fig.6a. One is derived by Kobayashi *et al.*(1998) using the exponential gamma distribution and another is the empirical relationship obtained by Ochi and Wang (1984) using extensive field data on the free surface elevation. The exponential gamma distribution tend to overpredict μ_4 and is almost the same as (23). The empirical equation (24) and that by Ochi and Wang (1984) give better agreement with the data. Fig.6b shows the comparisons among (23), (24), the laboratory data, the empirical relation and the field data by Ochi and Wang (1984). Eq.(24) with the measured μ_3 underpredicts the field data but (23) overestimates the field data.

Substituting (24) into (11) and (12) gives μ_3^* and μ_5^* as a function of μ_3 only:

$$\mu_3^* = \frac{1}{18\sqrt{2\pi}}\left[13\mu_3^2 + 9(\beta + 5)\right], \tag{25}$$

$$\mu_5^* = \frac{2}{9\sqrt{2\pi}}\left[95\mu_3^2 + 9(5\beta - 7)\right]. \tag{26}$$

The measured μ_3^* and μ_5^* are compared with (25) and (26) in Fig.7. The predicted μ_3^* and μ_5^* as a function of the measured μ_3 are reasonable and the agreement is similar to Fig.5b and 5c.

COMPARISON WITH FIELD DATA

Finally, (9), (11) and (12) using the measured μ_3 and μ_4 are compared with the field data on cross-shore(c) and longshore(l) velocities measured by Guza and Thornton (1985) at Torreys Pines Beach, San Diego, California, during

November 1978. The observed moments in Table 2 are spatially-averaged values. Table 2 also includes the predicted moments using (23), (25) and (26) with β=3 for the measured μ_3 only. Guza and Thornton (1985) analyzed their velocity data to estimate the sediment transport rates using the energetics model by Bailard (1981).

For the cross-shore velocities, the calculated moments μ_5, μ_3^* and μ_5^* using the measured μ_3 and μ_4 are in good agreement with the data on Nov.20th but the agreement is worse for Nov.17th. This is due to the unexpected combination of μ_3 and μ_4 for the Nov.17th data(high skewness and low kurtosis). For the alongshore velocities, the measured skewness was almost zero and the Gaussian distribution with μ_3=0 appears to be acceptable except for μ_5^*.

Table 2 Comparison between velocity moments field data by Guza and Thornton(1985) and theory with β=3.

	Data		Model				
moment	Nov.17th	Nov.20th	Nov.17th		Nov.20th		Gaussian
			μ_3&μ_4	μ_3	μ_3&μ_4	μ_3	μ_3=0
μ_3-c	0.55	0.50	–	–	–	–	0
μ_3-l	-0.04	0.01	–	–	–	–	0
μ_4-c	2.86	3.50	–	3.54	–	3.44	3.0
μ_4-l	3.41	3.44	–	3.00	–	3.00	3.0
μ_5-c	4.95	5.39	5.5	5.5	5.0	5.0	0
μ_5-l	-0.05	-0.52	-0.4	-0.4	0.1	0.1	0
μ_3^*-c	1.60	1.69	1.55	1.68	1.68	1.67	1.6
μ_3^*-l	1.68	1.67	1.68	1.60	1.68	1.60	1.6
μ_5^*-c	7.77	8.58	6.23	8.93	8.71	8.49	6.38
μ_5^*-l	8.06	8.56	8.02	6.40	8.14	6.38	6.38

c for cross-shore velocities; l for longshore velocities

CONCLUSION

Nonlinear wave statistics of irregular waves are examined in the shoaling and surf zones on a beach. First, the probability density function of the surface elevation and velocity can be represented by the Edgeworth expansion. Second, the analytical relationships among the order odd and even moments involved in the energetics-based sediment transport model are derived and verified using laboratory and field data. Third, semi-empirical relationship between the skewness and kurtosis is proposed to facilitate future applications.

REFERENCES

Bailard, J. A. (1981). An energetics total load sedment transport model for a plane sloping beach. *J. Geophys. Res., 86*(C11), 10938–10954.

Battjes, J. and J. Janssen (1978). Energy loss and set-up due to breaking wave. In *Proc., 16th Int. Conf. on Coastal Eng.*, Volume 1, pp. 596–587. ASCE.

Bitner, E. (1980). Non-linear effects of the statistical model of shallow-water wind waves. *Applied Ocean Res., 2*(2), 63–73.

Guza, R. and E. Thornton (1985). Velocity moments in nearshore. *J. Waterway, Port, Coast. and Ocean Eng., 111*(2), 235–256.

Haung, N. and S. Long (1980). An experimantal study of the surface elevation probaibility distribution and statistics of wind-generated waves. *J. Fluid Mech., 101*, 179–200.

Kendall, M. and A.Stuart (1963). *The advanced theory of statistics* (4th ed.)., Chapter 3.12-15 and 6.17-20. London: Charles Griffin.

Kobayashi, N., M. N. Herrmann, B. D. Johnson, and M. D. Orzech (1998). Probability distribution of surface elevation in surf and swash zones. *J. Waterway, Port, Coast. and Ocean Eng., 124*(3), 99–107.

Longuet-Higgins, M. (1963). The effect of non-linearities on statistical distirbutions in the theory of sea waves. *J. Fluid Mech., 17*, 459–480.

Mase, H. and N. Kobayashi (1991). Transformation of random breaking waves and its empirical numerical model considering surf beat. In *Proc., Coastal Sediments'91*, Volume 1, pp. 688–702. ASCE.

Mori, N. and T. Yasuda (1996). Weakly non-gaussian model of wave height distribution for random waves. In *Proc., 25th Int. Conf. on Coastal Eng.*, Volume 1, Orlando, pp. 412–426. ASCE.

Nwogu, O. (1993). An alternative form of the Boussinesq equations for nearshore wave propagation. *J. Waterway, Port, Coast. and Ocean Eng., 119*(6), 618–638.

Ochi, M. and K. Ahn (1994). Non-Gaussian probability distribution of coastal waves. In *Proc., 24th Int. Conf. on Coastal Eng.*, Volume 1, pp. 482–496. ASCE.

Ochi, M. and W. C. Wang (1984). Non-Gaussian characteristics of coastal waves. In *Proc., 19th Int. Conf. on Coastal Eng.*, Volume 1, pp. 516–531. ASCE.

Sharma, J. and R. Dean (1979). Developement and evaluation of a procedure for simulating a random derectional second order sea surface and associated wave forces. Technical Report 20, Dept. of Civil Eng., Univ. of Delaware, Newark, DE.

Shimizu, T. *et al.* (1996). Simutaneous measurements of wave field above sand beach in large wave flume. In *Proc., 43th Coastal Eng. Conf. in Japan*, Volume 1, pp. 491–495(in Japanese).

Srokosz, M. and M. Longuet-Higgins (1986). On the skewness of sea-surface elevation. *J. Fluid Mech., 164*, 487–497.

Tayfun, M. (1980). Narrow band nonlinear sea waves. *J. Geophys. Res., 85*(C3), 1548–1552.

Thornton, E. and R. Guza (1983). Energy saturation and phase speeds measured on a natural beach. *J. Geophys. Res., 87*(C12), 9499–9508.

Vinje, T. (1989). The statistical distribution of wave heights in a random seaway. *Appl. Ocean Research, 11(3)*, 143–152.

Winterstein, S., E. Bitner-Gregersen, and K. Ronold (1991). Statistical and physical models of nonlinear random waves. In *Proc., OMAE*, Volume 2, pp. 23–31. ASME.