CHAPTER 78

Offshore Sand Waves

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Abstract

A theoretical model for the equilibrium dimensions of offshore sand waves is presented. The model is an adaptation of the dune model by FredsØe (1982) to the marine environment, making use of the physical analogies between offshore sand waves and river dunes. The predicted sand wave heights and lengths conform well with the observations of sand wave formation reported in the literature. One of the results from the model is that for a given wave climate sand waves will only be formed under a certain range of mean current velocities. This range becomes narrower for increasing wave height or decreasing water depth and grain size.

1. Introduction

A sea bed on which active sediment transport is taking place will normally not remain plane. Different types of bed forms will be formed depending on the hydrodynamic environment and the characteristics of the bed material.

The dimensions of the bed forms are highly variable. Wave generated ripples may have wave lengths and heights of a few centimetres, while the linear tidal sand banks can have a length of tens of kilometres and a height of up to about 50 m. One of the important bed forms in connection with offshore structures and pipelines is the sand wave, which at the same time can be large enough and migrate fast enough to cause bed level changes around a structure which can give problems during its lifetime.

A comprehensive review of the characteristics of sand waves and other tidal bed forms has been presented by Stride (1982). In the following a brief resume of the conclusions is given, without reference to the original contributions forming the basis for the work by Stride.

Sand waves can be formed in a bed sediment ranging from fine sand to gravel. Sand waves can be symmetrical or asymmetrical depending on the relative strength of the opposing tidal currents. The asymmetric sand waves, generated where the tidal current from one direction is dominant, are the most common and will be treated in the following.

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An example, taken from Houbolt (1968), of an echo sounding showing a train of sand waves is given in Fig. 1. These asymmetric sand waves have a height of 5-7 m at 25-30 m water depth; the wave length is approx. 200 m.

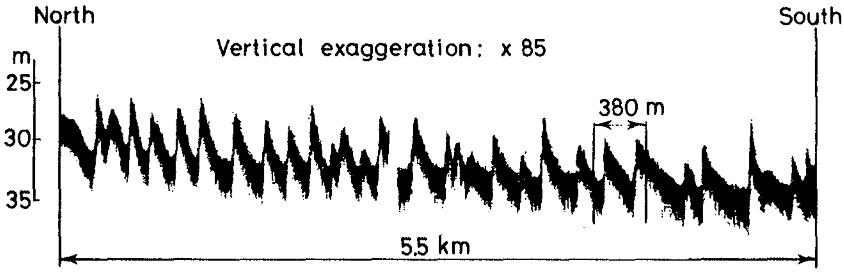

Figure 1. Example of echo sounding showing asymmetric sand waves, West of Imuiden, the Netherlands, from Houbolt (1968).

Sand waves are generally observed to migrate in the direction faced by the steep slopes, which is also the direction of the net sediment transport. The slopes of the steep fronts lie in the range of 5^o to 35^o, and generally the higher sand waves (> 2 m) have more gentle fronts, less than 20^o, and the smaller sand waves have slopes of more than 20^o. The gentle slopes lie in the range of 0.5^o-4^o; generally the more steep slopes are found on the active, migrating sand waves.

The height of the sand waves can vary considerably. It may be only a few centimetres, while the upper limit appears to be determined by the water depth. The highest sand waves are about one third of the water depth.

The sand wave length has also a large variaton, lying between 2 and 20 times the water depth, D, with an average of about 6 D. The ratio between the length and height of a sand wave is generally larger than 15 and may be as high as 100.

The formation and the dimensions of the sand waves will depend on the current velocities and the wave climate. According to Stride (1982) sand waves are not formed at very small current velocities. The lower limit for sand wave formation is given as $\bar{V}_m$ = 0.65 m/s, $\bar{V}_m$ being the mean spring tide near surface peak speed. Small sand waves can be formed at slightly lower speeds, and sand waves formed in gravel require larger flow velocities. It is supposed that a strong current can smooth out the sand waves similarly to the smoothing out of dunes in rivers, but the evidence from field observations is rather weak. The effect of waves is more well documented. The wave action during a storm can reduce the sand wave height at water depths of more than 20 m, and it can be observed that the relative height of the sand waves decreases in the shallower areas, where the near bed wave action increases, Terwindt (1971), Stride (1982). The smoothing out of sand waves by current or by waves is often assumed to be connected with the ratio between the suspended load and the bed load, which increases as the sediment transport rate increases.

The general behaviour of the sand waves, or the physical mechanisms behind the formation of sand waves seems not in any significant way to deviate from the processes associated with dunes formed under unidirectional current. The large sand waves formed by a tidal current can according to Stride (1982) be considered as unidirectional current formed ones, which have been modifed to a lesser or greater extent by the reversing current.

2. The Theory for Dune Dimensions, Current Only

A theory for the dimensions of sand waves (dunes) under steady current conditions has been presented by Fredsøe (1979) and Fredsøe (1982). A key point in this theory is the use of the continuity equation for the sediment :

$$\frac{\partial h}{\partial t} = - \frac{1}{1-n} \frac{\partial q}{\partial x} \qquad (1)$$

where h is the bed level, t is the time, q is the sediment transport rate, x is the coordinate in the transport direction, and n is the porosity of the bed sediment. By applying Eq. (1) to periodic, steadily migrating bed forms, the following relation emerges :

$$q = q_o + a\,(1-n)h \qquad (2)$$

where q_o is the sediment transport passing the trough of the sand waves and a is the migration velocity.

For dunes and sand waves a is determined by the amount af sediment, q_D, which is deposited at the front of the sand wave, Fig. 2 :

$$a = \frac{q_D}{(1-n)A} \qquad (3)$$

where A is the dune height.

The magnitude of q_D depends on the rate of sediment transport at the dune crest and the relative amount of suspended load, q_s, and bed load transport, q_b.

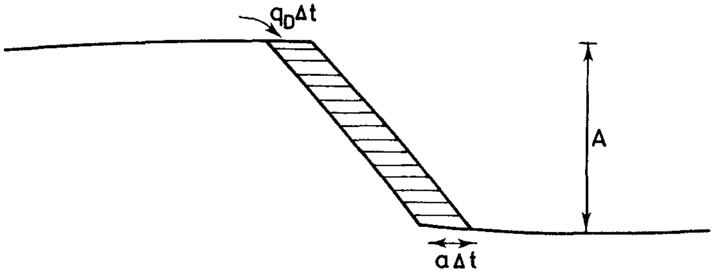

Figure 2. Migration of the dune front.

The bed load transport consists of the grains which move in almost continuous contact with the grains in the bed, and will be deposited at the front where flow separation occurs. The suspended sediment grains move above the bed without contact and will not settle immediately. A suspended grain will contribute to q_D and the dune migration if it by settling or diffusion moves into the separation 'bubble' before it is carried past the separation zone by the flow. Whether the grain is deposited at the front or not will thus depend on its distance from the bed when it passes the dune crest.

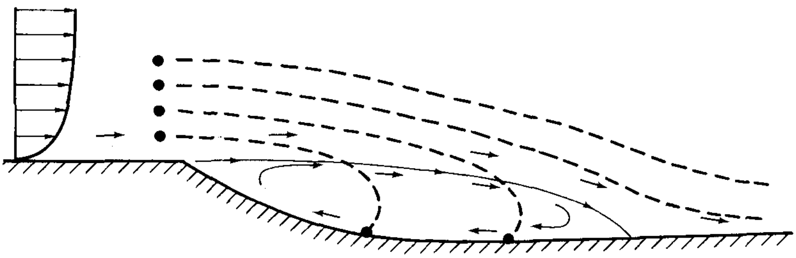

Figure 3. The probability for a suspended grain to settle in the trough depends on its height.

In the model by Fredsøe (1982) the detailed shape of the dune is determined from the requirement that Eq. (2) must fit with the sediment transport rate calculated from the shear stress distribution along the long, gentle upstream slope of the dune. The sediment transport is calculated by the model of Engelund and Fredsøe (1976). This sediment transport model calculates the suspended load and the bed load transport separately. The variation of the bed shear stress along the dune is determined on basis of flow measurements downstream of a rearward facing step, e.g. Smith (1970). In a recent paper McLean and Smith (1986) presented an analytical treatment of this flow situation.

The length of the separation zone at the steep front is about 6A. The bed shear stress increases gradually in the flow direction from zero at the reattachment point. The bed shear stress attains a local maximum at a distance of 16-20A from the step. Fig. 4 gives an example of the measured shear stress coefficient, $C_f = \tau/\rho V^2$, behind a rearward facing step, and a dune profile calculated by the theory by Fredsøe (1982).

In the following the details concerning determination of the dune shape will not be treated, the emphasis will be put on the main dimensions : the height, A, and the length, λ.

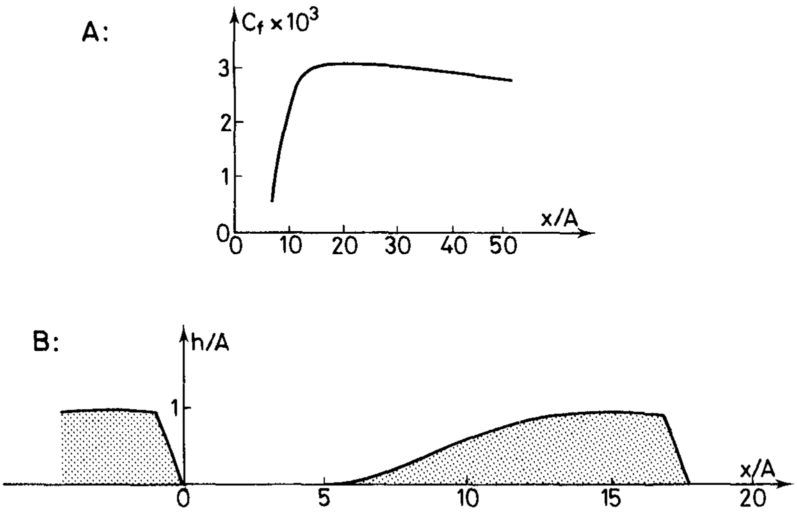

Figure 4. A: Shear stress coefficient downstream of a rearward facing step with the height. B: Example of dune profile calculated from the theory by Fredsøe (1982).

2.1 The Dune Height

Near the dune crest the influence from the preceeding dune is weak and the bed shear stress can be calculated by the logarithmic resistance law :

$$\frac{V}{U_f} = \frac{1}{\kappa}\left(\ln\left(\frac{30\ D}{k}\right)-1\right) \qquad (3)$$

where V is the local depth averaged flow velocity, U_f the shear velocity: $U_f = \sqrt{\tau_b/\rho}$, κ is V. Karman's constant, D is the local flow depth and k is the bed roughness taken to be 2.5 times d, the grain size of the bed sediment.

The sediment transport model of Engelund and Fredsøe (1976) can for a given bed sediment be expressed in dimensional form :

$$q = q(V, D) \tag{4}$$

The local change in the bed level near the crest can be expressed in two ways :

I) Purely geometrical :

$$\frac{\partial h}{\partial t} = - a \frac{\partial h}{\partial x} = \frac{-q_D}{A(1-n)} \frac{\partial h}{\partial x} \tag{5}$$

II) Through the continuity equation :

$$\frac{\partial h}{\partial t} = \frac{-1}{1-n} \frac{\partial q}{\partial x} = \frac{1}{1-n} \frac{dq}{dD} \frac{\partial h}{\partial x} \tag{6}$$

By combining Eq. (5) and (6) an expression for the dune height is obtained:

$$A = \frac{-q_D}{\frac{dq}{dD}} = \frac{-q_D}{\frac{\partial q}{\partial D} + \frac{\partial V}{\partial D}\frac{\partial q}{\partial V}} = \frac{-q_D}{\frac{\partial q}{\partial D} - \frac{V}{D}\frac{\partial q}{\partial V}} \tag{7}$$

This result for the dune height results in a migration velocity of the dunes given by :

$$a = \frac{1}{1-n} \frac{dq}{dD} \tag{8}$$

2.2 The Dune Length

According to Eq (2) the sediment transport attains a maximum at the top of the dune; immediately after the top the flow separates and the dune ends. At situations with dominant bed load transport the sediment transport is in local equilibrium with the hydraulic conditions, and the point of maximum transport coincides with the maximum in bed shear stress, τ_b. According to the measurements described previously the bed shear stress has a local maximum at a distance of about 16A downstream of the preceeding crest. At dominant bed load transport the dune length, λ, is thus given by :

$$\lambda = 16\ A \tag{9}$$

Fredsøe included the effect of gravity in his analysis. The gravity acting on the moving grains will tend to retard the bed load transport moving up the dune, modifying the transport relation which gives a predicted dune length larger than Eq. (9). The effect of gravity is notable at very low transport intensities.

The suspended load transport cannot be adjusted immediately to changes in the hydraulic conditions, as it takes time for the suspended sand grains to settle out from the concentration profile or be entrained into it. The maximum in the suspended load will therefore have a lag distance, L_s, behind the maximum in the bed shear stress. The dune length in case of both suspended and bed load transport can thus be expressed as :

$$\lambda = 16\ A + \frac{q_s}{q} L_s \tag{10}$$

3. Sand Wave Dimensions in Wave and Current

3.1 The Sand Wave Height

The physics behind the offshore sand waves are as described in the introduction very similar to the dunes in river, and the relations for dune dimensions can be applied directy if a model for sediment transport under the combined action of waves and current is introduced.

The present calculations are based on the sediment transport model by Fredsøe et al. (1985). This model converges towards the model by Engelund and Fredsøe (1976) for decreasing wave action.

In case of current without waves the first term in the denominator of Eq. (7) can be neglected because the flow resistance, Eq. (3), is insensitive to variation in D. In case of combined waves and current this term can be significant due to the relation between the water depth and the near bed orbital motion induced by waves of given height and period.

Sand wave heights predicted by Eq. (7) using the transport model by Fredsøe et al. (1985) are shown in Fig. 5. The calcualtions have been made for a water depth of 10 m and a bed material grain size of d = 0.20 mm. The direction of the wave motion is normal to the mean current, $\gamma = 90^o$. Two wave heights are considered : H = 2 m and 4 m with periods of 7.2 s and 10.1 s respectively.

Different assumptions for the settling of suspended sediment at the sand wave front is presented in Fig. 5 A, B and C.

In Fig. 5A it is assumed that all transported sediment is deposited at the sand wave front : $q_D = q_b + q_s$.

In Fig. 5B the sand wave height is calculated, assuming that only the bed load is deposited. For current only, the sand wave height, A, is the same for the lower current velocities because the bed load transport is dominant. For higher current velocities the suspended load transport becomes dominant and the sand wave height decreases if only the bed load is deposited. Under wave action sediment will be suspended in the wave boundary layer even for small current velocities, and the calculated sand wave heights are different in Fig. 5A and B for all current velocities.

In Fig. 5C the amount of suspended sediment which is deposited at the sand wave front is estimated on basis of the lag distance, L_s, and the length of the separation zone downstream of the front. q_D is calculated as :

$$q_D = q_b + q_s \exp(-L_s/6A) \tag{11}$$

The length scale for the lag of the suspended sediment, compared with the hydraulic conditions can be estimated from the concentration profile as :

$$L_s = \frac{U_c \cdot y_c}{w} \tag{12}$$

where y_c is the height of the centroid of the concentration profile, c(y) :

$$y_c = \frac{\int_a^D cy \, dy}{\int_a^D c \, dy} \tag{13}$$

U_c is the mean current velocity at the height y_c from the bed. w is the settling velocity of the suspended sediment.

For small current velocities L_s is small compared with A, and the calculated sand wave heights correspond to Fig. 5A. For increasing current velocities, L_s increases and the sand wave height converges towards the results in Fig. 5B.

3.2 The Flow Separation at the Sand Wave Front

The key assumption for the theory predicting the sand wave height is that the deposition of sediment at the front and thus the migration rate of the sand wave can be calculated directly on basis of the sediment transport at the crest of the sand wave. The basis for this assumption is that flow separation occurs at the dune front, leading to a vanishing sediment transport capacity at the toe of the dune front.

In case of combined wave and current motion the strong fluctuating pressure gradients associated with the wave motion can under certain circumstances be expected to suppress separation of the mean current.

In the present model a crude approximate criterion is used for the suppression of flow separation by the wave orbital motion. In steady

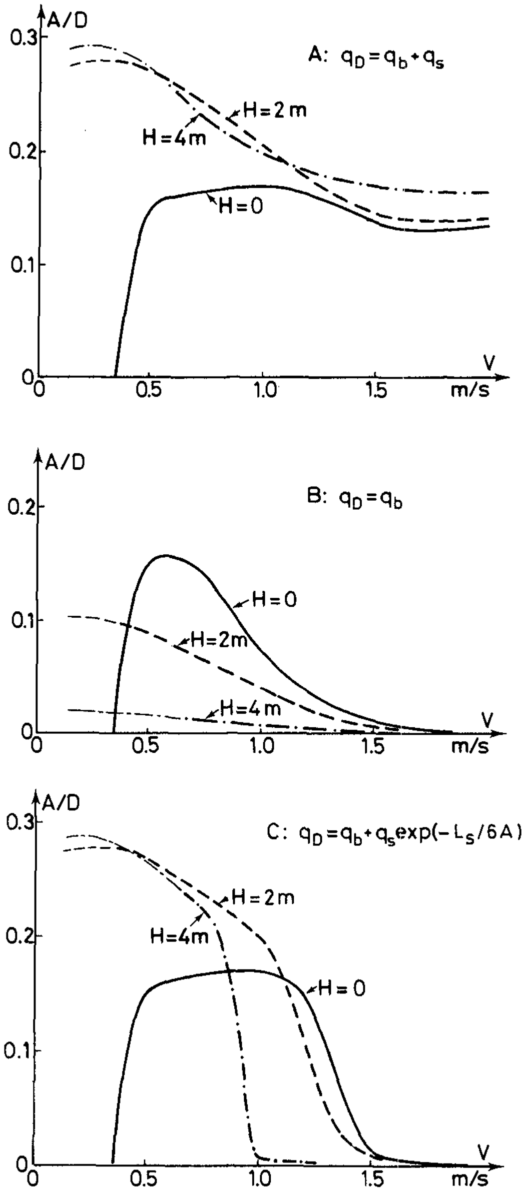

Figure 5. Sand wave height as function of V and H; d = 0.2 mm, D = 10 m, $\gamma = 90^\circ$.

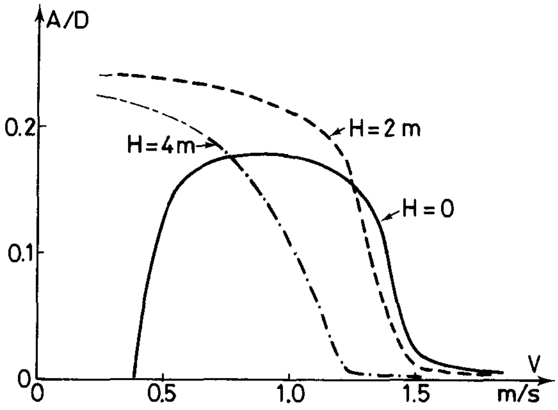

Figure 6. A/D as function of V and H, d = 0.2 mm, D = 20 m, $\gamma = 90^{\circ}$.

diffusor flow the largest possible diverging half angle without flow separation is about 5°. The sand wave front can be much steeper, close to the angle of repose of the bed material, about 30°.

The effect of the orbital motion is that the near bed water particles move along a much longer path than the straight line of the mean current. The 'effective' angle of divergence will therefore be reduced in combined wave-current motion. If the path travelled by the particle is about 7 times the straight course the average angle of divergence experienced by the water particles on the 30° slope will correspond to the 5° which causes no separation. The criterion for suppression of the flow separation has therefore been taken to be that the oscillating part of the shear velocity, U_{fw}, associated with the wave motion is larger than 7 times the shear velocity, U_{fo}, characterising the mean current in the wave boundary layer.

In the figures showing the results from the model, the part of the curves, where no flow separation is expected to take place due to the wave motion, is shown as thinly drawn lines.

3.3 Sensitivity to Changes in the Parameters

The number of parameters entering the description of the sand waves is too large to allow for the preparation of general dimensionless diagrams for prediction of sand wave heights.

The influence of varying a number of parameters is illustrated in Figs. 6, 7 and 8. In these figures the deposition of suspended sediment has been estimated by Eq. (11).

It can be seen from Fig. 6 that for similar near bed orbital motions the sand waves at 20 m have relative heights close to the sand wave height at 10 m. The transition due to the decrease in settling suspended sediment occurs at a higher current velocity at 20 m water depth because the sand waves are higher, and the ratio L_s/A is smaller for a given transport intensity.

The effect on A/D of changing the angle, γ, between the waves and the current is rather small, Fig. 7.

The most significant changes can be observed when the grain size, d, of the bed material is reduced to 0.15 mm. The range of current velocities, where sand waves are formed become more narrow than for the 0.2 mm sand. For the highest waves considered no sand waves are formed at all, Fig. 8.

For a given wave climate the range of current velocities under which sand waves are formed will gradually become narrower as the water depth decreases, because the near bed orbital motion and the suspended sediment load increase. This is in agreement with the observation that sand waves are decreasing or absent in the shallow parts of the southern North Sea, e.g. McCane and Langhorne (1982), Terwindt (1971) and Houbolt (1968).

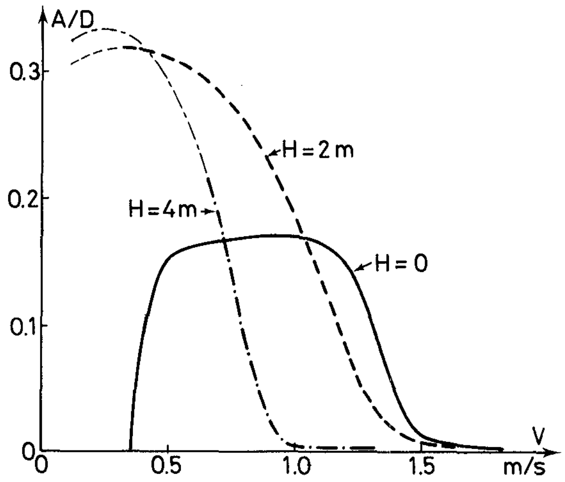

Figure 7. A/D as function of V and H, d = 0.2 mm, D = 10 m, $\gamma = 0^o$.

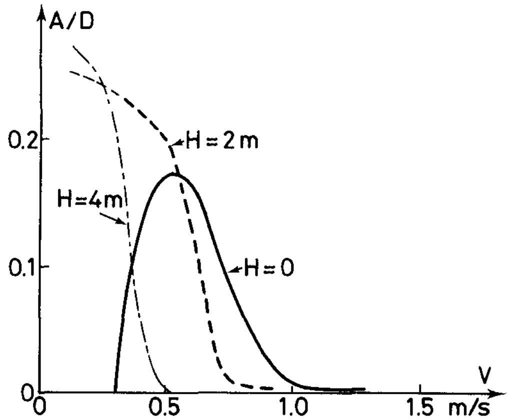

Figure 8. A/D as function of V and H.
$d = 0.15$ mm, $D = 10$ m and $\gamma = 90^{\circ}$.

3.4 The Sand Wave Length

As a first approximation the sand wave length is calculated by Eq. (10). The calculated steepness, A/λ, of the sand waves treated in Fig. 5C are shown in Fig. 9. As the suspended load increases the height decreases and the length increases. The sand wave steepness therefore drops rapidly for increasing current velocity. The effect of gravity on the bed load transport in case of pure current is indicated by the dotted line. In combined waves and current the suspended load is dominant even at small current velocities, and the gravity effect is not expected to be of importance.

Under field conditions the tidal current and the wave conditions will generally vary so fast that the sand waves at a given location reflect a temporal average of the different combinations of waves and current. When estimating this average each combination shall be weighted according to the strength it has in the process of forming the sand waves. A measure which can be used for such a weighting procedure is the quantity : dq/dD, which is proportional to the migration rate, a, of the sand waves. Fig. 10 shows the calculated migration rate for the sand waves treated in Fig. 5.

It can be seen how the wave condition, especially at moderate current velocities have strong influence on the formation of sand waves. It can also be noted that the relation between a and V is non-linear. The asymmetry in the tidal current can therefore be rather small and still produce asymmetric sand waves dominated by the direction of the strongest tidal current.

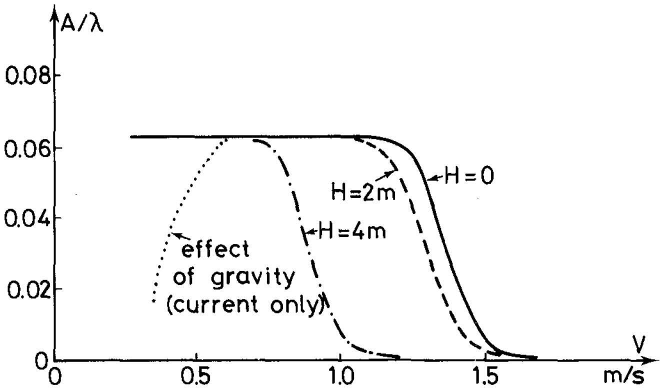

Figure 9. The sand wave steepness as function of V and H, d = 0.2 mm, D = 10 m, $\gamma = 90^{o}$.

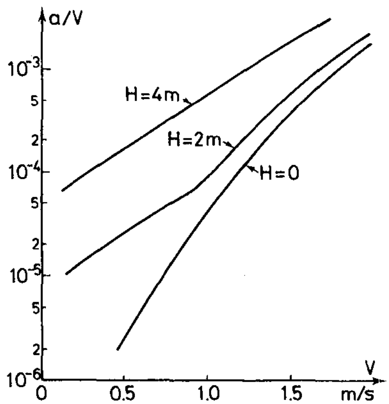

Figure 10. The sand wave migration rate as function of V and H. d = 0.2 mm, D = 10 m, $\gamma = 90^{o}$,

4. Conclusion

A theoretical model for the dimensions of offshore sand waves has been presented. The model is based on a model for dunes in rivers and a sediment transport model for combined waves and current.

At present no data has been published, which are comprehensive enough to allow for a detailed calibration of the different elements in the model. The predicted trends for the sand wave heights and lengths are, however, in general agreement with the observations reported in the literature.

According to the model there will for a given wave condition be a range of current velocities, under which sand waves will be formed. The upper limit for the sand wave formation is caused by the increasing amount of suspended sediment which passes the sand wave without being deposited, and thus does not contribute to the building-up of the sand wave.

For small water waves the lower velocity limit is associated with the initiation of sediment transport.

For large water waves the lower limit is caused by the suppression of flow separation at the front of the sand wave.

For a given wave climate the range of velocities where sand waves are formed become narrower for decreasing water depth or grain size of the bed material and at a certain water depth no sand waves may be formed at any current velocity at all.

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